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NON-ITERATIVE METHODS FOR DIGITAL IMAGE RESTORATION

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NEITERATIVNI METODI ZA DIGITALNU RESTAURACIJU SLIKA

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Chapter 1

Introduction

1.1 The notion of Image Restoration

Recording images is a very frequent event in everyday human life. Due to imperfections in the imaging and capturing process, the recorded image inevitably represents a degraded version of the original scene. The question of removing these imperfections is crucial to many of image analysis and image processing tasks. There exists a wide range of different degradations that need to be taken into account, covering for instance noise, geometrical degradations, illumination and color imperfections, and blur. Image restoration methods are aimed for the reconstruction of the original image from a degraded model.

The field of image restoration has seen a tremendous growth in interest over the last two decades. There are many excellent overview articles, journal papers, and textbooks on the subject of image restoration and identification [2, 5, 6, 26, 32, 43, 44]. A number of various algorithms have been proposed and intensively studied for achieving a fast-recovered and high-resolution reconstructed images see, e.g. [49, 50]. The recovery of an original image from degraded observations is of crucial importance and finds application in several scientific areas including medical imaging and diagnosis, military surveillance, satellite and astronomical imaging, reconstruction of poor-quality family portraits, and remote sensing.

Blurring is a form of bandwidth reduction of an ideal image owing to the imperfect image formation process. It can be caused by a relative motion between the camera and the original scene, or by an optical system that is out of focus. When aerial photographs are produced for remote sensing purposes, blurs are conditioned by atmospheric turbulence, aberrations in the optical system, and relative motion between the camera and the background. Such blurring is not confined to optical images; for example, electron micrographs are corrupted by spherical aberrations of the electron lenses, and computed tomography scans suffer from X-ray scatter.

The field of *image restoration* (also called as *image deblurring* or *image deconvolution*) is concerned with the reconstruction or estimation of the uncorrupted image from a blurred one [6]. Fundamentally, it tries to perform an operation on the image that is the inverse of the imperfections in the image formation system. In the use of image restoration methods, the attributes of the degrading system are assumed to be known a priori.

In practical situations, sometimes may not be able to obtain this information directly from the image formation process. The objective of blur identification is to estimate the attributes of the real imaging system from the observed degraded image itself prior to the restoration process. The combination of image restoration and blur identification is often referred to as *blind image deconvolution* [49].

Blind deconvolution algorithm based on the total variational (TV) minimization method is extremely effective for recovering edges of images as well as some blurring functions, e.g., motion blur and out-of-focus blur [11]. In the paper [99] the authors present anisotropic regularization techniques to exploit the piecewise smoothness of the image and the point spread function (PSF) in order to mitigate the severe lack of information encountered in blind restoration of shift-invariantly and shift-variantly blurred images. These techniques are demonstrated on linear motion blur and out-of-focus blur. Edge preserving regularization methods, in the context of image restoration and denoising, are presented in [73].

Images are aimed to memorize useful information, but unfortunately the presence of the blur is unavoidable. Motion blur is the effect caused by relative motion between the camera and the scene during image exposure time. Restoration of motion-blurred images has been a fundamental problem in digital imaging for a long time. We assume that the blurring function acts as a convolution kernel or point-spread function $h(n_1, n_2)$ and the image restoration methods that are described here fall under the class of linear spatially invariant restoration filters. It is also assumed that the statistical properties defined by the mean and correlation functions of the image do not change spatially.

Under these conditions the restoration process can be carried out by means of a linear filter of which the point-spread function is spatially invariant, i.e., it is constant throughout the image. These modeling assumptions can be mathematically formulated as follows. If we denote by $f(n_1, n_2)$ the desired ideal spatially discrete image that does not contain any blur or noise, then the recorded image $g(n_1, n_2)$ is modeled by the convolution which is determined using the two-dimensional point spread function $h(n_1, n_2)$ [6]:

$$\begin{aligned} g(n_1, n_2) &= h(n_1, n_2) * f(n_1, n_2) \\ &= \sum_{k_1=0}^{N-1} \sum_{k_2=0}^{M-1} h(k_1, k_2) f(n_1 - k_1, n_2 - k_2). \end{aligned} \tag{1.1.1}$$

The symbol $*$ denotes the convolution operation.
The objective of the image restoration is to make an estimate $f(n_1, n_2)$ of the ideal image, under the assumption that only the degraded image $g(n_1, n_2)$ and the blurring function $h(n_1, n_2)$ are given.

The goal of the Ph.D. dissertation is to develop an efficient and reliable methods for digital image restoration or image deblurring using mathematical models to analyze the process of blurring. In this way, we will focus on methods to remove blur caused by uniform and nonuniform motion. They are particularly important in applications related to the removal of blur from X-ray images, in automated number plate recognition systems, with bar code images, LCD televisors and monitors and other areas. This topic is intensively research in recent years, as evidenced by a large number of books, monographs, papers and computer implementation from this field.

One of our main motivations for developing the methods for digital image restoration is applicability in everyday life. Blurring images can be appropriate for generating background effects and image shadows. In our days, creating motion blur in images is something that many image artists, mainly photographers use in order to capture feigned movement. Moreover, two dimensional filtering based on the separable motion blur is also useful for smoothing the effects of the staircase like effect, known as 'aliasing'. The separable anti-alias filtering procedure is efficient on smoothing edges of images and can also round out features to produce highlighting effects.

Also, the process of the de-identification is an interesting application of the motion blur. Advances in imaging devices and web technologies have made it easy to capture and share large amounts of video data over the internet. The effluence of privacy information becomes an important issue in both academia and industry. Examples include the Google Street View, EveryScape, public and private surveillance video, the collection and distribution of medical face databases [30]. De-identification is intended for the elimination of identification information from images and videos, prior to sharing of the data, while keeping as much information on the action and its context. Recognition and de-identification are opposites, the recognition making use of all possible features to identify an object while the de-identification trying to obfuscate the features to thwart recognition. De-identification should be resistant to recognition by humans and algorithms [1]. Three types of videos need de-identification to not compromise the privacy of individuals [1]:

- *Casual videos* that are captured for other purposes and get shared. For example the images used by Google StreetView, EveryScape, the cameras setup in public spaces that can be viewed over the internet, videos or photos on sharing sites, etc. There is no need to know the identity of individuals who appear in these videos. All individuals should be de-identified irrevocably and early, perhaps at the camera itself.

- *Public surveillance videos* come from cameras watching spaces such as airports, streets, stores, etc. These type of videos usually are displayed on public monitors and a recorded version may be accessible to many people. The types of actions performed by individuals in these videos is important, but not their identities. Consequently de-identification is necessary.

- *Private surveillance videos* come from cameras placed at the entrances of semi-private spaces like offices. These type of videos usually are with higher quality and are likely to have a more detailed view of the individuals. De-identification may not be essential, but could be suggested to take care of possible viewing by non-authorized people.

Also automatic license plate de-identification is an important application [21]. According the above it is important to develop the automated methods for de-identifying individuals or items without affecting the context of the action in the image or in the video. Motion blur can be used for de-identification in the images or in the videos. Because we know the PSF and if necessary we can use the image deblurring methods in the process of identification of the people or actions in the recorded video.

Another practical example where we can use the image deblurring methods are barcode character recognition. Barcodes can be found on numerous items, such as packaged food, books, newspapers and more. There are different ways of reading these bar codes. One way is to use dedicated barcode readers. The second option is to acquire an image of the barcode using the camera that is anyway built into the device and process the image in order to decode the barcode [98]. Reading barcodes by image processing is slower and less reliable than using dedicated barcode scanners, but in some cases they are better. For example, dedicated systems based on reflected laser light do not work for reading barcodes on a monitor screen.

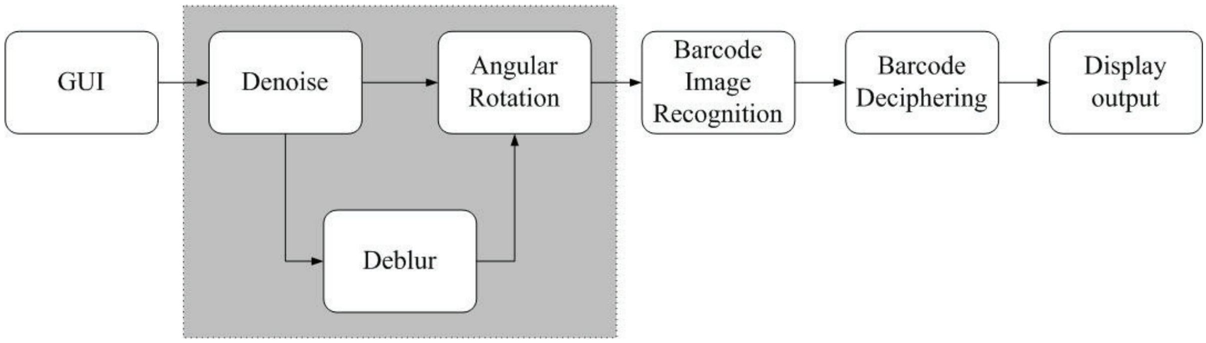

Figure 1.1.1: Barcode character recognition.

Barcode images acquired with cameras sometimes will not have the required quality to be recognized and decoded [98]. Reason for that could be motion blur. Our methods can be use for deblurring of the degraded images, and after that the deblurred image can be use in the image recognition process. The process of the recognition of the barcodes based on the image processing techniques is presented in the Figure 1.1.1. Some of the images may be too blurred and for successful deciphering must be implemented effective deblurred method.

The increasing consumption of liquid crystal displays (LCD) for computer monitors and home use has led to great interest in improvement of image quality especially when we have movements. If we compared with other types of displays as traditional cathode ray tube (CRT), plasma and projection displays, LCDs offer a lower cost, lower power consumption and higher resolution. Despite the great interest in solving the problem, LCDs still suffer from motion-blur the image. LCD motion blur is caused by two factors: the slow liquid crystal response time and the inherent sample-and-hold characteristic of LCD image formation.

With continuous improving of the physical properties of the liquid crystals and with using of method of overdrive is significantly mitigate response time [58]. By this way the problem of motion blur is reduced, but it's not eliminated. In [68, 69] are presented that when response time is $16ms$, 70% of the visible motion blur is part of the sample-and-hold property of the LCD display. Sample-and-hold motion blur will be present even with a zero response time. This blur is inherent to LCD image formation causes each pixel to emit approximately constant light through the frame period.

On Figure 1.1.2 is present the case when the object is moving horizontally with a constant velocity and a response time is zero [33]. Since the output at each pixel is held constant light through the frame period the displayed image does not match to the target trajectory predicted by Human Visual System (HVS). The dissimilarity between the eye tracking trajectory and the displayed data corresponds to the motion blur perceived by the human observer.

The methods for reduction of sample-and-hold LCD motion blur can be divided into several groups: back light flashing [24], frame rate doubling (black frame insertion [36], full frame insertion [51, 61]) and data pre-processing (motion-compensated inverse filters (MCIF) [47, 34]). The first method is used from Philips, it's consist of back light flashing at a faster rate than the frame period. With increasing the frame rate reduces the hold time and thus motion blur. MCIF engages estimating the motion and then apply a high pass filter.

Also we assume that the PSF(point spread function) is horizontal (if not, we can approximate the PSF with separable PSF and take its x component). The reason for this is because human eyes give much more consideration to horizontal motions from vertical motions [8].

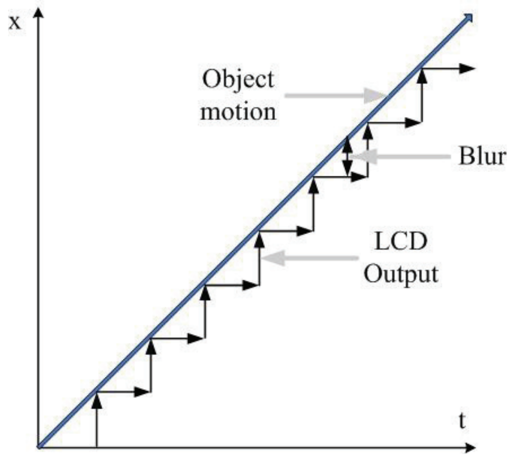

Figure 1.1.2: Position versus frame time.

In order to develop efficient algorithms and methods for reducing LCD motion blur is crucial to developing an accurate model of the occurrence of LCD blur. The model of LCD blurring is introduced in [9] and [68].

The process of degradation of the image is illustrated in Figure 1.1.3, where dynamic discrete content $I_d(x, y, t)$ is shown on the LCD display as $I_s(x, y, t)$. Firstly the image is degraded from the LCD display device, more accurate by sample-hold feature of the LCD device. After that the human visual system (HVS) will be degraded the displayed image and the perceived image is $I_o(x, y, t)$. The eye tracking and low pass filter (LPF) formed HVS.

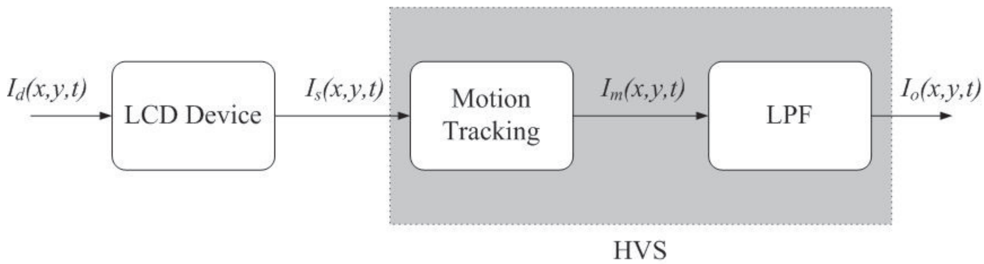

Figure 1.1.3: Process of the perception chain.

Our methods for image restoration can be used as pre-processing technique for reducing of the LCD motion blur. In this approach the signal is pre-processing before it is sent to the display. This means that the frame sampled at time t is $I_c(x, y, t)$ have to process with the method for image deblurring and we get the signal $I_d(x, y, t)$, that is input signal on Figure 1.1.3.

1.2 Organization of the Ph.D. dissertation

Generally, the Ph.D. dissertation is divided on three main parts: Chapter 2, Chapter 3 and Chapter 4. The first one is devoted to the modeling of the process of image formation and presentation of the standard methods of image restoration. The second one deals with the definition and description of the new non-iterative methods for image restoration. And finally, the third one presents experimental results and comparative analysis when we use the new non-iterative methods and standard methods for image restoration.

In the next chapter two commonly used filters for reconstructions of blurred image, namely Wiener filter and the constrained least-squares filter [6] are presented. After them, iterative nonlinear method for image restoration based on the Lucy-Richardson algorithm [26, 27] is shown. Also for image reconstruction we can use as well as the symmetric minimal rank (SMRS) solution of the inverse matrix problem [96]. The restoration methods based on moments (the Fourier and the Haar basis) [42] which are also used are described in this chapter. End of this chapter is devoted to the Truncated Singular Value Decomposition (TSVD) and Tikhonov (TIK) restoration methods [32].

The Chapter 3 consists of the five new non-iterative methods:

- The first method is a direct method for removing uniform linear motion blur from images. The method is based on a straightforward construction of the Moore-Penrose inverse of the blurring matrix for a given mathematical model. The computational load of the method is decreased significantly with respect to other competitive methods, while the resolution of the restored images remains at a very high level.

- The second method is based on an application of the partitioning method for determination of the Moore-Penrose inverse of a matrix augmented by a block-column matrix of arbitrary size. The adaptation of the partitioning method is applicable in the image restoration. The main contribution of the introduced method is a significant reduction in computational time required to calculate the Moore–Penrose inverse of a blurring matrix compared to other known methods for the pseudoinverse computation. The resolution of the restored image remains at a very high level.

- The next method generalizes image restoration algorithms which are based on the Moore–Penrose solution of certain matrix equations that define the linear motion blur. Our approach is based on the usage of least squares solutions of these matrix equations, wherein an arbitrary matrix of appropriate dimensions is included besides the Moore-Penrose inverse. In addition, the method is a useful tool for improving results obtained by other image restoration methods. Towards that direction, we investigate the case where the arbitrary matrix is replaced by the matrix obtained by the Haar basis reconstructed image. The method has been tested by reconstructing an image after the removal of blur caused by the uniform linear motion and filtering the noise that is corrupted with the image pixels. Quality of the restoration is observable by a human eye.

- The following method for reconstruction of blurred images damaged by a separable motion blur can be used after the application of currently developed image restoration algorithms. Our approach is based on the usage of least squares solutions of certain matrix equations which define the separable motion blur. The method uses appropriately selected matrices besides the Moore-Penrose inverse. The method is tested by reconstructing a set of images after the removal of blur caused by uniform and separable motion.

- Previously performed analyzes have confirmed that the method proposed in [83], can be used as a useful tool for improving restorations obtained by other image restoration methods. Continuing investigations in that direction, we investigate the case where arbitrary matrix is replaced by the matrix obtained by the Tikhonov regularization method or by the Truncated Singular Value decomposition method.

The Chapter 4 is devoted to experimental results and application of methods in some real cases. For comparison of the image restoration methods we use the following criteria: Improvement in Signal to Noise Ratio (ISNR) and Peak Signal to Noise Ratio (PSNR) [6, 26]. The methods are tested on standard images from MATLAB such as Lena, Balrbara, Cameraman etc., and real cases images: X-ray images and images from automated number plate recognition systems. The new non-iterative methods for digital image restoration improve the performance in terms of the quality of the restored images than the standard built-in image restoration methods in the software package MATLAB. Also, the new methods reduces the time required to complete the restoration process compared to conventional methods.

In the final chapter the conclusions with regard to the results obtained in the Ph.D. dissertation are presented. The following are comments on the results, their scientific and practical significance. At the end of this chapter a brief overview of the ideas for future work and possible further research in this and related areas is presented.

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Biografija

Igor Stojanović rođen je 01.02.1974 godine u Kumanovu, gde je sa odličnim uspehom završio osnovnu školu i Gimnaziju "Goce Delčev". Studije na smeru za Elektroniku i telekomunikacije na Elektrotehničkom fakultetu u Skoplju upisao je školske 1992/93 godine, a završio ih je 1997 sa odbranom diplomskog rada iz oblasti digitalne obrade signala.

Postdiplomske studije na Elektrotehničkom fakultetu u Skoplju , smer elektronika, upisao je u oktobru 1997 godina. Nakon polaganja predviđenih ispita sa prosecnom ocenom 10, magistrirao je u decembru 2002 sa odbranom magistarskog rada pod nazivom "Pretraživanje JPEG slika i lociranje objekata multirezolucijskom analizom" pod mentorstvom prof. dr Momcila Bogdanova.

Govori tečno engleski, a služi se i francuskim jezikom.

Posle diplomiranja radio je u Sektoru za informacione i komunikacione tehnologije u Upravi carina Republike Makedonije od septembra 1999 do decembra 2007 godina. Radio je na različitim pozicijama: sistem inženjer za operativne sisteme, baze podataka i pomoćnik direktora IKT sektora. Imao je obuke za baze podataka (Oracle i Informix u Sloveniji i Belgiji), Unix administracija (ECS Makedonija), administriranje mreže (Cisco, Belgija, Makedonija i Slovenija). Od Januara 2008 je zaposlen kao asistent na Fakultetu za informatiku pri Univerzitetu Goce Delčev - Štip.

Aktivno je učestvovao u realizaciji projekta "Procena rizika za carinu na zapadnom Balkanu". Projekat je finansiran sa strane Evropske komisije, "Information Society Technologies" - Sixth Framework Programme (2002-2006)).

Akademske doktorske studije iz oblasti Informatike (računarskih nauka) na Prirodno-matematičkom fakultetu u Nišu upisao je školske 2010/11. godine. Oblasti njegovog naučnog interesovanja su restauracija oštećenih slika, obrada slika, digitalna obrada signala, numerička linearna algebra.

Na Fakultetu za informatiku u Štipu izvodio je nastavu iz sledećih predmeta:

1. Objektno orijentisano programiranje
2. Digitalna logika
3. Operativni sistemi
4. Arhitektura kompjutera
5. Mrežni operativni sistemi

U međuvremenu je napisao i objavio sledeće naučne radove.

(a) Radovi u časopisima koji imaju Impact Factor:

1. P. Stanimirović, M. Miladinović, **I. Stojanović**, S. Miljković, *Application of partitioning method in removal of blur in images*, International Journal of Applied Mathematics and Computer Science, (IF 2012=1.008) (Accepted).

2. P. Stanimirović, **I. Stojanović**, S. Chountasis, D. Pappas, *Image Deblurring Process Based on Separable Restoration Methods*, Computational and Applied Mathematics, ISSN: 0101-8205 (Print) 1807-0302 (Online), DOI: 10.1007/s40314-013-0062-2 (IF 2012=0.452).

3. P. Stanimirović, S. Chountasis, D. Pappas, **I. Stojanović**, *Removal of blur in images based on least squares solutions*, Mathematical Methods in the Applied Sciences, Print ISSN: 0170-4214, Online ISSN: 1099-1476, DOI:10.1002/mma.2751 (IF 2012=0.778).

4. S. Miljković, M. Miladinović, P. Stanimirović, **I. Stojanović**, *Application of the pseudoinverse computation in reconstruction of blurred images*, Filomat **26(3)** (2012), 453-465, DOI: 10.2298/FIL1203453M (IF2012=0.714).

5. **I. Stojanović**, S. Markovski, C. Martinovska, A. Mileva, *Application of the progressive wavelet correlation for image recognition and retrieval from the collection of images*, Technics Technologies Education Management, ISSN: 1840-1503, **7(4)** (2012), 1550-1560 (IF 2012=0.414).

(b) Radovi u časopisima koji nemaju Impact Factor:

6. **I. Stojanović**, A. Mileva, D. Stojanović, I. Kraljevski, *Image Recognition by Using the Progressive Wavelet Correlation*, International Journal of Image, Graphics and Signal Processing (IJIGSP), ISSN: 2074-9074(Print), ISSN: 2074-9082 (Online), **4(9)** (2012), 1-7, DOI:10.5815/ijigsp.2012.09.01.

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9. **I. Stojanović**, P. Stanimirović, M. Miladinović, *Applying the Algorithm of Lagrange Multipliers in Digital Image Restoration*, FACTA UNIVERSITATIS, Series Mathematics and Informatics, ISSN 0352-9665, **27(1)** (2012), 41-54.