

## GA OPTIMISATION AND FEM ANALYSIS OF PWM INVERTER INDUCTOR FOR LCL FILTER

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**Abstract** – In this paper a mathematical model for design and optimisation of main inductor in a LCL filter using genetic algorithm is presented. The objective function for the optimisation is selected to be the efficiency of the inductor. Two different optimization search have been performed using the same objective function, but with different number of optimisation parameters. From these optimisations two different inductor solutions were realized with lower inductor losses and decreased temperature rise. Finite Element Method is used for further analysis of the two optimal designed solutions.

### Introduction

PWM converters have a wide application in adjustable-speed drives as well as in renewable energy resources. They have many advantages such as bidirectional power flow, controllable power factor and sinusoidal input current. The switching frequency of the power switches of the converter is in the range of 2-15 kHz. This causes a huge amount of high order harmonics around the switching frequency. To reduce the current harmonics around the switching frequency a high value of input inductance should be used. Above the applications of several kilowatts is quite expensive to realize high values of filter reactors. Therefore as an alternative a LCL filter is used. Still the presence of high order harmonics around the switching frequency is considerable which leads to increased filter losses and filter temperature rise. Therefore, a filter mathematical model is built and implemented in the genetic algorithm method used to minimise the filter losses [1]. The optimisation is performed for different switching frequencies in range from 2 kHz to 8 kHz. The electrical circuit of PWM converter with LCL filter is presented in Figure 1.

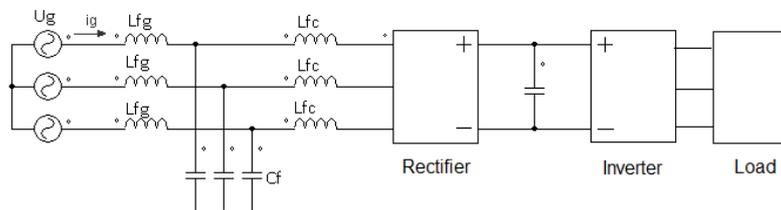


Fig. 1 PWM inverter with LCL filter

The rated values of the parameters of the filter are:  $U_g = 400$  V,  $f = 50$  Hz,  $I_g = 60$  A,  $L_{fg} = 0,6$  mH,  $R_{fg} = 8$  m $\Omega$ ,  $L_{fc} = 1.8$  mH,  $R_{fc} = 16$  m $\Omega$ ,  $C_f = 60$   $\mu$ F. The value of the DC link capacitance is  $C_{DC} = 2200$   $\mu$ F and of the DC link voltage is  $U_{DC} = 670$  V. In this paper the object of optimisation is the inductor that is placed on the converter side, also called main inductor  $L_{fc}$ . Considered PWM inverter with the LCL filter is presented in Figure 2.

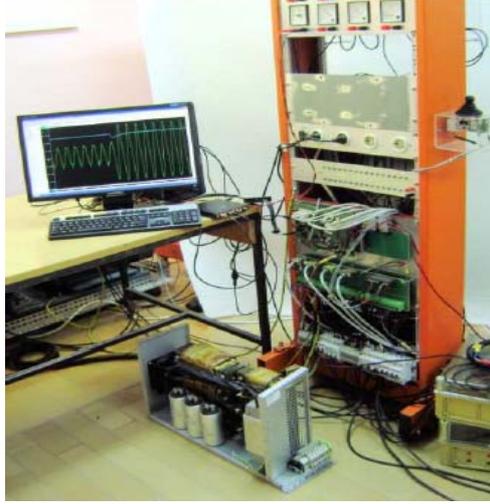


Fig. 2 PWM inverter with LCL filter

### **Inductor mathematical model**

The object of the investigation and optimization is the converter side inductor  $L_{fc}$ . In the inductor model the number of turns in inductor windings, as well as the geometrical parameters are the input variables. Different design equations are used for calculation of inductor magnetic flux density  $B_m$ , length of air gap  $l_g$  and winding resistance  $R_{coil}$  for each phase of the inductor. Copper losses from fundamental current harmonic in each phase are found from:

$$P_{cu1} = \frac{1}{2} R_{coil} i_{m1}^2 \quad (1)$$

where  $i_{m1}$  is the maximal value of fundamental current in each phase. The copper losses, due to high order harmonics, are calculated using equations from reference [2]:

$$P_{cu\_fs} = k_p R_{coil} i_{m\_fs}^2 \quad (2)$$

where  $k_p$  is the proximity factor of the winding at switching frequency and  $i_{m\_fs}$  is the current amplitude at switching frequency (ripple current) and it is found from measurements. The proximity factor  $k_p$  can be found from:

$$k_p = \frac{1}{3} \frac{h_{wire}}{\delta_{cu}} (2N_{sl}^2 + 1) \quad (3)$$

where  $h_{wire}$  is the height of copper wire conductor,  $N_{sl}$  is the number of conductors in one layer of the winding and  $\delta_{cu}$  is the skin depth of the winding at switching frequency. The core losses of the inductor are found from:

$$P_{feh} = k_c V_{core} (f_1^\alpha B_m^\beta + f_s^\alpha B_{m\_fs}^\beta) \quad (4)$$

where  $k_c$ ,  $\alpha$ ,  $\beta$  are the constants for the lamination,  $V_{core}$  is the overall volume of the material,  $f_1$  is operating frequency and  $f_s$  is the switching frequency of the converter. Flux density of the ripple current component is found from:

$$B_{m\_fs} = B_m \frac{i_{m\_fs}}{i_{m1} + i_{m\_fs}} \quad (5)$$

Air gap losses generated by fringing flux around the gap are found from:

$$P_{gap} = k_g E_x (f_1 B_m^2 + f_s B_m^2 - f_s) \quad (6)$$

where value of the constant  $k_g$  is 0.155 and  $E_x$  is the width of inductor leg [3]. Inductor temperature rise is calculated from:

$$T = 450(\psi)^{0.826} \quad (7)$$

where  $\psi$  is calculated using the following equation:

$$\psi = \frac{P_\gamma}{A_T} \quad (8)$$

$A_T$  is the inductor surface area. Inductor efficiency factor is found from:

$$\eta = \frac{P_1 - \sum P_\gamma}{P_1} \quad (8)$$

where  $P_\gamma$  is the sum of all losses in inductor due to the fundamental current component, as well as due to higher order harmonics. Input power  $P_1$  is found from:

$$P_1 = U_L I_g \quad (9)$$

where  $U_L$  is the voltage drop on the inductor.

During numerical calculation of main inductor, as well as during optimization certain constraints regarding proper operation of LCL filter must be observed. One of them is resonance frequency which must be avoided at low or high order switching frequencies.

Resonance frequency should be within following limits:

$$10f_n \leq f_{res} \leq 0.5f_{sw} \quad (10)$$

and it is calculated from [4]:

$$\omega_{res}^2 = L_T Z_{LC} / L_{fc} \quad (11)$$

where  $L_T = L_{fc} + L_{fg}$  and  $Z_{LC}$  is found from

$$Z_{LC}^2 = (L_{fg} C_f)^{-1} \quad (12)$$

To achieve good filter effect at switching frequency harmonic attenuation rate- $d$  must be calculated. In theory the harmonics are less when  $d$  is less. In practical aspects  $d$  should be below 0.2 [4].

$$d = \frac{Z_{LC}^2}{|\omega_{res}^2 - \omega_{sw}^2|} \quad (13)$$

### **Genetic algorithms optimisation results**

The main task of the optimisation procedure is to improve the characteristics of the investigated object by simple design modifications. The optimisation procedure applied in this research is the genetic algorithm (GA), which in its nature is always searching for the maximum value of the objective

function. The objective function for this optimisation search is selected to be the efficiency of the investigated inductor. In this research work two different optimisation approaches of the inductor are performed, such as: model 1 (M1) where the number of turns in the inductor windings is assigned as an optimisation variable and model 2 (M2) where the number of turns in inductor winding and the number of steel lamination in the inductor core are selected as optimisation variables. A variety of models are developed for wide range of switching frequencies from 2÷8 kHz. In Table 1 the optimised parameters, the value of the objective function and some other important parameters for the inductor at rated switching frequency of 2.5 kHz are presented. In Fig. 3 and 4 the copper losses and the iron losses, for different switching frequencies are presented, while in Fig.5 total inductor losses in function of the different switching frequencies are also presented. The inductor efficiency factor for all models is presented in Fig. 6. From the presented results it can be concluded that both optimised inductor models have increased efficiency compared to the basic inductor model, decreased losses ( $P_{loss}$ ) and temperature rise ( $T_{rise}$ ) as well as good harmonic attenuation rate- $d$ . In theory the harmonics have smaller value when the value of  $d$  is smaller. In practice  $d$  should be below 0.2.

Table 1 Optimisation results for switching frequency 2.5 kHz

	$N_{turns}$ (l)	$N_{lam}$ (l)	$P_l$ (W)	$P_{cu}$ (W)	$P_{fe}$ (W)	$P_{gap}$ (W)	$P_\gamma$ (W)	$\eta$	$\psi$ (°)	$f_{res}$ (Hz)	$d$
BM	54	130	2034	219	38	11,07	268,8	0,868	97	969	0,136
M1	48	130	1643	172	31,78	8,94	212,9	0,87	80,5	997	0,134
M2	48	150	1895	183,68	36,66	8,94	229,3	0,88	81,8	977	0,133

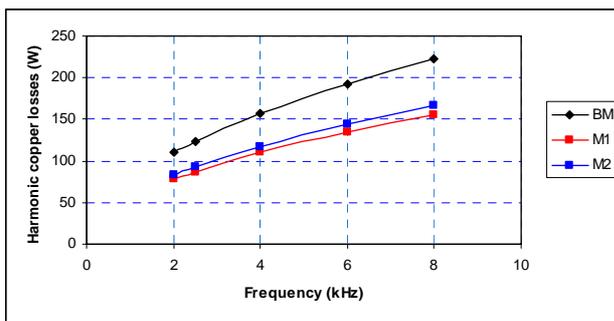


Fig. 3 Harmonic copper losses at different switching frequencies

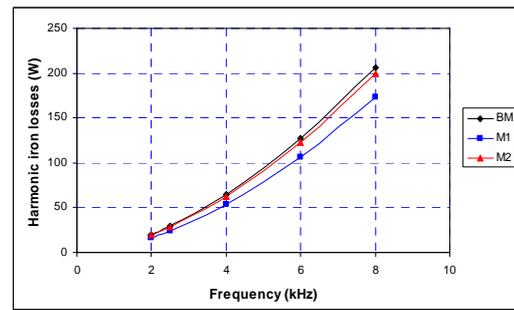


Fig. 4 Harmonic iron losses at different switching frequencies

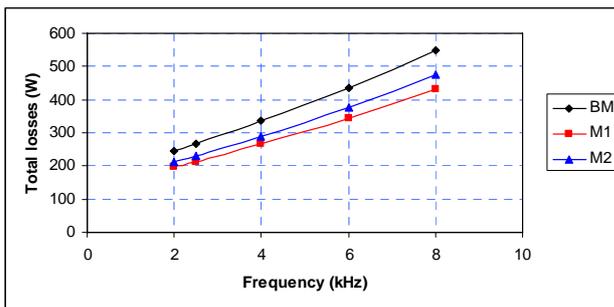


Fig. 5 Total losses at different switching frequencies

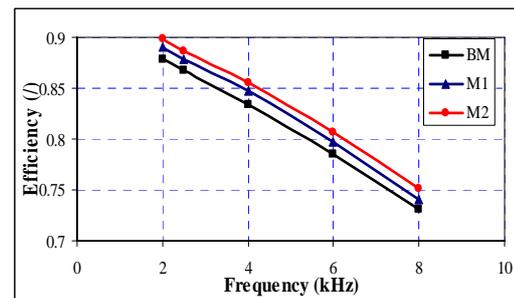


Fig. 6 Efficiency at different switching frequencies

Table 2 Ranges of variation of input variables for switching frequency 2500 Hz

	Ranges of variation	BM	M1	M2
$N_{turns}$	[48÷60]	54	48	48
$N_{lam}$	[110÷150]	130	130	130

In Table 2 are presented ranges of variation of optimization variables and their outputs from GA optimization program.

### Finite Elements Method Results

Finite Element Method (FEM) is a numerical method used for solving relatively complex electromagnetic problems where material nonlinearity and anisotropy is included in analyzed domain. Method involves discretization of the whole analyzed domain in small triangle surfaces, which are called finite elements. By applying Maxwell's equations into FEM it is possible to calculate the stationary distribution of magnetic field inside electrical devices. The FEM analysis of the inductor is divided into three parts: pre-processing, processing and post-processing part. In pre-processing part the inductor geometry, as well as the inductor boundary conditions are defined. For this specific inductor model are chosen Dirichlet boundary conditions e.g.  $A=0$ . The most common use of Dirichlet-type boundary conditions in magnetic problems is to define  $A=0$  along a boundary to keep the magnetic flux from crossing the boundary. Properties of all materials are input in inductor model. For each user defined magnetic material the magnetization curve and its values, and also the lamination fill factor of the magnetic material should be inserted. In order the value of the magnetic vector potential  $A$  to be determined it is necessary the whole domain i.e. inductor's cross-section to be divided into a certain number of elements. The number of elements is problem dependent and for the inductor model the finite element mesh is consisted of  $N=2721$  nodes and  $E=5233$  elements. For harmonic problems the permeability  $\mu$  should be considered as a constant value. In the processing part the Maxwell's equation are solved. For the analysis of the inductor a time-harmonic approach is implemented. Consequently the following partial equation is used and solved numerically:

$$\nabla x \left( \frac{1}{\mu(B)} \nabla x A \right) = -\sigma A + J_{src} - \sigma \nabla V \quad (14)$$

where  $J_{src}$  represents the applied current sources. The additional voltage gradient  $\nabla V$  in 2-D field problems is constant over the conduction body [5]. However, FEM retains a nonlinear relationship in the harmonic formulation, allowing the program to approximate the effects of saturation on the phase and amplitude of the field distribution. FEM also allows for the inclusion of complex and frequency-dependant permeability in time harmonics. These features allow the program to model materials with thin laminations and approximately model hysteresis effects. Program is run at constant frequency  $f=50$  Hz. Results from the FEM analysis for the basic model are presented in Fig. 7 and 8, where the magnetic flux distribution and the temperature distribution for basic the model of inductor at grid frequency  $f=50$  Hz are presented, respectively.

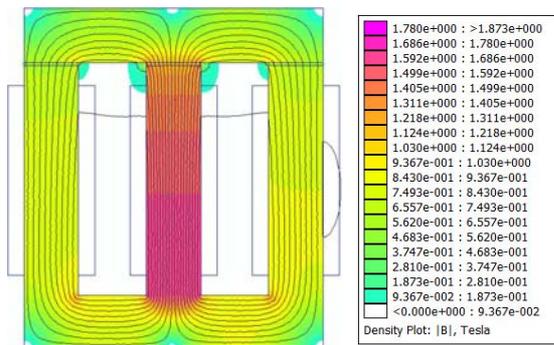


Fig. 7 Magnetic flux distribution-BM

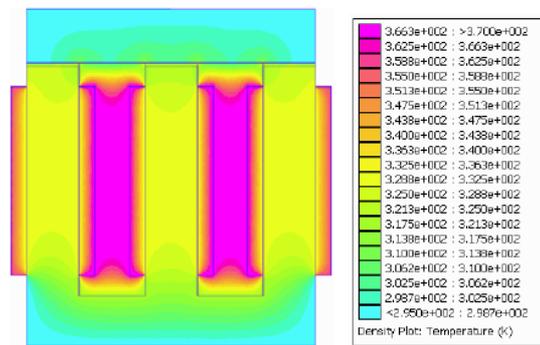


Fig. 8 Temperature distribution-BM

FEM analysis is also performed for the two optimized motor models. For both optimized motor models identical results are obtained regarding magnetic flux density distribution and temperature distribution. Therefore they are presented as one model in Fig. 9 and 10, respectively.

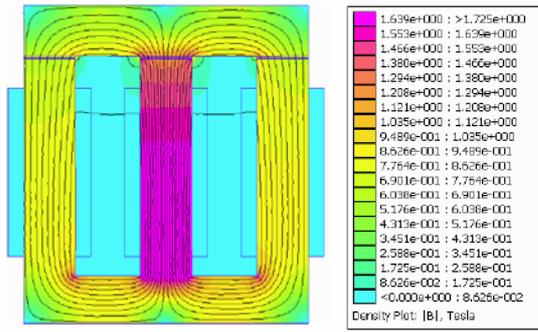


Fig. 9 Magnetic flux distribution-M1 & M2

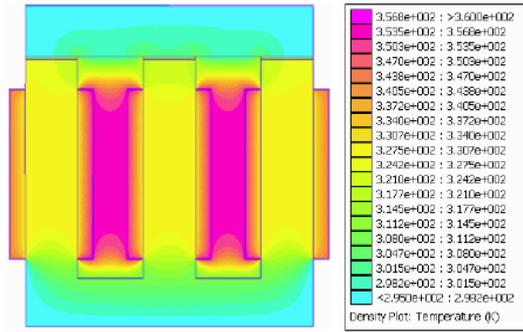


Fig. 10 Temperature distribution- M1 & M2

Magnetic flux distribution in the middle of the air gap in the second inductor leg for the basic, as well as optimized motor models is presented in Fig. 11 and 12 respectively.

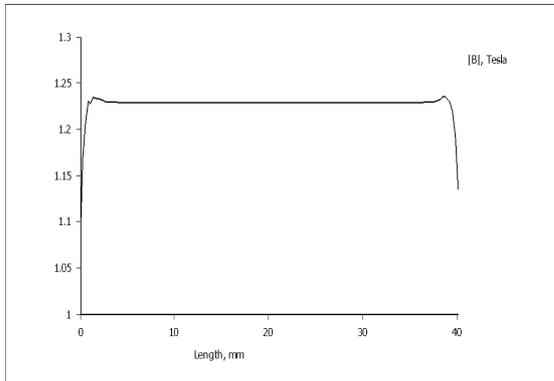


Fig. 11 Magnetic flux distribution in air gap-BM

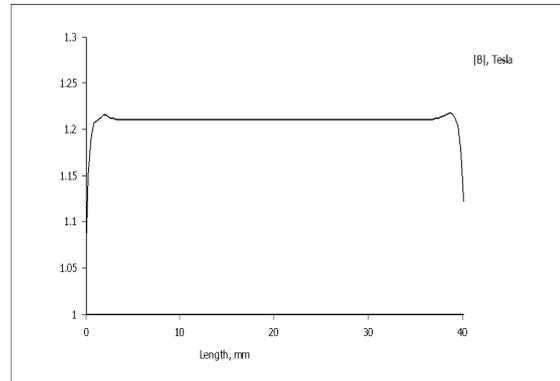


Fig. 12 Magnetic flux distribution in air gap-M1&M2

### Conclusion

In recent years a large increase in the use of electrical loads supplied through power electronics has taken place. Most of the electronic circuits use line filters as an interface between the power electronics circuits and the grid. The switching of the self commutated power electronic converters generates a current consisted of components at main frequency together with harmonics around the switching frequency and multiples of the switching frequency. The switching frequency of the converter can be high, some tens of kHz, which impose the inductor a wide current spectrum. To minimize the power losses in the inductor generated by the harmonic current the design of the inductor should be made with care.

This paper evaluates optimal design of main inductor for LCL filter in a three phase PWM converter. The Method of Generic Algorithm is applied as optimization method with the efficiency of the inductor as an objective function. The main purpose of the optimization procedure is to decrease inductor losses, as well as the temperature rise. Two optimal models are developed by varying the number of winding turns in the first one and the number of winding turns and steel laminations in the second one. Both optimal inductor models have decreased total losses of 212 and 229 W, which are lower than the 269 W of basic model, as a result of which the efficiency rises from 0.868 of the basic model to 0.87 and 0.88 of the optimized ones. Optimal inductor models have decreased temperature rise of 80.5° and 81.8° compared to the temperature of the basic model which is 97°C. In order to obtain the magnetic flux distribution, as well as the temperature distribution, a FEM analysis is performed on all inductor models. After comparison analysis of the obtained FEM results it can be

concluded that the optimal models have decreased flux density compared to the basic one (1.639 T to 1.789 T) and lower temperature distribution in the inductor cross section. The quality of the developed models can be also proved through the values of the temperature in the basic, as well as optimized models. The value of flux density obtained from the numerical calculations of 1.17 T is quite comparable with the value of magnetic flux density of 1.21 T, as shown in Figs. 7 and 8, which also proves the accuracy of the developed mathematical model for inductor losses calculation.

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