Speed-up for 3D Finite Element Analysis by Using Multigird Method

Ryutaro Tokuda, So Noguchi, Hideo Yamashita,
Hiroshima University, Kagamiyama 1-4-1, Higashihiroshima, Japan

Vlatko CingoskiElectric Power Company of Macedonia, 11 October, 9, 1000, Skopje, Macedonia

Abstract. In this paper, the efficiency of the multigrid method for electromagnetic field computations is presented. We apply the multigrid method to nodal and edge finite element analyses. A comparison of the computational time between the multigrid method and the ICCG method, which is commonly used a solution method in the finite element analysis, is also presented. It is obvious that the efficiency of the multigrid methods is better than that of the ICCG method especially for solving larger systems of equations.

1. Introduction

Recently, with the tremendous developments in the computer software and hardware technology a large-scale simulation is feasible. But, it needs a long computation time because we have to solve a large system of equations. Therefore, we need to shorten the computation time in order to solve a large system of equations for which various techniques have already been proposed. Among them, the multigrid method that is a fast solution method for a large system of equations is the most promising.

In finite element analysis, a nodal finite element has been traditionally used. Naturally, the multigrid method was initially developed for the nodal finite element analysis [1]-[4]. Recently, an edge finite element method is widely used for electromagnetic analysis because of its good computational properties. Respectively, we applied the multigrid method not only to a nodal finite element method but also for an edge finite element method. In this paper, we applied the mutigrid method to the 3D magnetostatic problems, and investigated its efficiency. The convergence criterion was set to 10^{-6} for all analysis cases. For the analyses, we used a personal computer with Alpha processor CPU (21164A, 633MHz).

2. Multigrid Method

In analysis using the multigrid solution method, it is necessary to prepare several meshes with different mesh densities. The stationary iterative methods (*e.g. Gauss-Seidel method*) can eliminate the high frequency components of an error in several iteration steps, but it requires many iteration steps to eliminate the low frequency components of the same error. This process causes very slow convergence of the iteration process. On the other hand, in the multigrid method, the high frequency components of an error are eliminated on the finer mesh, and the low frequency error components are eliminated on the coarser mesh. This procedure is called *"coarse gird correction"* and is a basic process of the multigrid method. Fig. 1 shows the flowchart of the coarse grid correction.

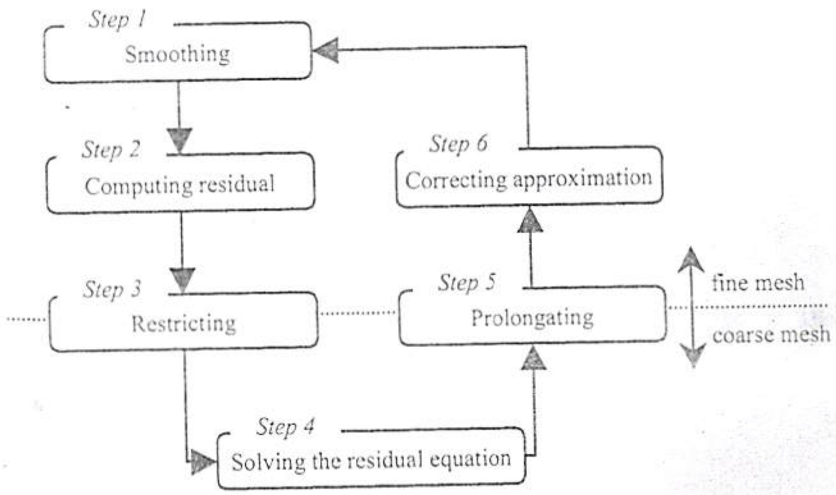

Figure 1: Flowchart of coarse grid correction

Step 1 – Relax a few times on $K_2 x_2 = f_2$ to obtain an approximation solution $\widetilde{x}_2$. The matrix K_2 is the system matrix, and vectors x_2 and f_2 are the unknown vector and the source vector, respectively. This step is called "*smoothing*".

Step 2 –Compute the residual vector $r_2 = f_2 - K_2 \widetilde{x}_2$.

Step 3 – Project the residual vector onto a coarser mesh, $r_1 = R\ r_2$, using the restriction operator R.

Step 4 – Exactly solve the residual equation $K_1 e_1 = r_1$ to obtain an error approximation e_1.

Step 5 – Interpolate the approximation error to the finer mesh, $e_2 = P\ e_1$, using the prolongation operator P.

Step 6 –Compute the improved solution $x_2 = \widetilde{x}_2 + e_2$.

Step 7 –Continue into a new iterative cycle by starting again from smoothing.

At *step 4*, it is possible to apply the coarse grid correction scheme recursively. Then, the number of meshes is unrestricted and several iteration schemes are possible. In that case, it is called *"A Multigrid Method"*.

3. Applications

We applied the multigrid method to the 3D magnetostatic model shown in Fig. 2, where the permeability of iron is 1000 and the source current is 1000 At.

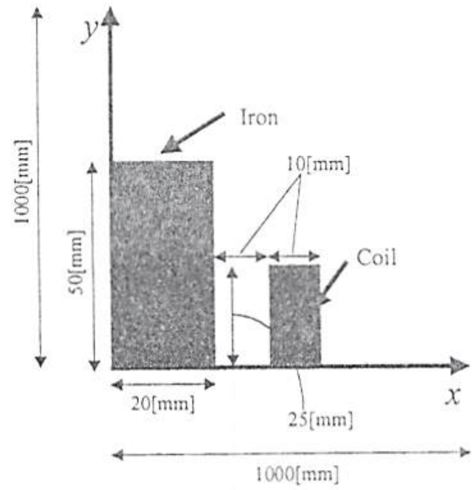

Figure 2: Analysis model

3.1. *Nodal element*

First, the multigrid method is applied to the tetrahedral nodal finite element. Four meshes with different mesh density are prepared for the multigrid method. The number of nodes, elements and unknowns for each mesh are shown in Table I. Subdividing one element of the coarser mesh into 8 finer elements generates the finer mesh. Fig. 3 shows the comparison of the multigrid method and the ICCG method. For 30,000 unknowns, the computation time of the multigrid method is almost equal to that

Table 1: Number of nodes for nodal elements

Level	Nodes	Elements	Unknowns
1	232	591	928
2	1284	4728	5136
3	8215	37824	32860
4	57933	302592	231732

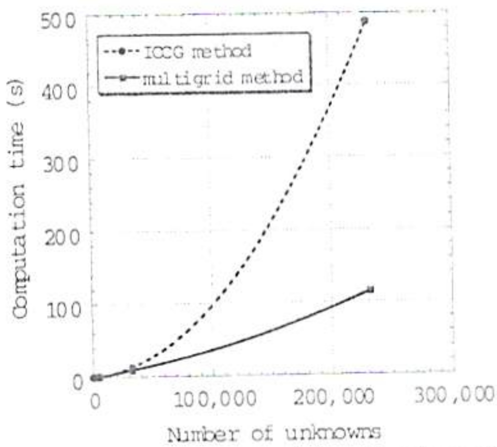

Figure 3: Number of unknowns vs. computation time for nodal element

of the ICCG method. But, as the number of unknowns increases, the computation time of the ICCG method increases rapidly. On the contrary, the computation time of the multigrid method increases slowly as the number of unknowns increases. Therefore, as the number of unknowns increases, the multigrid method is faster than the ICCG method. For about 230,000 unknowns, the multigrid method is 4 times faster than the ICCG method.

3.2. *Edge element*

Next, the multigrid method is applied to the brick edge finite element. Table II shows the number of nodes, elements and edges. Fig. 4 shows the comparison of the multigrid method and the ICCG method. As the number of unknowns increases, the computation time of the ICCG increases rapidly but the computation time of the multigrid method increases modestly. In this example, the multigrid method is 4 times faster than the ICCG method for 350,000 unknowns.

Table 2: Number of nodes for edge elements

Level	Nodes	Elements	Edges
1	480	210	1138
2	2697	1680	7012
3	17385	13440	48088
4	123057	107520	353392

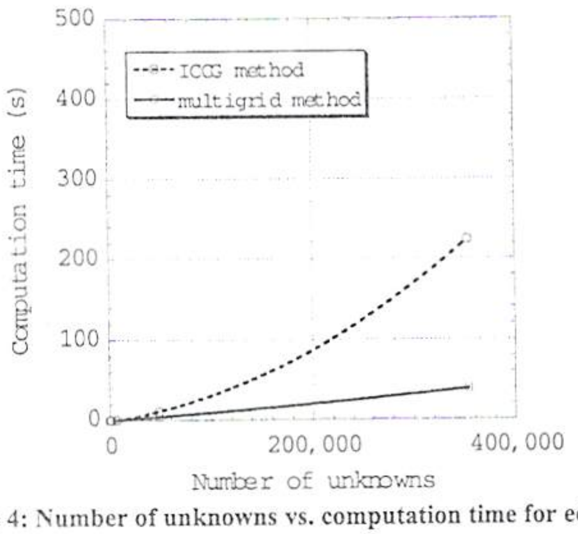

Figure 4: Number of unknowns vs. computation time for edge element

Therefore, it is revealed that the multigrid method is equally effective for nodal and for edge finite element analyses. Especially, the edge finite element method using the multigrid method is the fastest in this example.

4. Conclusions

We presented an investigation of the efficiency of the multigrid method for nodal and for edge finite element analyses. The multigrid method improves strongly the convergence rate and reduces the computation time in comparison with the ICCG method. The multigrid method is very effective for the large-size problem.

References

[1] *William L. Briggs, "A Multigrid Tutorial", SIAM, 1987.*

[2] *I. Tsukerman, "Fast finite element solvers for problems with magnetic materials", IEEE Trans. on Magnetics, September 1993.*

[3] *Vlatko Cingoski and Hideo Yamashita, "First Electromagnetic Field Computations Using Multigrid Method Based on Nested Finite Element Meshes", The 7th International Journal of Theoretical Electromagnetics, Ed, A. Kost, in press, 1999.*

[4] *Vlatko Cingoski, Katsumi Tsubota and Hideo Yamashita, "Investigation of the Efficiency of the Multigrid Method for Finite Element Electromagnetic Field Computations Using Nested Meshes", submitted to INTERMAG '99, 1999.*

[5] *J. C. Nedelec, "Mixed Finite Elements in R^3", Numerical Math., Vol. 50, No. 5, pp. 315-341.*