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What Mathematics School Beginners Know and Can Do – a Matter of Importance or Not

Slagjana Jakimovik[1], Irena Trajanovska[1], Valentina Gogovska[1]
and Tatjana Atanasova Pachemska[2]
[1] *"Ss Cyril and Methodius" University in Skopje,*
[2] *University "Goce Delchev" in Shtip*

Abstract

The determination of the levels of mathematical knowledge and skills is of essential importance when planning and organizing any form of work with students as well as in the process of creating mathematics curriculum. A question emerges: What kind of mathematical knowledge and skills are children equipped with when entering the first grade of primary school? In the Republic of Macedonia, written mathematics testing of children at this age (approximately 6 years) is not conducted. At the national level there are no data on this issue.

At the beginning of the school year 2011/2012 a research study was conducted in primary schools in the Republic of Macedonia, based on a previous research conducted in the Netherlands. The results on five of the given problems are discussed and compared with the results obtained from similar studies conducted previously in other European countries. Another question consequently emerges: Does the national mathematics curriculum for the first grade give full credit to children's actual mathematics competences and does it provide optimal conditions for their further advancement?

***Key words**: first grade pupils; mathematical competences; national curriculum; primary education.*

Introduction

At each level of education, further development of pupils' mathematical competences is preconditioned by the pupils' current cognitive abilities as well as previously acquired knowledge and skills. The determination of the actual levels of mathematical competences serves multiple purposes: setting up appropriate curriculum goals, proposing optimal strategies for reaching these goals, planning and organizing effective mathematics instruction, measuring the effects of various factors which influence the outcomes, etc. Building positive attitudes towards learning mathematics by the enhancement of pupils' self-confidence is one of the least discussed goals of mathematics education in the Republic of Macedonia. Proper recognition and explicit acknowledgement of children's informal knowledge and self-constructed strategies for approaching and solving problems as well as making children's abilities recognizable to both children themselves and teachers/parents contributes greatly towards the fulfilment of this goal. How can suitable situations be created in which the children's competences will emerge and be given the chance to develop to their full potential if there is no relevant information on the current state of their development? At the national level there are no published research data on this issue.

In 1987 in the Netherlands, the OW&OC research group (renamed Freudenthal Institute in 1991) in collaboration with other departments at the Utrecht University began the MORE research project (Van den Heuvel-Panhuizen, 1996). The MORE research project was designed and conducted to answer questions about the implementation and effects of the realistic approach in mathematics education (RME) in the Netherlands (for a brief overview, see Chapter 2 in Van den Heuvel-Panhuizen, 1996). Considering the existing paper-and-pencil tests as ill-suited for measuring young learners' mathematical abilities and strategies, a new set of tests was designed. The 'entry test', developed for evaluating the beginning first-graders competences before systematic mathematics instruction has begun, consequently served as a basic tool for research studies in Germany, conducted in Northrhine-Westphalia (Selter, 1993; Selter,1995), in Switzerland (Hengartner and Röthlisberger, 1995, as cited in Van den Heuvel-Panhuizen, 1996), again in Germany, conducted in Berlin and Brandenburg (Grassmann et al., 1995), in the Czech Republic and in Slovakia (Hošpesová et al., 1995).

In each of the consequent studies, the original Dutch 'entry test' was replicated as closely as possible given the restrictions imposed by the specifics of each country's educational system, the wider cultural contexts that influence young children's informal knowledge and understanding, as well as the researcher's own beliefs. Hence, the original Dutch scenario gives a fairly good overview of how the test was administered. Since the mathematical competences of a large number of children had to be evaluated, individual interviews were not deemed feasible, although they would have been much better suited for young children. Only a small sample of children was interviewed individually. The teachers read the questions and instructions out loud since children at this age are not expected to know how to read and write. All relevant numerical information was given on the test page so that children's short term memory did not interfere as a factor on their ability to answer the questions. The original test consisted of 26 problems as more than one problem on each topic was included in order to rule out responses provided by chance. The final version of the MORE 'entry test' is given in the Appendix at the end of Chapter 5 (Van den Heuvel-Panhuizen, 1996).

Methods

The research was conducted at the beginning of the school year 2011/12 in 16 primary schools in the Republic of Macedonia. The choice of schools represents fairly well the diversity of the Macedonian educational system: urban and rural schools, ethnically diverse and ethnically homogenous schools, with 780 students tested or interviewed. Using pictures of situational stories, 18 problems were introduced in order to investigate first graders' understanding of relational concepts, knowledge of number symbols up to 20, knowledge of the counting sequence up to 20, and addition and subtraction skills up to 10 within a given context. Unlike the original version of the test, the 'option' problems, where each child chose the numbers she/he wanted to work with, were not included. Teachers were instructed to read the questions aloud to the whole class and to give the children sufficient time to mark (circle or scratch) the answer on the pictorial depiction of the problem. The pictures represented the problems in contexts familiar to children, but for some problems teachers read 2-3 short sentences to introduce the question through a story. The test was administered within half an hour. As in the Dutch case, responses that were not entirely according to the instructions but were correct, were considered correct.

Results and Discussion

In this paper, five of the problems (see the Appendix) were selected to make a comparative analysis of the results obtained in the various studies conducted in: the Netherlands, Germany I (the first study, conducted in Northrhine-Westphalia), Switzerland, Germany II (the second study, conducted in Berlin and Brandenburg), the Czech Republic, Slovakia and Macedonia. The percent of correct responses to each problem observed in each country is represented by the diagram in Figure 1. The total number of children tested in each country is given in brackets in the legend of Figure 1.

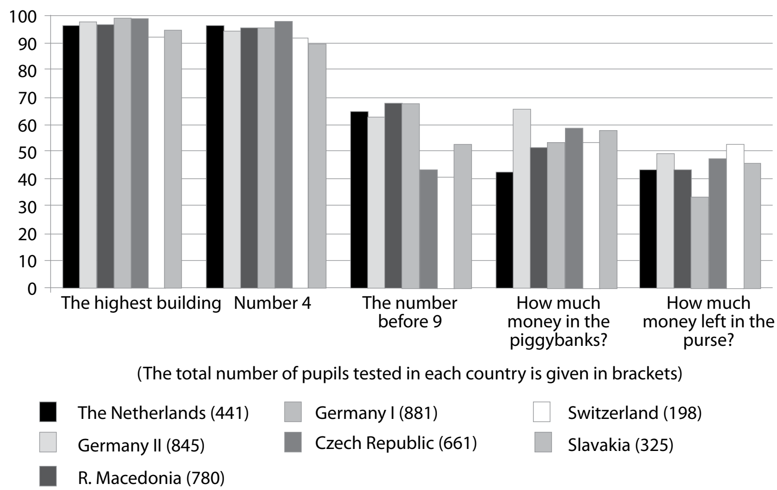


Figure 1 .*The percent of correct responses to each problem, by country*

Furthermore, for each country the observed frequencies of erroneous responses to each of the first four problems and the observed frequencies of correct responses to the fifth problem are presented and compared with the expected frequencies of erroneous and correct responses, respectively. The reasons behind this decision are based on the results, and on the mechanism for the calculation of the Chi-Square test, $\chi^2 = \Sigma [(O - E)^2 / E]$, where O is the observed value, E is the expected value, and the summation is made over all categories. Obviously, higher χ^2 value is generated by higher difference between an observed value O and an expected value E if divided by a low expected value E, which in cases of high success rate appears for the erroneous responses. The country's expected value of correct/erroneous responses is calculated by dividing the product of the raw total (the total number of correct/erroneous responses across all countries considered) and the column total (which is the same as the total number of valid responses for that country) by the total number of valid results across all countries considered.

The first problem, originally referred to as 'The highest building' by Van den Heuvel-Panhuizen and Gravemeijer (1991), appeared in each of the above mentioned studies. Children were asked to mark the highest building. The total number of responses to the first problem considered in each country, except Macedonia, is given in the legend in Figure 1. For Macedonia the number of valid responses for this problem was 764, not 780, due to test administration problems encountered by one teacher. More than 90 % of the children tested in each country answered the question correctly (Figure 1). Pearson's chi-square test showed the existence of statistically significant difference among countries, $\chi^2 = 65.08$, $p < .001$ (with 6 degrees of freedom, N = 4115). The number of erroneous responses was lower than expected in the second German study and in the Czech Republic, as opposed to the results in Slovakia and Macedonia, which can be traced on the diagram given in Figure 2.

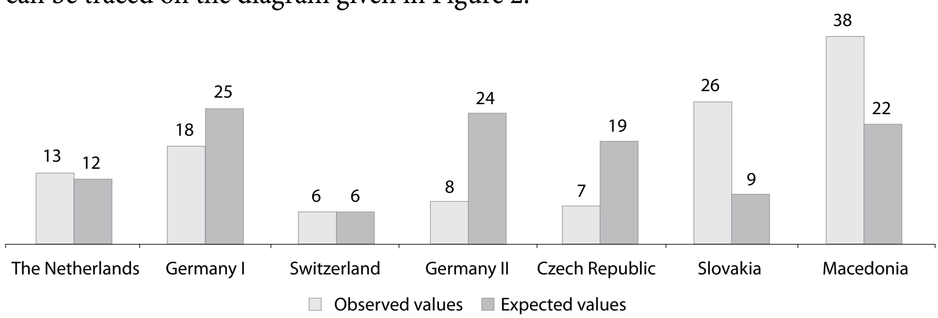


Figure 2. *The number of erroneous responses on 'The highest building'*

In the second problem, referred to as 'Number 4', children in Macedonia were asked to mark the car with number 4. Number 3 was to be marked by the children in Switzerland and number 5 by the children in the other countries. Again, less than 10 % of children in each country responded to the question incorrectly (Figure 1). Pearson's chi-square test showed the existence of statistically significant differences among countries, $\chi^2 = 62.83$, $p < .001$ (with 6 degrees of freedom, N = 4131). The reasons originated mainly from the observed number of erroneous responses obtained in Macedonia, which was much higher than the expected value, (see Figure 3).

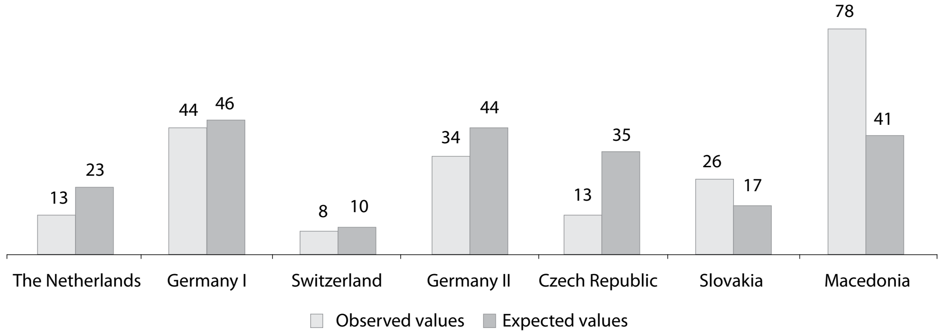


Figure 3. *The number of erroneous responses to 'Number 4'*

The third problem evaluated children's knowledge of the backwards counting sequences, that is, the predecessors of numbers. In order to answer the question which number comes before 9 in the present study, and before 8 in the other studies, a drawing of a space rocket launching was presented and children were informed that counting down was taking place, as depicted by the numbers 10, 9, … at the bottom of the drawing. The wording of the question varied among countries from "Which number is next?" in the Dutch study, and "Which number comes before 9?" in the Macedonian study. The latter choice of words proved to be an inappropriate one since many students marked 10, referring to the number placed before 9 in the sequence 10, 9, … Unlike in the Dutch study, in the Macedonian study counting down the numbers 10 and 9 out loud by the teacher was not explicitly prohibited. This decision was based on the opinion that the benefits from enabling the children to understand the question accurately carry greater weight than the danger of children getting carried away and continuing counting out loud. The number of children whose responses to this problem were considered valid was 631, not 780, due to a technical error in the translation of the question from Macedonian to Albanian language in some of the schools. The percent of correct responses significantly varied among countries, from 41 % in Slovakia to 68 % in Switzerland and in the second German study. Pearson's chi-square test showed the existence of statistically significant differences among countries, $\chi^2 = 160.12$, $p < .001$ (6 degrees of freedom, N = 3982). The huge χ^2 value originated from the following findings. The differences between the observed frequencies of erroneous responses and the expected ones were approximately 7 %, 5 %, 10 %, 10 % of the sample in the Netherlands, in the first German study, in Switzerland, and the second German study, respectively, where children did much better than in the other three countries. The differences between the observed frequencies of erroneous responses and the expected ones were approximately 14 %, 17 % and 5 % of the sample in Czech Republic, Slovakia and Macedonia, respectively (see Figure 4).

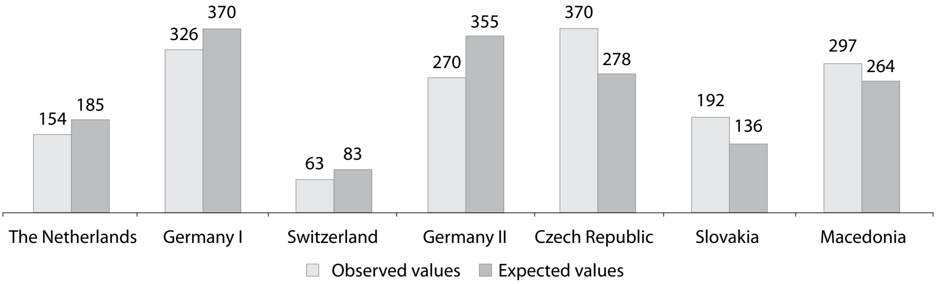


Figure 4. *The number of erroneous responses to 'The number before 9'*

Whether children are able to add small numbers, like 3 and 2 (in the Macedonian study) or 4 and 3 (in the other studies), was investigated using the fourth problem, which was referred to in this study as the 'How much money in the piggy banks? (3+2)'. The context of the original Dutch problem, a pinball machine with two pin balls already being played, depicts a completely unknown game for the children in Macedonia. Collecting money from two piggybanks was deemed as a far more appropriate context and it connected nicely with the following shopping problem. At the same time, the drawing did not offer the possibility to count the individual coins and thus calculate the sum by counting. Around 66 % of the children tested in the first German study responded correctly – a result far beyond the experts' expectations (Selter, 1995). Even the comparatively lower results obtained in the Dutch study (44 % correct responses) exceeded the Dutch experts' predictions (Van den Heuvel-Panhuizen, 1996). The results in the other countries ranged from 52 % of correct responses in Switzerland up to 59 % in Czech Republic (Figure 1). Pearson's chi-square test showed the existence of statistically significant differences among countries, $\chi^2 = 71.13$, $p < .001$ (6 degrees of freedom, N = 4131).

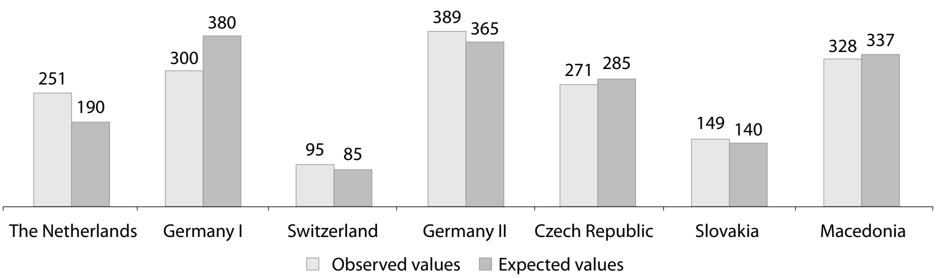


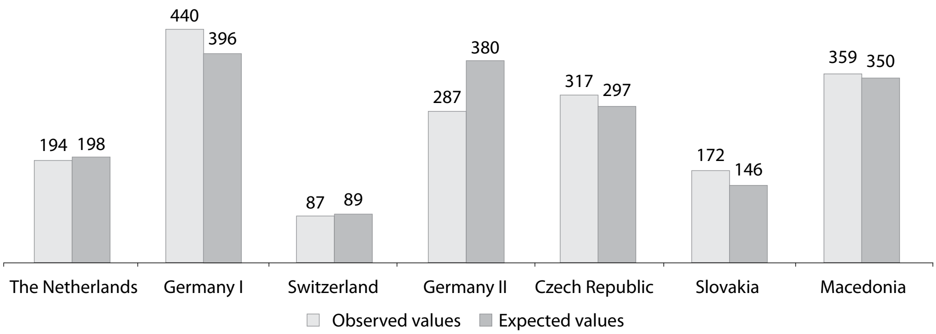
Figure 5. *Number of erroneous responses to 'How much money in the piggy banks?'*

As can be traced on the diagram in Figure 5, the positive difference between the observed and the expected frequency of erroneous responses in the Dutch study, equal to 14 % of the country's sample, together with the negative difference between the observed and the expected frequency of erroneous responses equal to 10 % of the first German sample, were the dominant factors which contributed to the high χ^2 value.

"You have 10 denars in your purse. If you buy a lollypop (sunglasses) priced 8 denars, how much money will you be left with?" was the context presented to the children with the last of the chosen problems. As in the previous problem the context was familiar, yet the drawing offered no possibility for counting the coins to calculate the difference 10 – 8. Between 40 % and 50 % of the children tested in each study, except in the second German study (34 %) and in Slovakia (53 %), responded correctly (Figure 1). Pearson's chi-square test showed the existence of statistically significant differences among countries, $\chi^2 = 61.47$, $p < .001$ (6 degrees of freedom, N = 4131). Since in almost all of the studies less than 50 % of the responses were correct, the major contributors to the high χ^2 value are to be found among the differences between the observed and the expected frequencies of correct responses (Figure 6).

Figure 6. *Number of correct responses to 'How much money left in the purse?'*

The highest absolute difference appeared in the second German study where the difference between the observed and the expected frequency of correct answers was equal to 11 % of the sample.



Conclusions

Although there were statistically significant differences among the results obtained in the seven different studies, there were also some general similarities. More than 90 % of children in each sample responded correctly to the relational concept problem. The same is true for the number symbol problem. Higher variation among different countries appeared in the results obtained on the backward counting sequence problem. There can be many explanations for this variation. For one, the wording of the question is a matter of crucial importance as previously noted by Van den Heuvel-Panhuizen (1996). The context of the problem is another matter of significance. It is fair to expect that young children from rural areas in Macedonia were not very familiar with the context of a space shuttle launching accompanied by counting down due to lesser exposure to images and media information connected with this issue. In almost all of the studies more than half of the children accurately added small numbers within a given context, even when the possibility to count the objects was not available. Between one third and one half of the children in the studies were able to correctly subtract numbers smaller than 10 without having to rely on counting the objects in a pictorial depiction of the situation.

In stark contrast to these findings stand the national curriculum goals for the first grade mathematics (Наставни програми по математика за I одделение деветгодишно основно образование, 2007). The cognitive levels aimed at within the number content area are bare recognition of number symbols up to 10, counting up to 10, adding 1 and subtracting 1. Since the national curriculum is interpreted as a restrictive framework, neither teachers in their practice nor textbooks in use offer anything beyond the boundaries of this framework. Quite a significant body of research has been accumulated and published worldwide supporting the fact that young children understand the meaning of quantities and acquire skills for managing numbers in everyday situations before they start receiving organized instruction in schools (Carpenter and Moser, 1984; Ginsburg, 1975; Jordan et al., 2009). Ginsburg (1975) concluded that children possess informal knowledge of mathematics before entering school, which they use as a referential frame for adopting and adjusting mathematics taught at school. Jordan et al. (2009) provide evidence that before entering the first grade children develop foundational number competence that supports more complex mathematics and thus demonstrate the importance of kindergarten number competence for setting children's developmental learning trajectories in primary school mathematics. Failure to capitalize on young learners' experiential knowledge has been deliberated by many authors (Carpenter and Moser, 1975; Resnick, 1989). According to Hiebert (1984), the lack of establishment of links between the children's prior knowledge and mathematics taught at school results in children replacing their own spontaneous reasoning with formal procedures without gaining any meaningful insight. Positive reinforcement of children's self-confidence comes from the acknowledgement of and assigning proper weight to their self-constructed strategies for solving problems, which consequently increases their motivation for studying mathematics.

The recommendations which emerged from this study are not aimed at introducing more content matter in the early childhood curriculum. The first-grade mathematics curriculum should reflect young learners' actual abilities. Also, teachers need to evaluate children's knowledge on each topic before offering instruction and create flexible situations or pose 'problems with elasticity' (Van den Heuvel-Panhuizen, 1996), which would allow each student to work them out at her/his own level and at the same time allow the teacher to collect valuable information on each student's level of mathematical competence (see, for example, the 'Restaurant lesson' in Van den Heuvel-Panhuizen, 2001). In conclusion, more windows for making informed decisions in everyday school practice and in curriculum design can be opened.

References

Carpenter, T.P., & Moser, J.M. (1984). The acquisition of addition and subtraction concepts in grades one through three. *Journal for Research in Mathematics Education, 15* (3), 179-202.

Ginsburg, H. (1975). Young Children's Informal Knowledge of Mathematics. *The Journal of Children's Mathematical Behaviour*, *1* (3), 4-11.

Grassmann, M., Mirwald, E., Klunter, M., & Veith, U. (1995). Arithmetische Kompetenz von Schulanfängern – Schluszfolgerungen für die Gestaltung des Anfangsunterrichtes – mehr Fragen als Antworten [The arithmetical competence of beginning first graders – conclusions for education in the early grades – more questions than answers]. *Sachunterricht und Mathematik in der Primärstufe, 23* (7), 302-321.

Hengartner, E., & Röthlisberger, H. (1995). Rechenfähigkeit von Schulanfängern [Students' arithmetical abilities at the beginning of primary school]. In H. Brügelmann, H. Balhorn, and I. Füssenich (Eds): *Am Rande der Schrift. Zwischen Sprachenvielfalt und Analphabetismus*. (pp. 66-86). Lengwil am Bodensee: Libelle Verlag / Deutsche Gesellschaft für Lesen und Schreiben.

Hiebert, J. (1984). Children's mathematics learning: The struggle to link form and understanding. *The Elementary School Journal, 84* (5), 496-513.

Hošpesová, A., Kuřina, F., & Tichá, M. (1995). Kennen wir die Kenntnisse unsere Schüler? [Do we know the knowledge of our students?] A paper presented at the 29th Bundestagung für Didaktik der Mathematik in Kassel.

Jordan, N.C., Kaplan, D., Ramineni, C., & Locuniak, M. (2009). Early Math Matters: Kindergarten Number Competence and Later Mathematics Outcomes. *Developmental Psychology*, *45* (3), 850-867.

Наставни програми за I одделение во деветгодишно основно образование (2007). Скопје: Биро за Развој на Образованието. Retrieved on 1st September 2012 from http://www.bro.gov.mk/docs/Iodd_nastavna_programa_MK-ALB_za_web.pdf

Resnick, L.B. (1989). Developing Mathematical Knowledge. *American Psychologist*, *44* (2), 162-169.

Selter, C. (1993). Die kluft zwischen den arithmetischen kompetenzen von erstklaszlern und dem pessimismus der experten [The gap between the arithmetical competence of first-graders and the pessimism of experts]. In: Betriege zum Mathematikunterricht 1993 (pp.350-353). Hildesheim: Franzbecker.

Selter, C. (1995). Zur Fiktivität der 'Stunde Null'im arithmetischen Anfangsunterricht [The fiction of 'starting from scratch' at the beginning of arithmetic education]. *Mathematische Unterrichtspraxis, 16* (2), 11-19.

Van den Heuvel-Panhuizen, M. (1990). Realistic arithmetic/mathematics instruction and tests. In K. Gravemeijer, M. van den Heuvel-Panhuizen, and L. Streefland, *Contexts, Free Productions, Tests and Geometry in Realistic Mathematics Education*. Utrecht: OW&OC

Van den Heuvel-Panhuizen, M. (1996). *Assessment and Realistic Mathematics Education*, Utrecht: CD-β Press, Center for Science and Mathematics Education.

Van den Heuvel-Panhuizen, M. (2001). A learning-teaching trajectory description as a hold for mathematics teaching in primary schools in The Netherlands. In M. Tzekaki (Ed.), *Didactics of Mathematics and Informatics in Education, 5th Panhellenic Conference with*

International Participation (pp. 21-39) Thessaloniki: Aristotle University of Thessaloniki / University of Macedonia/ Pedagogical Institute.

Van den Heuvel-Panhuizen, M., & Gravemeijer, K. (1991). Tests are not all bad. An Attempt to Change appearance of written tests in mathematics instruction at primary school level. In L. Streefland (Ed.), *Realistic Mathematics Education in Primary School* (pp.133-155). Utrecht: CD-β Press / Freudenthal Institute.

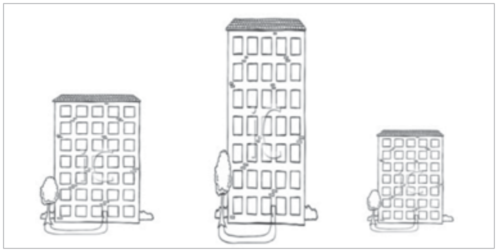
Slagjana Jakimovik
"Ss Cyril and Methodius" University in Skopje
"St Kliment Ohridski" Faculty of Pedagogy
Bul. Partizanski Odredi bb, 1000 Skopje, Republic of Macedonia
slagjana.jakimovik@gmail.com

Irena Trajanovska
"Ss Cyril and Methodius" University in Skopje
"St Kliment Ohridski" Faculty of Pedagogy
Bul. Partizanski Odredi bb, 1000 Skopje, Republic of Macedonia
irenatrajanovska@hotmail.com

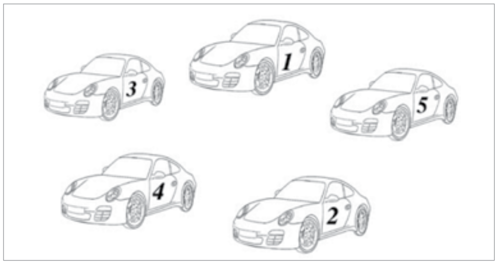
Valentina Gogovska
"Ss Cyril and Methodius" University in Skopje
Faculty of Natural Sciences and Mathematics
"Gazi Baba" bb, 1000 Skopje, Republic of Macedonia
valentinagogovska@gmail.com

Tatjana Atanasova Pachemska
University "Goce Delchev" in Shtip
"Krste Misirkov" bb, 2000 Shtip, Republic of Macedonia
tatjana.pacemska@ugd.edu.mk

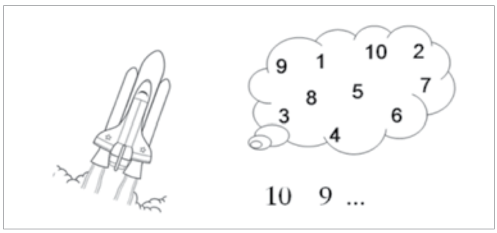
Appendix



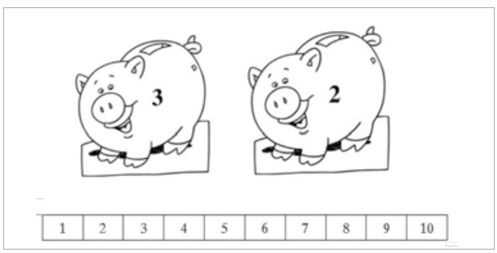
'The Highest Building'



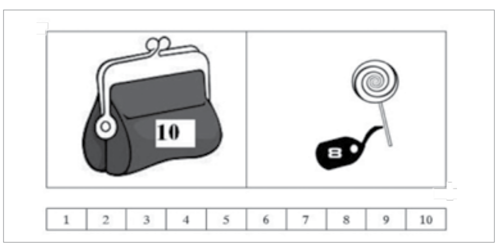
'Number 4'



'The number before 9'



'How much money in the piggy banks?'



'How much money left in the purse?'

Što na početku prvoga razreda učenici znaju i mogu iz područja matematike – je li to važno ili nije?

Sažetak

Određivanje razina matematičkog znanja i vještina od ključne je važnosti prilikom planiranja i organiziranja rada s učenicima, kao i prilikom stvaranja kurikula iz područja matematike. Pitanje koje se postavlja jest: Kakvo znanje i vještine iz matematike posjeduju učenici na početku prvog razreda osnovne škole? U Republici Makedoniji ne provode se pismeni testovi iz matematike za šestogodišnjake. Na nacionalnoj razini ne postoje podatci povezani s tom problematikom.
Na početku šk. god. 2011./2012. u Republici Makedoniji provedeno je istraživanje u osnovnim školama koje se oslanja na istraživanje u Nizozemskoj. Provedena je analiza rezultata i njihova usporedba s rezultatima sličnih istraživanja u europskim zemljama. Na kraju istraživanja postavlja se pitanje: Uzima li nacionalni kurikul za matematiku za prvi razred u obzir u potpunosti matematičku kompetenciju djece i pruža li im optimalne uvjete za daljnji napredak?

Ključne riječi: *matematičke kompetencije; nacionalni kurikul; primarno obrazovanje; učenici prvoga razreda;*