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Estimation of minimal path vectors of multi state two terminal networks with cycles control [1]

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One of the hardest problems in two terminal networks reliability theory is to obtain minimal path or cut vectors of such a network. Moreover, the bigger problem is appeared when we have a network with cycles. Here we give one algorithm solution for such a problem.

1. Introduction

Two-terminal reliability (2TR) is a well-known problem in the area of network binary reliability. For the binary network it is assume that a whole network and its components can be in two states: working or failed state. However, the binary approach does not completely describe some networks. These networks and its components may operate in any of several intermediate states and better results may be obtained using a multi-state reliability approach.

In this paper we propose an algorithm for obtaining minimal path vectors of multi-state two terminal network. Some algorithms for obtaining minimal path or cut vectors are given in [1], [2], [4] and [5], but these algorithms give candidates for minimal cut or path vectors that are not minimal, so you must do additional calculations to eliminate them. In fact these vectors are eliminated by mutual comparison; when some candidate for minimal path (cut) vector is greater (smaller) than another candidate, then it is not a minimal, and it is eliminate. This procedure is relatively expensive, because the number of minimal

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path vectors is much greater than the number of nodes and links in the network. Therefore, we want to find a way of determining whether any given path vector is a minimal path vector, without vector comparison. For that reason, in this paper we improve the algorithms given in [1] and [2], so that we analyze the properties of the minimal path vectors, in order to separate them from those vectors that are not minimal path vectors.

Further in this section we will give some definitions for multi-state two-terminal networks. More of these definitions are given in [4] and [5]. Let n is a number of link in the network. A **multi-state link** is defined as an arc of a network having a set of states $\{0, s_1, s_2, \ldots, M_i\}$, $0 < s_1 < s_2 \ldots < M_i$, $1 \leq i \leq n$. Let set $\mathbf{M} = (M_1, M_2, \ldots, M_n)$ to be the vector of maximal state of the network.

A vector $\mathbf{x}$ that reflects the state of a component is called a **state space set**. For every multi-state link, its **capacity state set** is obtained as the product of full capacity of the component and its states. For entire system is defined **system capacity state set** S, as the set of all available capacities from source to sink. A vector $\mathbf{X}$ that describes the state of all the system's components is called a **state vector**. The set of all state vectors is denoted by E, $E = S_1 \times, \ldots, \times S_n$ (where S_i is capacity vector of the i-th link).The **structure function** $\phi(\mathbf{x}) : E \to S$, maps the state vector into a system state. In fact, $\phi(\mathbf{x})$ is available capacity from the source to the sink under state vector $\mathbf{x}$. The vector $\mathbf{x}$ is a **minimal path vector to level** d ($\mathbf{MPV}_d$), if $\phi(\mathbf{x}) \geq d$ and for every other $\mathbf{y} < \mathbf{x}$, $\phi(\mathbf{y}) < d$.

2. Main results

Suppose that the network works in the state $\mathbf{x}$, where $\mathbf{x}$ is a minimal path vector to level d. Then, in order to deliver d units from the source to the sink, each link is used only in one direction, [3]. Let us regard only the structure of the network, without the capacity of its links, i.e. the unweighted graph. Suppose that the links are oriented as they are used, and the other links are removed from the graph. The obtained subgraph is acyclic oriented graph, [3]. This graph we will denote by $G_{\mathbf{x}}(V, E_{\mathbf{x}})$ and the corresponding accessibility matrix for $G_{\mathbf{x}}$ we will denote by $A_{\mathbf{x}}$.

Using these, we may define ordering of the nodes in respect to $\mathbf{x}$.

Definition 1. Let $\mathbf{x}$ be a minimal path vector to level d. For two nodes u and v we will say that $u <_{\mathbf{x}} v$ if there is a path from u to v in the graph G. This ordering will be called: ordering of the nodes in respect to $\mathbf{x}$.

In fact, the relation $<_{\mathbf{x}}$ is a accessibility relation for $G_{\mathbf{x}}$. Note that this relation is an ordering only in the case when the $\mathbf{x}$ is a minimal path vector for some level d.

Example 1 Let us consider the network of Figure 1 a). The vector (1,1,1,2,0,0,0,2) is a minimal path vector of level 2. Ordering of the nodes in respect to (1,1,1,2,0,0,0,2), i.e. $G_{(1,1,1,2,0,0,0,2)}$ is shown in Figure 1 b).

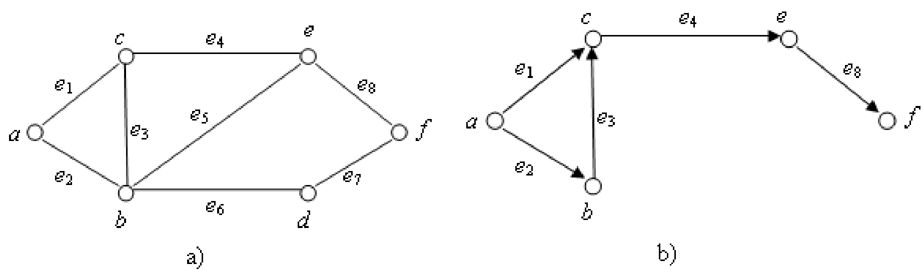

Figure 1

Definition 2. Let $\mathbf{x}$ and $\mathbf{y}$ are paths in G for some levels d and d'. We define a graph $G_{\mathbf{x+y}} = (V, \widetilde{E}_{\mathbf{x+y}})$, where $(u, v) \in \widetilde{E}_{\mathbf{x+y}} \Leftrightarrow (u, v) \in E_{\mathbf{x}}$ or $(u, v) \in E_{\mathbf{y}}$.

Proposition 1. *Let* $\mathbf{x}$ *and* $\mathbf{y}$ *are minimal path vectors in* G *for some levels* d *and* d' *and* $\mathbf{x} + \mathbf{y} < \mathbf{M}$. *If* $G_{\mathbf{x+y}}$ *is an acyclic graph, then the vector* $\mathbf{x} + \mathbf{y}$ *is a minimal path vector for level* $d + d'$.

Proof: It is clear that $\mathbf{x} + \mathbf{y}$ is a path vector for level $d + d'$. Let us suppose that it is not a minimal path vector, i.e. there is a smaller minimal path vector $\mathbf{z}$.

Since $\mathbf{z}$ is a minimal path vector for level $d + d'$, from the Kirchhoff's Current law follows that when the network is in the state $\mathbf{z}$, exactly $d + d'$ units get out from the source node and come into the sink node and for other nodes, the number of units that get into the node is equal to the number of units that get out from it.

Let us regard the vector $\mathbf{x} + \mathbf{y} - \mathbf{z}$. If the network is in this state, then there no units that gets out from the source node and comes into the sink node. Also, for the other nodes, the number of units that get into the node is equal to the number of units that get out from it, and there are nodes in which at last one unit gets in and out. If the links are oriented in respect to $\mathbf{x} + \mathbf{y} - \mathbf{z}$, this will be possible only if there exist at last one cycle, which is in contradiction with our assumption that $G_{\mathbf{x+y}}$ is an acyclic graph.

From the last Proposition we can conclude that in order to check whether a sum of d minimal path vectors of level 1 is the minimal path vector of level d, it is sufficient to check whether the corresponding graph is acyclic. It can be seen from the accessibility matrix of the graph. If on the diagonal in this matrix at least one element is equal to 1, then the graph has a cycle. It is clear that the accessibility relation is a transitive closure of the adjacency relation. But, the procedure for finding the transitive closure, in general case, has great complexity. Actually, to get the accessibility matrix, the adjacency matrix should be multiplied by itself at most $|V|$ times. But it will be proven that it can be obtained by 3 multiplying of a $|V| \times |V|$ matrix. To prove that we define the relation

$$u \ \widetilde{\alpha}_{\mathbf{x}+\mathbf{y}} \ v \Leftrightarrow u <_{\mathbf{x}} v \ or \ u <_{\mathbf{y}} v$$

and the relation $\alpha_{\mathbf{x}+\mathbf{y}}$ as a transitive closure of the $\widetilde{\alpha}_{\mathbf{x}+\mathbf{y}}$. It is clear that $\alpha_{\mathbf{x}+\mathbf{y}}$ is a accessibility relation for $G_{\mathbf{x}+\mathbf{y}}$.

E x a m p l e 2 The vectors (1,0,0,1,1,1,1,0) and (0,1,1,1,0,0,0,1) are a minimal path vector of level 1 for the network of Figure 1 a).

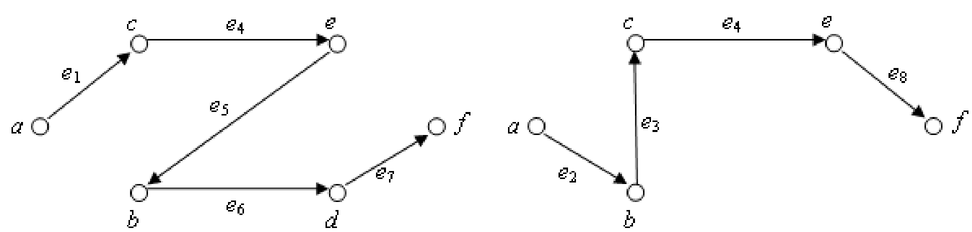

Figure 2

By adding these vectors the vector (1,1,1,2,1,1,1,1) is obtained. This vector is not a minimal path vector because it is greater than the vector (1,1,0,1,0,1,1,1). Figure 3 shows that the graph which is obtained by adding these vectors contains a cycle.

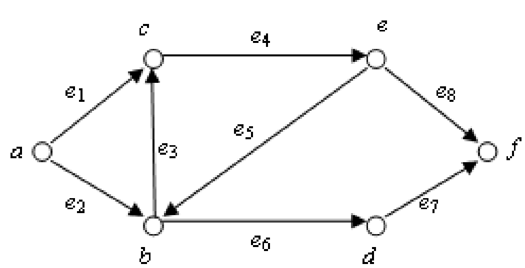

Figure 3

Let $\widetilde{A}_{\mathbf{x+y}} = A_{\mathbf{x}} \oplus A_{\mathbf{y}}$ where $\oplus$ is a binary and (i.e. $0 \oplus 0 = 0, 0 \oplus 1 = 1, 1 \oplus 0 = 1, 1 \oplus 1 = 1$). It is clear that $A_{\mathbf{x+y}} = \sum_{i=1}^{|V|} \widetilde{A}^{i}_{\mathbf{x+y}}$, where the adding operation is a binary or (i.e. $\oplus$) and the multiplication operation is a binary and, $\otimes$ (i.e. $0 \otimes 0 = 0, 0 \otimes 1 = 0, 1 \otimes 0 = 0, 1 \otimes 1 = 1$).

To reduce the number of operations for obtaining the matrix $A_{\mathbf{x+y}}$, we give some statements.

Definition 3. Minimal binary path for multi state network with graph $G = (V, E)$ is a any sequence of links $(u_0, u_1), (u_1, u_2), \ldots, (u_{k-2}, u_{k-1}), (u_{k-1}, u_k)$, where $u_i, u_{i+1} \in E$, u_0 is the source and u_k is the sink of the network, and $u_i \neq u_j$ for $i \neq j$.

For each minimal binary path $(u_0, u_1), (u_1, u_2), \ldots, (u_{k-2}, u_{k-1}), (u_{k-1}, u_k)$ we define minimal binary path vector such that if one link is in the sequence $(u_0, u_1), (u_1, u_2), \ldots, (u_{k-2}, u_{k-1}), (u_{k-1}, u_k)$, then the corresponding coordinate is equal to 1, otherwise it is equal to 0.

Example 3 One binary minimal path for a network in the Figure 1 a) is $(a, c), (c, e), (e, f)$. And appropriate binary minimal path vector is a vector (1,0,0,1,0,0,0,1).

In fact, the minimal binary path vector for multi state network is a minimal vector for a binary network that has the same structure with multi state networks. It is clear that for a binary minimal path vector $\mathbf{x}$, ordering $<_{\mathbf{x}}$ is a strict linear ordering. So we have the next Proposition:

Proposition 2. *Let* $\mathbf{x}$ *is a binary minimal path vector and* $u, v, u_1, v_1 \in V$, *then from* ($u <_{\mathbf{x}} u_1$ *or* $u_1 <_{\mathbf{x}} u$) *and* ($v <_{\mathbf{x}} v_1$ *or* $v_1 <_{\mathbf{x}} v$) *follows that* $u <_{\mathbf{x}} v$ *or* $v <_{\mathbf{x}} u$

Proposition 3. *Let* $\mathbf{x}$ *is a minimal path vector to level* d, $\mathbf{y}$ *is a binary minimal path vector and* $u <_{\mathbf{y}} v <_{\mathbf{x}} w <_{\mathbf{y}} r$ *then either* $u <_{\mathbf{y}} r$ *or on the diagonal in the matrix* $\widetilde{A}^2_{\mathbf{x+y}}$ *at least one element is equal to 1.*

Proof. Let $u <_{\mathbf{y}} v <_{\mathbf{x}} w <_{\mathbf{y}} r$. Because $\mathbf{y}$ is a binary minimal path vector from Proposition 2 we have that $u <_{\mathbf{y}} w$ or $w <_{\mathbf{y}} u$.

If $u <_{\mathbf{y}} w$, then $u <_{\mathbf{y}} w <_{\mathbf{y}} r$ and because $<_{\mathbf{y}}$ is transitive relation, $u <_{\mathbf{y}} r$.

If $w <_{\mathbf{y}} u$, then $w <_{\mathbf{y}} u <_{\mathbf{y}} v$ and because $<_{\mathbf{y}}$ is transitive relation, $w <_{\mathbf{y}} v$. Now, $w <_{\mathbf{y}} v$ and $v <_{\mathbf{x}} w$ from where $w \, \widetilde{\alpha}_{\mathbf{x+y}} \, v$ and $v \, \widetilde{\alpha}_{\mathbf{x+y}} \, w \Rightarrow w \widetilde{\alpha}^2_{\mathbf{x+y}} w$, i.e. on the diagonal in the matrix $\widetilde{A}^2_{\mathbf{x+y}}$ at least one element is equal to 1. ∎

Corollary 1 *Let* $\mathbf{x}$ *is a minimal path vector to level* d *and* $\mathbf{y}$ *is a minimal binary path vector and* $u <_{\mathbf{x}} v <_{\mathbf{y}} w <_{\mathbf{x}} r <_{\mathbf{y}} m$. *Then either* $u \widetilde{\alpha}_{\mathbf{x+y}} m$ *or on* *the diagonal in the matrix* $\widetilde{A}^2_{\mathbf{x}+\mathbf{y}}$ *at least one element is equal to 1.*

Proof. Let $u <_{\mathbf{x}} v <_{\mathbf{y}} w <_{\mathbf{x}} r <_{\mathbf{y}} m$ then from $v <_{\mathbf{y}} w <_{\mathbf{x}} r <_{\mathbf{y}} m$ and Proposition 2 either $v <_{\mathbf{y}} m \Rightarrow u <_{\mathbf{x}} v <_{\mathbf{y}} m \Rightarrow u\widetilde{\alpha}_{\mathbf{x}+\mathbf{y}}m$, or on the diagonal in the matrix $\widetilde{A}^2_{\mathbf{x}+\mathbf{y}}$ at least one element is equal to 1. ∎

Theorem 1 *Let* $\mathbf{x}$ *is a minimal path vector to level* d *and* $\mathbf{y}$ *is a binary path vector and* $G_{\mathbf{x}+\mathbf{y}}$ *does not have cycles . Then*

$$\widetilde{\alpha}^4_{\mathbf{x}+\mathbf{y}} \subseteq \widetilde{\alpha}_{\mathbf{x}+\mathbf{y}} \cup \widetilde{\alpha}^2_{\mathbf{x}+\mathbf{y}} \cup \widetilde{\alpha}^3_{\mathbf{x}+\mathbf{y}}$$

Proof. Let $u\ \widetilde{\alpha}^2_{\mathbf{x}+\mathbf{y}}\ v$. We have

$$\begin{aligned} u\ \widetilde{\alpha}^2_{\mathbf{x}+\mathbf{y}}\ v \ &\Rightarrow\ (\exists w \in V) u\ \widetilde{\alpha}_{\mathbf{x}+\mathbf{y}}\ w\ \widetilde{\alpha}_{\mathbf{x}+\mathbf{y}}\ v \\ &\Rightarrow\ (\exists w \in V)(u <_{\mathbf{x}} w \vee u <_{\mathbf{y}} w) \wedge (w <_{\mathbf{x}} v \vee w <_{\mathbf{y}} v) \\ &\Rightarrow\ (\exists w \in V)(u <_{\mathbf{x}} w <_{\mathbf{x}} v) \vee (u <_{\mathbf{y}} w <_{\mathbf{x}} v) \\ &\quad\ \vee (u <_{\mathbf{x}} w <_{\mathbf{y}} v) \vee (u <_{\mathbf{y}} w <_{\mathbf{y}} v). \end{aligned}$$

Because the relations $<_{\mathbf{x}}$ and $<_{\mathbf{y}}$ are transitive follows

$$(0.1) \qquad (\exists w \in V)\ u <_{\mathbf{x}} v\ \vee\ u <_{\mathbf{y}} w <_{\mathbf{x}} v\ \vee\ u <_{\mathbf{x}} w <_{\mathbf{y}} v\ \vee\ u <_{\mathbf{y}} v$$

Let $u\ \widetilde{\alpha}^3_{\mathbf{x}+\mathbf{y}}\ v$. From (0.1) we have

$$\begin{aligned} u\ \widetilde{\alpha}^3_{\mathbf{x}+\mathbf{y}}\ v \ &\Rightarrow\ (\exists w_1 \in V_1) u\ \widetilde{\alpha}^2_{\mathbf{x}+\mathbf{y}}\ w_1\ \widetilde{\alpha}_{\mathbf{x}+\mathbf{y}}\ v \\ &\Rightarrow\ (\exists w_1 \in V)\,((\exists w \in V)(u <_{\mathbf{x}} w_1) \vee (u <_{\mathbf{y}} w <_{\mathbf{x}} w_1) \\ &\quad\ \vee (u <_{\mathbf{x}} w <_{\mathbf{y}} w_1) \vee (u <_{\mathbf{y}} w_1))) \wedge ((w_1 <_{\mathbf{x}} v) \vee (w_1 <_{\mathbf{y}} v)) \\ &\Rightarrow\ (\exists w, w_1 \in V)\,((u <_{\mathbf{x}} w_1 <_{\mathbf{x}} w) \vee (u <_{\mathbf{x}} w_1 <_{\mathbf{y}} w) \\ &\quad\ \vee (u <_{\mathbf{y}} w <_{\mathbf{x}} w_1 <_{\mathbf{x}} v) \vee (u <_{\mathbf{y}} w <_{\mathbf{x}} w_1 <_{\mathbf{y}} v) \\ &\quad\ \vee (u <_{\mathbf{x}} w <_{\mathbf{y}} w_1 <_{\mathbf{x}} v) \vee (u <_{\mathbf{x}} w <_{\mathbf{y}} w_1 <_{\mathbf{y}} v) \\ &\quad\ \vee (u <_{\mathbf{y}} w_1 <_{\mathbf{x}} w \vee u <_{\mathbf{y}} w_1 <_{\mathbf{y}} w))\,. \end{aligned}$$

Since the relations $<_{\mathbf{x}}$ and $<_{\mathbf{y}}$ are transitive, we have

$$\begin{aligned} (\exists w, w_1 \in V)\quad & (u <_{\mathbf{x}} v) \vee (u <_{\mathbf{x}} w_1 <_{\mathbf{y}} v) \vee (u <_{\mathbf{y}} w <_{\mathbf{x}} v) \\ & \vee (u <_{\mathbf{y}} w <_{\mathbf{x}} w_1 <_{\mathbf{y}} v) \vee (u <_{\mathbf{x}} w <_{\mathbf{y}} w_1 <_{\mathbf{x}} v) \\ & \vee (u <_{\mathbf{x}} w <_{\mathbf{y}} v) \vee (u <_{\mathbf{y}} w_1 <_{\mathbf{x}} v) \vee (u <_{\mathbf{y}} v). \end{aligned}$$

From Proposition 3 and since there are not cycles in $G_{\mathbf{x}+\mathbf{y}}$,

$$\begin{aligned} (\exists w, w_1 \in V)\quad & (u <_{\mathbf{x}} v) \vee (u <_{\mathbf{x}} w_1 <_{\mathbf{y}} v) \vee (u <_{\mathbf{y}} w <_{\mathbf{x}} v) \vee (u <_{\mathbf{y}} v) \\ & \vee (u <_{\mathbf{x}} w <_{\mathbf{y}} w_1 <_{\mathbf{x}} v) \vee (u <_{\mathbf{x}} w <_{\mathbf{y}} v) \vee (u <_{\mathbf{y}} w_1 <_{\mathbf{x}} v). \end{aligned}$$

So, we obtain

$$(0.2) \qquad (\exists w, w_1 \in V)(u\ \widetilde{\alpha}^2_{\mathbf{x}+\mathbf{y}}\ v) \vee (u <_{\mathbf{x}} w <_{\mathbf{y}} w_1 <_{\mathbf{x}} v).$$

At the end, let $u\ \widetilde{\alpha}^4_{\mathbf{x+y}} v$. From (0.1) and (0.2) we have

$$\begin{aligned} u\ \widetilde{\alpha}^4_{\mathbf{x+y}} v &\Rightarrow (\exists w_2) u\ \widetilde{\alpha}^3_{\mathbf{x+y}}\ w_2\ \wedge\ w_2\ \widetilde{\alpha}_{\mathbf{x+y}}\ v \\ &\Rightarrow (\exists w, w_1, w_2 \in V)((u\ \widetilde{\alpha}^2_{\mathbf{x+y}}\ w_2) \vee (u <_{\mathbf{x}} w <_{\mathbf{y}} w_1 <_{\mathbf{x}} w_2)) \\ &\quad \wedge (w_2 <_{\mathbf{x}} v \vee w_2 <_{\mathbf{y}} v) \\ &\Rightarrow (\exists w, w_1, w_2 \in V)((u\ \widetilde{\alpha}^3_{\mathbf{x+y}}\ v) \vee (u <_{\mathbf{x}} w <_{\mathbf{y}} w_1 <_{\mathbf{x}} v) \\ &\quad \vee (u <_{\mathbf{x}} w <_{\mathbf{y}} w_1 <_{\mathbf{x}} w_2 <_{\mathbf{y}} v). \end{aligned}$$

From Proposition 3 and because there are not cycles in $G_{\mathbf{x+y}}$ we obtain

$$u\ \widetilde{\alpha}^4_{\mathbf{x+y}} v \Rightarrow u \widetilde{<}^3_{\mathbf{x+y}} v \vee u \widetilde{<}^2_{\mathbf{x+y}} v$$

Which means that $\widetilde{\alpha}^4_{\mathbf{x+y}} \subseteq \widetilde{\alpha}_{\mathbf{x+y}} \cup \widetilde{\alpha}^2_{\mathbf{x+y}} \cup \widetilde{\alpha}^3_{\mathbf{x+y}}$. ■

This theorem give a way to obtain transitive closure on the adjacency relation and that relation is accessibility relation. If we found accessibility relation than we can find according accessibility matrix. That is importance of this theorem and can be seen in following corollary.

Corollary 3 *Let* $\mathbf{x}$ *is a minimal path vector to level* d *and* $\mathbf{y}$ *is a minimal path vector then* $A_{\mathbf{x+y}} = \widetilde{A}_{\mathbf{x+y}} \oplus \widetilde{A}^2_{\mathbf{x+y}} \oplus \widetilde{A}^3_{\mathbf{x+y}}$.

Corollary 4 *If* $G_{\mathbf{x+y}}$ *have a cycle, then* $A_{\mathbf{x+y}}$ *has element* $a_{uu} = 1$.

3. Algorithm for networks with component capacity state set {0,1,2, ..., M_i}

In this section we give an algorithm that works for networks with capacity state set of the i-components {0,1,2, ..., M_i}. Steps of the algorithm:

1) Obtaining all binary minimal path vectors.
2) Find the matrix $A_{\mathbf{x}}$, for each binary minimal path vector $\mathbf{x}$.
3) Initialize $\mathbf{MPV}^{'}_{d+1} = \emptyset$.
4) Find the matrix $\widetilde{A}_{\mathbf{x+y}}$, for each minimal path vector $\mathbf{x}$ of level d and each binary minimal path vector $\mathbf{y}$.
5) $A_{\mathbf{x+y}} = \widetilde{A}_{\mathbf{x+y}} \oplus \widetilde{A}^2_{\mathbf{x+y}} \oplus \widetilde{A}^3_{\mathbf{x+y}}$
6) If all the elements on the diagonal of $A_{\mathbf{x+y}}$ are 0, then $\mathbf{MPV}^{'}_{d+1} = \mathbf{MPV}^{'}_{d+1} \cup \{\mathbf{x+y}\}$
7) The set $\mathbf{MPV}_{d+1}$ is obtain from the set $\mathbf{MPV}^{'}_{d+1}$ by elimination the element that appear more then once.

8) Repeat steps 3, 4, 5,6 for all $d \leq M$.

The following example illustrates the work of the algorithm

Example 4 Let us have the network in Figure 4.

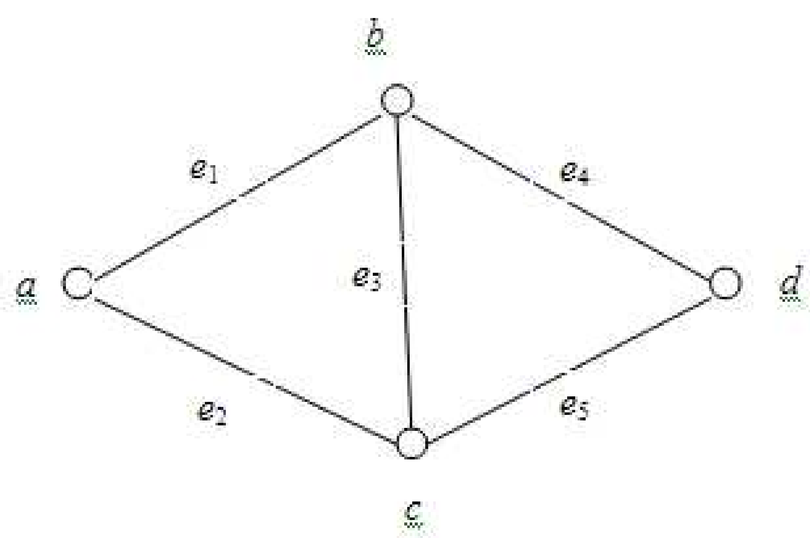

Figure 4

The set of minimal path vectors to level 1 is

$$\mathbf{MPV}_1 = \{(1,0,1,0,1),(1,0,0,1,0),(0,1,1,1,0),(0,1,0,0,1)\}$$

In the Table 1 are given minimal path vectors to level 2. From the table,we can see that accessibility matrix $A_{\mathbf{x}+\mathbf{y}}$ for the vector $\mathbf{x}+\mathbf{y}$=(1,0,1,0,1) +(0,1,1,1,0) = (1,1,2,1,1) have element 1 on the diagonal. The vector (1,1,2,1,1) is not a minimal path vector because it is greater than the vector (1,1,0,1,1).

$\mathbf{MPV}_1$	$\mathbf{MPV}_1$	$A_{\mathbf{x+y}}$	$\mathbf{MPV}_2$
1 0 1 0 1	1 0 1 0 1	0 1 1 1 0 0 1 1 0 0 0 1 0 0 0 0	2 0 2 0 2
1 0 1 0 1	1 0 0 1 0	0 1 1 1 0 0 1 1 0 0 0 1 0 0 0 0	2 0 1 1 1
1 0 1 0 1	0 1 1 1 0	0 1 1 1 0 1 1 1 0 1 1 1 0 0 0 0	1 1 2 1 1 not $\mathbf{MPV}_2$
1 0 1 0 1	0 1 0 0 1	0 1 1 1 0 0 1 1 0 0 0 1 0 0 0 0	1 1 1 0 2
1 0 0 1 0	1 0 0 1 0	0 1 0 1 0 0 0 1 0 0 0 0 0 0 0 0	2 0 0 2 0
1 0 0 1 0	0 1 1 1 0	0 1 1 1 0 0 0 1 0 1 0 1 0 0 0 0	1 1 1 2 0
1 0 0 1 0	0 1 0 0 1	0 1 1 1 0 0 0 1 0 0 0 1 0 0 0 0	1 1 0 1 1
0 1 1 1 0	0 1 1 1 0	0 1 1 1 0 0 0 1 0 1 0 1 0 0 0 0	0 2 2 2 0
0 1 1 1 0	0 1 0 0 1	0 1 1 1 0 0 0 1 0 1 0 1 0 0 0 0	0 2 1 1 1
0 1 0 0 1	0 1 0 0 1	0 0 1 1 0 0 0 0 0 0 0 1 0 0 0 0	0 2 0 0 2

Table 1: Minimal path vectors to level 2

4. Conclusion

In this paper we proposed algorithm for obtaining minimal path vector. Advantage of this algorithm is that with this algorithm only minimal path vectors are obtained. In this algorithm, we check that all elements on diagonal on the accessibility matrix are zero. This control take less time than operation comparing of the vector , what are used for eliminating on the candidate for minimal path vector which are not minimal. Now, we want to consider complexity of the set of minimal path vector. Candidate for minimal path can be each subsets of set of nodes on the graph, i.e each ordered m-tuple of nodes, where —V— is a number of nodes. Since, the complexity of the set of minimal path vectors is O($2^{|V|}$).

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