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r-Adaptive Method for Mesh Improvements Using Directly Magnetic Flux Density as an Error Norm Estimator

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Abstract

'A posteriori' error estimation technique using directly magnetic flux density at each node in correlation with previously proposed method for error norm estimation by Zienkiewicz and Zhu is presented in this paper. The unknown exact solution for magnetic flux density at each node for error norm estimation is being replaced with the approximated value computed inexpensively by means of a simple averaging procedure. In this paper, the effectiveness of the presented error estimator for finite element mesh improvements utilizing r-adaptive method is investigated and its usefulness is evaluated using a simple test model.

1. Introduction

Controlling the mesh density in the areas where the physical quantity of interest is changing rapidly is very important in finite element analysis in order to reduce the computation time and to improve the accuracy of the results. For this purpose, various adaptive methods for mesh improvements have been investigated [1], [2], [3]. Recently, posterior error estimators in connection with adaptive methods for mesh generation and mesh improvements using the continuity of the tangential component of electric field [2], the normal component of magnetic flux density [3] and the uniformity of magnetic energy [4] have been reported. However, error estimation must be independent of the type and class of problems solved and must operate uniformly over materials with different electromagnetic properties.

The main idea is to obtain accurate values of a physical quantity which is of the primary importance for the analysts. For example, for magnetic field problems it is desirable to estimate the error norm using directly magnetic flux density values, not magnetic potential.

In this paper, we applied the estimation of the error norm using directly magnetic flux density values to improve the quality of the finite element solution by means of r-adaptive method in magnetic field computation. The usefulness of the proposed approach is verified using numerical data for a simple test model.

2. Outline of the proposed algorithm

In Fig. 1, an outline of the proposed r-adaptive algorithm is presented. After the

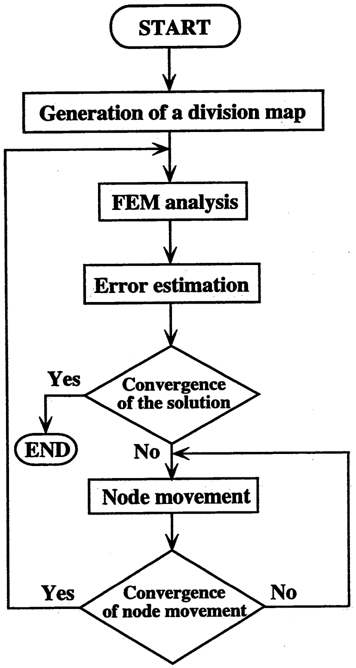


Fig. 1 Flow chart.

element method is performed. Next, the error estimation is carried out and the conve gence of the solution is investigated by computing the convergence of the error nor Then, node movement according to the results of the error estimation and the values magnetic flux density is executed continuously until the largest movement of each no separately becomes less than 1 % of the size of the objective area. Finally, finite el ment analysis is performed again, this time using a newly generated division map. Wh the convergence of the finite element solution is obtained, the iteration process is stoppe

3. Error norm estimation

An error norm $||e||_e$ for arbitrary finite element e is calculated by the following equati

$$||e||_e = \sqrt{\int_{\Omega_e} (\bar{\mathbf{B}} - \mathbf{B})^T \cdot \nu(\bar{\mathbf{B}} - \mathbf{B}) d\Omega_e} \ , \qquad ($$

where $\bar{\mathbf{B}}$ is the exact value of magnetic flux density, $\mathbf{B}$ is the approximated value of magnetic flux density calculated directly from the finite element solution $\mathbf{A}$, such that $\mathbf{B} = \nabla \times \mathbf{A}$, ν is the reluctivity and Ω_e is the area of element e.

However, since the exact value of magnetic flux density $\bar{\mathbf{B}}$ is unknown, we replace the exact value with some approximated value of magnetic flux density. Initially, an average value of magnetic flux density $\hat{\mathbf{B}}$ is computed at each node i according to the following equation

$$\hat{\mathbf{B}}_i = \frac{1}{L}\sum_{j=1}^{L} \mathbf{B}_{ej} , \qquad (2)$$

where $\mathbf{B}_{ej}$ is the magnetic flux density of element j and L is the total number of elements that has node i in their list of nodes. Next, the exact value of magnetic flux density $\bar{\mathbf{B}}$ is approximated using the following expression

$$\bar{\mathbf{B}} \approx \sum_{i=1}^{m} N_i \hat{\mathbf{B}}_i , \qquad (3)$$

where m is the total number of nodes per element and N_i is the shape function for node i.

4. Application of the proposed r-adaptive method

The movement of each finite element node is performed according to the following equations

$$x_N^{(k)} = \frac{\sum_{i=1}^{L} x_{Ci}^{(k-1)} \, ||e||_i^{(k-1)}}{\sum_{i=1}^{L} ||e||_i^{(k-1)}} \quad ; \quad y_N^{(k)} = \frac{\sum_{i=1}^{L} y_{Ci}^{(k-1)} \, ||e||_i^{(k-1)}}{\sum_{i=1}^{L} ||e||_i^{(k-1)}} , \qquad (4)$$

where $x_N^{(k)}$ and $y_N^{(k)}$ are x and y coordinates for each node at k-th iteration, while $x_{Ci}^{(k-1)}$ and $y_{Ci}^{(k-1)}$ are x and y coordinates at iteration $(k-1)$ for the center of each element i which surrounds node N. Node movement is performed in turns starting from node number 1 till the last node of the finite element mesh until the process converges. It is important to notice that all boundary nodes can move only along the boundary line in order to preserve the physical properties and the geometry of the analyzed domain.

The error norm $||e||_i^{(k)}$ after the k-th node movement is performed is estimated according to the following equation

$$||e||_i^{(k)} = \left(\sqrt{\frac{S_k}{S_0}}\right)^{n-p+1} \left(\sqrt{\frac{S_k}{S_0}}\right)^{\frac{D}{2}} ||e||_j^{(0)} . \qquad (5)$$

In the above equation S_0 and S_k are the initial finite element area and the finite element area after performing the k-th node movement respectively, n is the rank of the differential operator of the error norm, p is the order of the shape functions, and D is the dimension of the problem (e.g. for two-dimensional problem $D = 2$) [5].

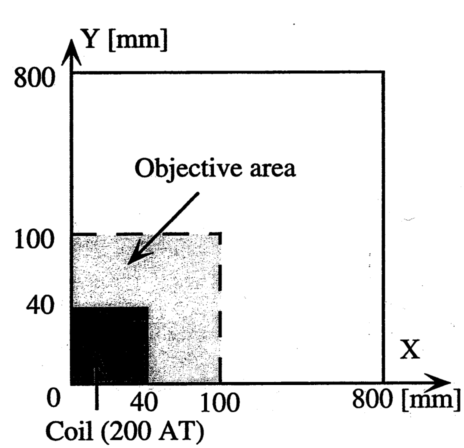


Fig. 2 Application model.

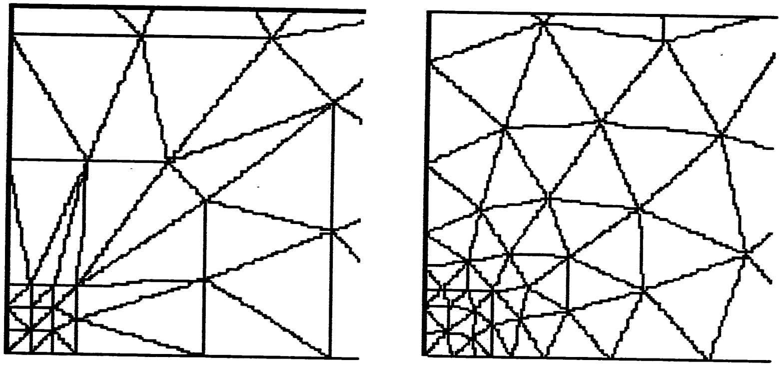
5. Application model and results

In order to demonstrate the usefulness of the proposed method, we treat a simple moc shown in Fig. 2 with theoretically known solution. The initially developed division m: and the division map obtained after using the r-adaptive method with the proposed err norm estimator are shown in Fig. 3. The number of iterations in Eq. 4 was $k = 12$.

In Fig. 4 the comparison between the numerically obtained results for the initial me and for the final mesh are given. It can be seen from Fig. 4 that in case of the initi mesh more than 60 % of the objective area (see Fig. 2) has absolute error larger than ([Gauss]. At the same time, using the proposed method which directly utilizes the valu of magnetic flux density as an error norm estimator less than 10 % of the objective ar has the absolute error larger than 0.5 [Gauss]. Therefore, the proposed r-adaptive meth using directly magnetic flux density as an error norm estimator provides more accura results with less computational effort.

6. Conclusion

We proposed a new r-adaptive method that utilizes directly the physical variables whi are of primary importance for the analysts. We utilized directly magnetic flux densi as an error norm estimator in 2-D magnetic field analysis using r-adaptive finite eleme method. In the future, we intend to expand the proposed method by its connection wi h-adaptive method, therefore developing a mixed r- and h-adaptive method again bas on direct use of magnetic flux density as an error norm estimator.



a) Initial division map

b) Final division map

Fig. 3 Division maps.

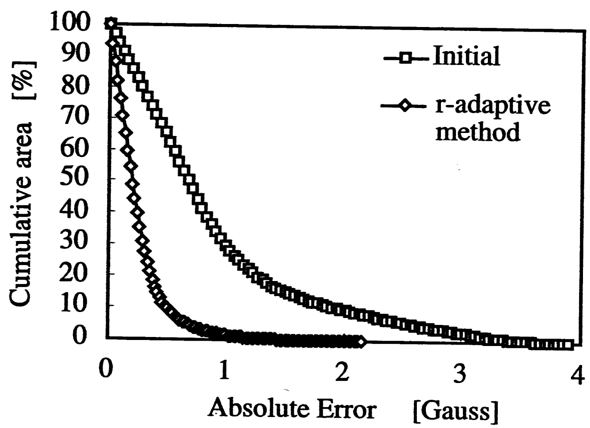


Fig. 4 Cumulative area vs. absolute error.

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