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DTC-SBTC MODEL: OPTIMAL POLICIES, WAGE PREMIUM AND CREDIT RATIONING

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Abstract

In this paper: Financial frictions distort the direction of technical change, credit rationing plus wage premium jointly determines direction of innovation, credit policy is more powerful than subsidies alone. In skill premium vs skill ratio analysis there are multiple equilibria and the dominant stand here is that: Non-monotonic skill demand generates multiple steady states. Multiple equilibria with wage premium and financial frictions there are indeed multiple equilibria but not all of them are stable. And in the skill premium vs skill ratio analysis there are more stable equilibria. Ramsey-Mirrlees-Pigou in DTC-SBTC model analysis showed that taxes (environmental and income) are decreasing skill premium while clean subsidies are increasing inequality. Effect of carbon taxes is ambiguous and optimal subsidy: higher carbon tax lowers optimal clean subsidy, higher financial frictions or worse credit conditions, lower optimal clean subsidy, effects of social preferences are ambiguous. If clean sector creates more jobs government may increase subsidies, if subsidies may benefit skilled workers or capital owners, government may reduce subsidies to limit inequality. Policy tools interact, taxes substitute subsidies, credit frictions weaken them, and inequality reshapes them.

Keywords: Directed Technological change, Skill-Biased Technical Change, Wage inequality, Credit rationing, Carbon Taxes, Clean Subsidies

JEL:H2, H23, J31, N5, O3, O44

1.Introduction

This paper will utilize canonical CES model of skill differentials by Katz, L. F., & Murphy, K. M. (1992), to find equilibrium with CES function[1]. Central model in this paper also is Directed technological change model by Acemoglu et al. (2012),i.e. DTC-AABH model. Economists have long before recognized that the direction of innovation responds to economic incentives see Hémous, D., & Olsen, M. (2021) and Hicks J. (1932), Kennedy (1964)[2]. Induced innovation is macroeconomic hypothesis first proposed by John Hicks see Hicks (1932),where Hicks proposed: "a change in the relative prices of the factors of production is itself a spur to invention, and to invention of a particular kind—directed to economizing the use of a factor which has become relatively expensive. Hicks (1932) also wrote:" A change in the relative prices of the factors of production is itself a spur to invention, and

[1] The Katz-Murphy model (1992) is a seminal economic framework explaining U.S. wage inequality through a supply-and-demand. It shows that rising college wage premiums from 1963–1987 were driven by steady demand growth for skilled labor outpacing its supply, and that fluctuations in skill supply explain decade-to-decade changes in wage inequality

[2] There is large literature on the determinants of aggregate technical progress exists and which does not address the issues of direction and bias of technological change exists see Romer (1990), Aghion, Howitt (1992), Grossman, Helpman, (1991).

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to invention of a particular kind- directed to economizing the use of a factor which has become relatively expensive, see Acemoglu et al.(2015). Key tenets of the "New view" were: that it would be more plausible to assume that technological progress is localized and improves the productivity of techniques currently being used and some techniques with neighboring capital-labor ratios, rather than all other techniques regardless of how far are they from current practice[3] Hicks's orthodoxy was followed by Drandakis Phelps (1966),Samuelson (1965), Ahmad, S. (1966). Skill-Biased Technical Change is a shift in the production technology that favors skilled over unskilled labor by increasing its relative productivity and, therefore, its relative demand, see Violante (2008). Skill-Biased technical change (SBTC hereafter) is a shift in production technology that favors skilled over unskilled labor, by increasing its relative productivity , and therefore its relative demand[4]. SBTC induces a rise in the skill premium $\frac{w_h}{w_l}$, the ratio of skilled to unskilled labor. Technical change is viewed as a change in production function i.e. change in given output for given inputs. Measure for technical change introduces by Solow (1957),is aggregate total factor productivity (TFP). Solow defines total factor productivity as an increase in output that leaves MRT [5]untouched for a given input; thus a change in TFP is a form of factor-neutral technical change[6]. Blackorby et al.(1976), defines Hicks neutrality (HN), implicit Hicks neutrality(IHN), and extended Hicks neutrality (EHN)[7].To investigate policies clean subsidies and taxes we build on endogenous growth model building on two sector model in Acemoglu et al. (2012). Many other papers build on this model as it allows investigation of policies that affect production and research, see Greaker et al.(2018),Acemoglu et al.(2014), Durmaz, T., & Schroyen, F. (2020), Lemoine, D. (2017). Credit rationing in the context of a directed technological change model refers to situations where financial constraints prevent firms from borrowing enough to invest in new technologies, despite being willing to pay the prevailing interest rate. Credit rationing does not just affect the direction of change but also the overall pace of innovation. If firms are constrained from borrowing, the overall rate of technological improvement and, consequently, economic growth can be diminished. Here we apply Stiglitz-Weiss (1981) credit rationing model. One study by Guiso, L. (1998),finds that: Innovative firms undertake high-risk-high-return projects which are likely to be little understood by financial intermediaries. As a consequence, they may end-up allocating too large a share of funds to traditional, low-risk-low-return projects. Credit rationing asymmetry is at play here: because of adverse selection (banks cannot distinguish between high and low-risk borrowers) and moral hazard (borrowers may take excessive risks). High-tech projects often suffer from this, as they are hard to collateralize, making lenders reluctant to fund them.There are many studies that use SBTC to explain the expanding wage inequality and wage premium see Juhn et al., (1993), Autor et al.(1998), Autor et al.(2003), Berman et al., (1998), Pi, J., & Zhang, P. (2018). Card, D., & DiNardo, J. E. (2002),found that rise in wage inequality was episodic event in US and it cannot be attributed to SBTC, i.e. they found other mundane candidate factors such as fall in the real value of minimum wage of women. Then to test for optimal policies we will use Ramsey-Mirrlees-Pigou in DTC-SBTC model. We will apply marginal tax rates

[3] Previously it was assumed that technological progress was neutral: Hicks neutral: $\tilde{F}\,[K\,(t), L\,(t), A\,(t)] = A\,(t)\,F\,[K\,(t), L\,(t)]$, Harrod-neutral technological progress: $\tilde{F}\,[K\,(t), L\,(t), A\,(t)] = F\,[K\,(t), A\,(t)\,L\,(t)]$, and Solow neutral technological progress: $\tilde{F}\,[K\,(t), L\,(t), A\,(t)] = F\,[A\,(t)\,K\,(t), L\,(t)]$

[4] Relative demand refers to the ratio of the demand for one good (or service) compared to another, usually expressed as a downward-sloping curve based on relative prices,see

[5] The Marginal Rate of Transformation (MRT) measures the quantity of one good that must be sacrificed to produce an additional unit of another good, representing the opportunity cost along the Production Possibility Frontier (PPF),$MRT = \frac{\Delta Y}{\Delta X}$.

[6] Factor-neutral technical change refers to an advancement in technology that increases the productivity of all factors of production (like labor and capital) at the same rate

[7] Hicks neutrality (HN) requires that the MRS between each pair of inputs be independent of technical change, implicit Hicks neutrality (IHN) requires that the marginal rate of substitution between each pair of inputs be independent of technical change at constant factor proportion, and Extended Hicks neutrality (EHN) refers to a situation in which the production function can be multiplicatively decomposed into a function of inputs only and another function of technical change only, see Blackorby et al.(1976).

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from Mirrlees (1971) (unobservable ability),and Ramsey (1927), optimal taxes rates for different factors, with Pigou (1920) corrective environmental taxes.

2. The Canonical CES model of Skill Differentials

Here we are presenting workhorse model by Katz, L. F., & Murphy, K. M. (1992). with two types of workers skilled and unskilled or high and low education and they are imperfect substitutes. We suppose there are $L(t)$ unskilled and $H(t)$ skilled workers. The production function for the aggregate economy takes the constant elasticity of substitution (CES) form:

equation 1

$$Y(t) = \left[\left(A_l(t)L(t)\right)^{\rho} + \left(A_h(t)H(t)\right)^{\rho}\right]^{\frac{1}{\rho}}$$

Where $\rho < 1$ and $\rho \in (-\infty, 1)$ .Aggregate production function :$Y\ (t)\ =\ \left[\left(A_l(t)L(t)\right)^{\rho}\ +\ \left(A_h(t)H(t)\right)^{\rho}\ \right]^{\frac{1}{\rho}}$,and elasticity of substitution is:

equation 2

$$\sigma \equiv \frac{1}{1-\rho}, \rho \in (-\infty, 1)$$

σ is %Δ in relative demand for low (high) skill workers per %Δ in relative wage of high (low) skill workers. Three cases are observed here:

σ is %Δ in relative demand for low (high) skill workers per %Δ in relative wage of high (low) skill workers.

- $\sigma\ \rightarrow\ 0\ (or\ \rho\ \rightarrow\ -\infty)$: Skilled and unskilled workers are Leontief[8]. Fixed proportions. ‘Perfect complements’,see Allen, R. G. D. (1968).
- $\sigma\ \rightarrow\ \infty\ (or\ \rho\ \rightarrow\ 1)$: Skilled and unskilled workers are perfect substitutes. Changes in aggregate supplies affect the price of skill overall. Relative wage of skilled vs. unskilled $\left(\frac{w_H}{w_L}\right)$ is constant
- $\sigma\ \rightarrow\ 1\ (or\ \rho\ \rightarrow\ 0)$: Production function is Cobb Douglas, with fixed shares paid to each factor

Key distinction here is:

- $\sigma\ <\ 1$: Gross complements. A reduction in supply of one input reduces demand for the other
- $\sigma\ >\ 1$: Gross substitutes. A reduction in supply of one input raises demand for the other.

A more general production function with skill replacing technologies[9] is given as:

[8] Leontief functions represent fixed-proportion, "perfect complement" relationships where inputs (production) or goods (utility) must be combined in specific, inflexible ratios. . They are characterized by L-shaped isoquants/indifference curves and a function ($q = \min\left\{\frac{z_1}{a}, \frac{z_2}{b}\right\}$), meaning additional units of one input are useless without more of the other.The mathematical form is:$f(z_1, z_2) = \min\left\{\frac{z_1}{a}, \frac{z_2}{b}\right\}$, representing the minimum of the scaled inputs, where q is the quantity of output produced, z_1 and z_2 are the utilised quantities of input 1 and input 2 respectively, and a and b are technologically determined constants.

[9] Skill-replacing technologies refer to advancements that automate, render obsolete, or diminish the value of specific human skills, particularly routine cognitive and manual tasks. Unlike past revolutions that often replaced manual labor, current technological changes—driven by Artificial Intelligence (AI) and robotics—are impacting mid-level, white-collar skills, such as routine writing, basic coding, data entry, and accounting.

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equation 3

$$Y\,(t) \;=\; K^{\alpha}\,\left\{\left(1\;-\;b\,(t)\right)[A_l(t)L(t)+\;B_l(t)]^{\rho}\;+\;b\,(t)[A_h(t)H(t)+\;B_h(t)]^{\rho}\,\right\}^{\frac{1-\alpha}{\rho}}$$

Here, B_l , B_h are directly skill-replacing technologies. And b_t a technology that shifts the allocation of tasks among factors A_l, A_h terms are 'intensive' technical changes, augmenting without reallocating, K is capital: enters in Hicks-neutral form [10] above, no bearing on skill premium. Let's note that if $\sigma\;\rightarrow\;1$, the $b\,(t)$ terms limit to the exponents in the Cobb-Douglas production function

1. Only one good, skilled and unskilled workers are imperfect substitutes in its production.
2. Two-good economy: Consumers have utility function $\left[Y_l^{\rho}\;+\;Y_h^{\rho}\,\right]^{\frac{1}{\rho}}$ with elasticity of substitution $\sigma\;=\;1/\,(1\;-\;\rho)$, Good Y_h is produced with $Y_h\;=\;A_hH$, Good Y_l is produced with $Y_l\;=\;A_lL$.
3. A mixture of these two where different sectors produce goods that are imperfect substitutes, and high and low education workers are employed in all sectors

Since labor market is competitive, wages are set according to their marginal products:

equation 4

$$w_L = \frac{\partial Y}{\partial L} = A_l^{\rho}\left[A_l^{\rho} + A_h^{\rho}\left(\frac{H}{L}\right)^{\rho}\right]^{\frac{1-\rho}{\rho}}$$
$$w_H = \frac{\partial Y}{\partial H} = A_h^{\rho}\left[A_h^{\rho} + A_l^{\rho}\left(\frac{H}{L}\right)^{-\rho}\right]^{\frac{1-\rho}{\rho}}$$

Combining the wage equations to get skill premium π:

equation 5

$$\pi = \frac{w_H}{w_L} = \left(\frac{A_h}{A_l}\right)^{\rho}\cdot\left(\frac{H}{L}\right)^{-(1-\rho)} = \left(\frac{A_h}{A_l}\right)^{\frac{\sigma-1}{\sigma}}\cdot\left(\frac{H}{L}\right)^{-\frac{1}{\sigma}}$$

By taking logs we have:

equation 6

$$\ln\pi = \left(\frac{\sigma-1}{\sigma}\right)\ln\left(\frac{A_h}{A_l}\right) - \frac{1}{\sigma}\ln\left(\frac{H}{L}\right)$$

Now :

equation 7

$$(\partial\ln\pi)\;/\;\partial\ln\left(\frac{H}{L}\right)\;=-\frac{1}{\sigma}<0$$

For given "skill bias," A_h/A_l, an increase in relative supplies $\frac{H}{L}$ lowers relative wages with elasticity σ.On aggregate production functions and where do they come from see: Jones, Charles. (2005),and Houthakker, H. S. (1955), and $\sigma\;\in\;[1,2]$. By differentiation we obtain:

[10] When technology enters in Hicks-neutral form, the production function is typically represented as:$Y = A(t)\cdot F(K,L)$. The marginal product of capital (MPK) and the marginal product of labor (MPL) increase at the same rate. Therefore, the ratio of marginal products,$\frac{MPK}{MPL}$, remains constant for given K/L capital to labor ratio, see Hicks (1963). According to Hicks, a neutral invention is "An invention which raises the marginal productivity of labour and capital in same proportion".

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equation 8

$$\frac{\partial ln\omega}{\partial \ln\left(\frac{A_h}{A_l}\right)} = \frac{\sigma - 1}{\sigma}$$

For an exogenous rise in $\left(\frac{H}{L}\right)$ wage premium $\pi = \frac{w_H}{w_L}$ falls. Wages of unskilled workers rise, wages of skilled workers decrease. Average wage rise provided the skill premium is positive:

equation 9

$$\bar{w} = \frac{Lw_L + Hw_H}{L + H} = \frac{\left[(A_l)^\rho + \left(\frac{A_h H}{L}\right)^\rho\right]^{1/\rho}}{1 + \frac{H}{L}}$$

Increasing $\frac{H}{L}$ provided skill premium is positive: $\pi > 0$, $A_h^\rho \left(\frac{H}{L}\right)^\rho - A_l^\rho > 0$.

- $\omega = W_H / W_L$ rises if $\sigma > 1$, falls if $\sigma < 1$, and is unchanged if $\sigma = 1$
- Average wages rise if σ > 0.
- Wages of L workers rise if $\sigma < \infty$.
- Both W_H and W_L rise if $\sigma \geq 1$.

equation 10

$$ln\pi = \ln\left(\frac{W_h}{W_l}\right) = \left(\frac{\sigma - 1}{\sigma}\right)\ln\left(\frac{A_h}{A_l}\right) - \frac{1}{\sigma}\ln\left(\frac{H}{L}\right)$$

The technological parameter is $\frac{\sigma-1}{\sigma}\ ln\ \left(\frac{A_h}{A_l}\right)$ or simply $(\sigma - 1)\ ln\ \left(\frac{A_h}{A_l}\right)$ since denominator just a scalar

- We can observe $\frac{H}{L}$ and $\pi = ln\frac{W_h}{W_l}$
- If we knew σ, could infer $\Delta\ ln\ \left(\frac{A_h}{A_l}\right)$

This approach pioneered by Katz and Murphy (1992).

equation 11

$$ln\pi = \frac{\sigma - 1}{\sigma}\ln\left(\frac{A_h}{A_l}\right) - \frac{1}{\sigma}\ln\left(\frac{H}{L}\right)$$

Previous was structural equation. Now we will add time subscripts to everything except for σ:

equation 12

$$ln\, \pi_t = \gamma_0 + \gamma_1 t + \gamma_2\, ln\left(\frac{H}{L}\right)_t + e_t\, .$$

In previous Unknowns are σ and $\left(\frac{A_h}{A_L}\right)_t$, γ_0 is a constant, γ_1 gives the time trend on $\left(\frac{\sigma-1}{\sigma}\right)\ ln\left(\frac{A_{Ht}}{A_{Lt}}\right)$, and $\hat{\gamma}_2$ is an estimate of $\frac{1}{\sigma}$.

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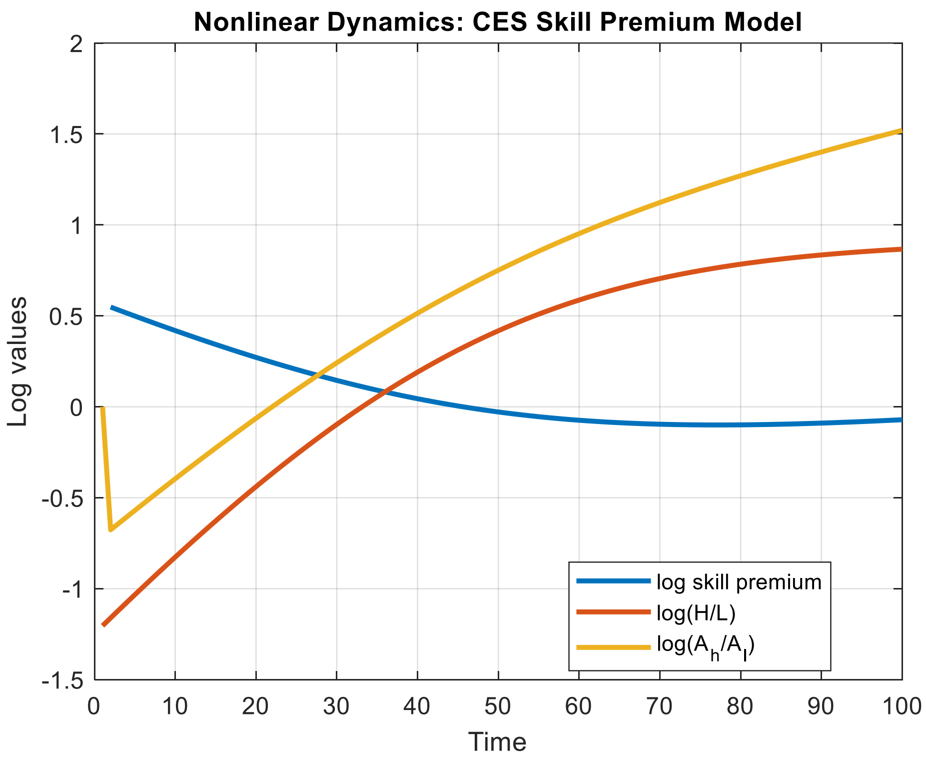
Identification assumptions: (1) $ln\left(\frac{H}{L}\right)t$ is exogenous or quasi-fixed[11];(2) $\partial \frac{ln\left(\frac{A_h}{A_L}\right)}{\partial t}$ is approximately linear. Krusell, P., Ohanian, L. E., Ríos-Rull, J.-V., & Violante, G. L. (2000) (KORRV-model) offer

equation 13

$$G\ (k_{st}, k_{et}, L_t\ , H_t) =\ k_{st}^{\alpha}\left[\beta L_t^{\delta}\ +\ (1\ -\ \beta)\left(\lambda k_{et}^{\rho}\ +\ (1\ -\ \lambda) H_t^{\rho}\ \right)^{\frac{\delta}{\rho}}\right]^{\frac{1-\alpha}{\delta}}$$

k_{st} is structures capital, k_{et} is equipment capital, α is structure share of output (note: Cobb-Douglas) and β is the extensive margin technological parameter, $\sigma_e\ = \frac{1}{(1-\rho)}$ is elasticity between H labor and equipment capital k_e , $\sigma_u\ = \frac{1}{(1-\delta)}$ is elasticity between $H\ +\ K_e$ aggregate and L. Key condition here is $\sigma_u > \sigma_e$, which implies skills-complementarity. If elasticity between $H\ +\ K_e$ versus L greater than elasticity between K_e and H, then Ke is a relative complement to H.

Figure 1 Nonlinear Dynamics: CES Skill Premium Model



Source: Author's calculations

[11] In standard empirical labor economics models, particularly the Katz-Murphy (1992) framework used to estimate skill-biased technical change (SBTC), (the relative s upply of skilled to unskilled labor) is typically assumed to be quasi-fixed or treated as exogenous at any given point in time

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2.1 Equilibrium with CES production function

This is based on Krusell, P., Ohanian, L. E., Ríos-Rull, J.-V., & Violante, G. L. (2000) , we are assuming that production requires high skilled labor H , low skilled labor L and capital K:

equation 14

$$Y_t = \left[\mu(e_{lt}L_t)^\sigma + (1-\mu)\big(\lambda K_t^\rho + (1-\lambda)(e_{ht}H_t)^\rho\big)^{\frac{\sigma}{\rho}}\right]^{\frac{1}{\sigma}}$$

Here μ, λ are time invariant distribution parameters, and e_{lt}, e_{ht} are time variant efficiency parameters. Wage premium $\omega = \frac{w_H}{w_L}$ as a function of production function parameters and the ratios of factor supplies is given as:

equation 15

$$w_H = \frac{\partial Y_t}{\partial H_t} = (1-\mu)(1-\lambda)e_{ht}^\rho H_t^{\rho-1}\big(\lambda K_t^\rho + (1-\lambda)(e_{ht}H_t)^\rho\big)^{\frac{\sigma}{\rho}-1} Y_t^{1-\sigma}$$

$$w_L = \frac{\partial Y_t}{\partial L_t} = \mu e_{lt}^\sigma; L_t^{\sigma-1} Y_t^{1-\sigma}$$

equation 16

$$\omega = \frac{w_H}{w_L} = \frac{(1-\mu)(1-\lambda)}{\mu}\frac{e_{ht}^\rho}{e_{lt}^\sigma}\frac{H_t^{\rho-1}}{L_t^{\sigma-1}}\big(\lambda K_t^\rho + (1-\lambda)(e_{ht}H_t)^\rho\big)^{\frac{\sigma}{\rho}-1}$$

$$= \frac{(1-\mu)(1-\lambda)}{\mu}\left(\frac{e_{ht}}{e_{lt}}\right)^\sigma \left(\frac{H_t}{L_t}\right)^{\sigma-1} \left(\lambda \left(\frac{K_t}{e_{ht}H_t}\right)^\rho + (1-\lambda)\right)^{\frac{\sigma}{\rho}-1}$$

If we log-linearize previous:

equation 17

$$\frac{\partial lnX}{\partial t} = \frac{1}{X}\frac{\partial X}{\partial t} = \frac{\dot{X}}{X} = gx$$

$$\frac{\partial \ln\left(\frac{X}{Y}\right)}{\partial t} = \frac{\dot{X}}{X} - \frac{\dot{Y}}{Y} = g_X - g_Y$$

And wage premium equation re expressed:

equation 18

$$\omega = \frac{(1-\mu)(1-\lambda)}{\mu}\left(\frac{e_{ht}}{e_{lt}}\right)^\sigma \left(\frac{H_t}{L_t}\right)^{\sigma-1} \left(\lambda \left[\left(\frac{K_t}{e_{ht}H_t}\right)^\rho - 1\right] + 1\right)^{\frac{\sigma}{\rho}-1}$$

By taking logs we get :

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equation 19

$$
\begin{aligned}
ln\omega &= \ln\left(\frac{(1-\mu)(1-\lambda)}{\mu}\right) + \sigma ln\left(\frac{e_{ht}}{e_{lt}}\right) + (\sigma-1)\ln\left(\frac{H_t}{L_t}\right) + \frac{\sigma-\rho}{\rho} ln\left(\lambda\left[\left(\frac{K_t}{e_{ht}H_t}\right)^{\rho} - 1\right] + 1\right) \\
&\simeq ln\left(\frac{(1-\mu)(1-\lambda)}{\mu}\right) + \sigma ln\left(\frac{e_{ht}}{e_{lt}}\right) + (\sigma-1)\ln\left(\frac{H_t}{L_t}\right) + \frac{\sigma-\rho}{\rho}\left(\lambda\left[\left(\frac{K_t}{e_{ht}H_t}\right)^{\rho} - 1\right]\right) \\
&= ln\left(\frac{(1-\mu)(1-\lambda)}{\mu}\right) + \frac{\sigma-\rho}{\rho}\lambda + \sigma ln\left(\frac{e_{ht}}{e_{lt}}\right) + (\sigma-1)ln\left(\frac{H_t}{L_t}\right) \\
&+ \frac{\sigma-\rho}{\rho}\lambda\left(\frac{K_t}{e_{ht}H_t}\right)^{\rho}
\end{aligned}
$$

If we denote $\frac{\dot{x}}{x} \equiv g_x$ we have:

equation 20

$$
g_v \simeq (1-\sigma)(g_L - g_H) + \sigma(g_{eh} - g_{el}) + (\sigma-\rho)\lambda\left(\frac{K_t}{e_{ht}H_t}\right)^{\rho}(g_K - g_H - g_{el})
$$

Now, looking at the right hand side of the above expression, the first term reflects the influence of changes in relative supply, the second term is the skill-bias effect, reflecting changes in the efficiency of different types of labor, and the final term is the effect of complementarity between capital and skilled labor. Now,we assume that the relative supply of skilled labor is endogenous, that is, responds to the skill premium. In particular, assume

equation 21

$$
\left(\frac{H_t}{L_t}\right) = c\omega^{\theta}, \theta > 1
$$

Here CES is :

equation 22

$$
\frac{\partial \ln\left(\frac{H}{L}\right)}{\partial ln\omega} = \theta
$$

Now we will solve for equilibrium ω:

equation 23

$$
ln\omega = k + \sigma\ln\left(\frac{e_{ht}}{e_{lt}}\right) + (\sigma-1)[lnc + \theta ln\omega] = \frac{1}{1+\theta(1-\sigma)}\left(k + \sigma ln\left(\frac{e_{ht}}{e_{lt}}\right) + (\sigma-1)lnc\right)
$$

Now :

equation 24

$$
\frac{\partial \ln\left(\frac{H}{L}\right)}{\partial ln\omega} = \frac{\sigma}{1+\theta(1-\sigma)} > 0, for\ \sigma > 0
$$

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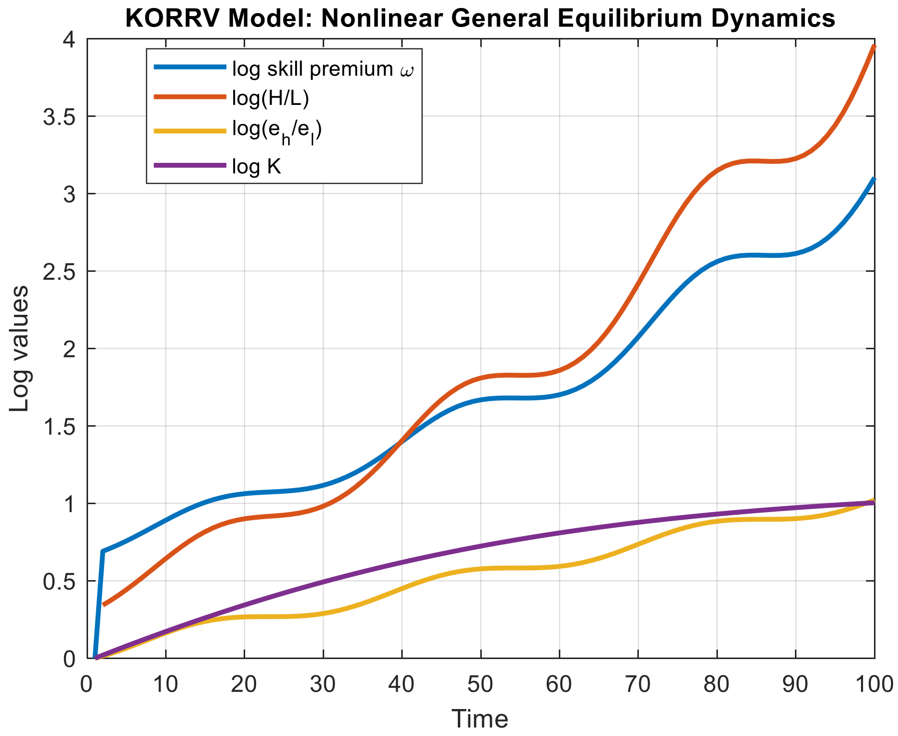
Figure 2 KORRV Model: Nonlinear General Equilibrium Dynamics



Source: Author's calculations

3. DTC-AABH model

This model is due to Acemoglu et. al (2012) . On the production side:

equation 25

$$Y = \left(Y_C^{\frac{\varepsilon-1}{\varepsilon}} + Y_D^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon-1}{\varepsilon}}$$

Final good Y produced competitively with a clean intermediary input Y_c, and a dirty input Y_d.When, $\varepsilon > 1$ two inputs are substitutes. Now, for $j \in \{C, D\}$ input Y_j produced with labor L_j and a continuum of machines x_{ij} :

equation 26

$$Y_j = L_j^{1-\alpha} \int_0^1 A_{ji}^{1-\alpha} x_{ji}^{\alpha} di$$

And : $\sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} u(C_t, S_t)$ where S is environmental stock. And u is increasing and concave, with:

equation 27

$$\lim_{S \to 0} u(C, S) = \infty, \frac{\partial u}{\partial S}(C, \bar{S}) = 0$$

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Production of dirty inputs depletes environmental stock S[12]:

equation 28

$$S_{t+1} = -\xi Y_{dt} + (1 + \delta)S_t, S \in (0, \bar{S})$$

This reflects at the upper bound $\bar{S}(< \infty)$,baseline unpolluted level of environmental quality. Absorbing at lower bound $S = 0$. The parameter ξ measures the rate of environmental degradation resulting from the production of dirty inputs, and δ is the rate of "environmental regeneration."[13]. Scientists at the beginning of each period are working to innovate in clean or dirty sector. In Acemoglu et al.(2012) model each scientist is randomly allocated to one machine in their target sector. Probability of success of each scientist is η_j .If there is successful proportional improvement of quality $\gamma > 0$ each scientist gets one period monopoly rights:$A_{jit} = (1 + \gamma)A_{ijt-1}$. There is knowledge externality in own sector (dirty or clean technologies).]

Assumption 1

inequality 1

$$A_{d0} \gg A_{c0}{}^{14}$$

Laissez-faire equilibrium: Here scientists choose the sector with higher expected profits Π_{jt}:

equation 29

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{\eta_c}{\eta_d}\left(\frac{p_{ct}}{p_{dt}}\right)^{\frac{1}{1-\alpha}}\left(\frac{L_{ct}}{L_{dt}}\right)\ \frac{A_{ct-1}}{A_{dt-1}}$$

Where $\left(\frac{p_{ct}}{p_{dt}}\right)^{\frac{1}{1-\alpha}}$ represents the price effect, $\frac{L_{ct}}{L_{dt}}$ is the market size effect, $\frac{A_{ct-1}}{A_{dt-1}}$ is direct productivity effect. The direct productivity effect pushes towards innovation in the more advanced sector. The price effect towards the less advanced, price effect stronger when ε is smaller. And $\frac{L_{ct}}{L_{dt}}$ market size effect towards the more advanced when $\varepsilon > 1$. Now let :

equation 1

$$\varphi \equiv (1 - \alpha)(1 - \varepsilon)$$

And :

equation 30

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{\eta_c}{\eta_d}\left(\frac{1 + \gamma\eta_c s_{ct}}{1 + \gamma\eta_d s_{dt}}\right)^{-\varphi-1}\left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{-\varphi}$$

Now equilibrium production levels are :

[12] S is general quality of environment, inversely related to CO2 concentration

[13] In ecology regeneration is the ability of an ecosystem – specifically, the environment and its living population – to renew and recover from damage.In this case δ measures amount of pollution that can be absorbed without extreme adverse consequences

[14] This is capturing the fact that currently fossil-fuel technologies are more advanced than alternative energy/clean technologies.

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equation 31

$$Y_d = \frac{1}{(A_c^{\varphi} + A_d^{\varphi})^{\frac{\alpha+\varphi}{\varphi}}} A_c^{\alpha+\varphi} A_d$$

$$Y = \frac{A_c A_d}{\left(A_c^{\varphi} + A_d^{\varphi}\right)^{\frac{1}{\varphi}}}$$

And $\varphi \equiv (1-\alpha)(1-\varepsilon)$. Since $A_{d0} \gg A_{c0}$,Y_d will always grow without bound in laissez-faire. And if $\varepsilon > 1$, then all scientists are directed at dirty technologies, so $g_{Y_d} \to \gamma\eta_d$

Definition 1

An equilibrium is given by sequences of wages (w_t), prices for inputs (p_{jt}), prices for machines (p_{jit}), demands for machines (x_{jit}), demands for inputs (Y_{jt}), labor demands (L_{jt}) by input producers $j \in \{c,d\}$, research allocations (s_{dt}, s_{ct}), and quality of environment (S_t) such that, in each period t: (pjit,xjit) maximizes profits by the producer of machine i in sector j; L_{jt} maximizes profits by producers of input j; Y_{jt} maximizes the profits of final good producers; (s_{dt}, s_{ct}) maximizes the expected profit of a researcher at date t; the wage w_t and the prices p_{jt} clear the labor and input markets respectively; and the evolution of S_t is given by:$S_{t+1} = -\xi Y_{dt} + (1+\delta) S_t$

equation 32

$$\left\{w_t, (p_{jt}) j \in \{c,d\}, (p_{jit}) j \in \{c,d\}, i \in [0,1], (x_{jit}) j, i, (Y_{jt}) j, (L_{jt}) j, (s_{ct}, s_{dt}), S_t\right\}_{t=0}^{\infty}$$

Each monopolistic machine producer (j,i)solves[15]:

equation 33

$$\max_{p_{jit}, x_{jit}} \quad \pi_{jit} = (p_{jit} - \psi)\, x_{jit}$$

Optimal machine pricing (standard Dixit–Stiglitz (1977)) is $p_{jit} = \frac{\psi}{\alpha}$ and machine demand[16]:

equation 34

$$x_{jit} = \left(\frac{\alpha^2 p_{jt} A_{jt}^{1-\alpha}}{\psi}\right)^{\frac{1}{1-\alpha}}$$

Labor demand conditions are:$w_t = (1-\alpha)\, p_{jt} \frac{Y_{jt}}{L_{jt}}$.Input demand condition is:$p_{jit} = \alpha p_{jt} \frac{Y_{jt}}{x_{jit}}$.Researchers allocate effort:$s_{ct} + s_{dt} = 1$with technology evolution:$A_{j,t+1} = A_{jt}(1+\eta s_{jt}), j \in \{c,d\}$and optimal allocation:

equation 35

$$\frac{s_{ct}}{s_{dt}} = \frac{\mathbb{E}_t[V_{ct+1}]}{\mathbb{E}_t[V_{dt+1}]}$$

[15] subject to derived demand from input producers: $x_{jit} \in \arg\max_{x \geq 0} \left\{p_{jt} A_{jt}^{1-\alpha} \left(\int_0^1 x_{jit}^{\alpha}\, di\right) - \int_0^1 p_{jit}\, x_{jit}\, di\right\}$

[16] Here ψ are constant marginal cost of producing one unit of any machine.

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Type equation here.where V_{jt} is the value of an innovation in sector j.Environmental quality evolves as:

equation 36

$$S_{t+1} = (1-\delta)S_t - \phi Y_{dt} + \chi Y_{ct}$$

Lemma 1

Under laissez-faire, it is an equilibrium for innovation at time t to occur in the clean sector only when $\eta_c A_{ct-1}^{-\varphi} > \eta_d(1+\gamma\eta_c)^{(\varphi+1)} A_{dt-1}^{-\varphi}$, in the dirty sector only when $\eta_c(1+\gamma\eta_d)A_{ct-1}^{-\varphi}$, and in both sectors when $\eta_c(1+\gamma\eta_d s_{dt})^{\varphi+1}$, $A_{ct-1}^{-\varphi} = \eta_d(1+\gamma\eta_c s_{ct})^{\varphi+1}$ and $s_{ct} + s_{dt} = 1$

Proof: Let V_{jt}denote the expected value of a successful innovation in sector $j \in \{c,d\}$.Under monopolistic competition and CES demand, the value of a new machine is proportional to:$V_{jt} = \eta_j(A_{jt})^{-\phi}(\Pi_{jt+1})$,where : $\eta_j$is research productivity,$A_{jt}^{-\phi}$reflects diminishing returns to innovation,$\Pi_{jt+1}$is next-period monopoly profit.Productivity evolves:$A_{j,t} = A_{j,t-1}(1+\gamma\eta_j s_{jt})$, where:s_jtis the share of research labor allocated to sector j,γ>0 scales spillovers.Thus next-period profits satisfy:$\Pi_{jt+1} \propto (1+\gamma\eta_j s_{jt})^{\phi+1}$.[17] Researchers allocate effort to maximize expected innovation value: $\max_{s_{ct}, s_{dt}} \; s_{ct}V_{ct} + s_{dt}V_{dt}$ s.t. $s_{ct} + s_{dt} = 1$.This is a linear program, so the solution is a corner unless values are equal. Clean-only innovation ($s_{ct} = 1, s_{dt} = 0$) is optimal if:

inequality 2

$$V_{ct} > V_{dt} \text{ evaluated at } s_{ct} = 1.$$

Substituting:

inequality 3

$$\eta_c A_{c,t-1}^{-\phi} > \eta_d(1+\gamma\eta_c)^{\phi+1} A_{d,t-1}^{-\phi}.$$

Hence innovation occurs only in the clean sector. Similarly, dirty-only innovation ($s_{dt} = 1$) requires:

inequality 4

$$V_{dt} > V_{ct} \text{ evaluated at } s_{dt} = 1,$$

which yields:

inequality 5

$$\eta_d A_{d,t-1}^{-\phi} > \eta_c(1+\gamma\eta_d)^{\phi+1} A_{c,t-1}^{-\phi}.$$

When both sectors receive R&D, the first-order condition requires indifference: $V_{ct} = V_{dt}$
Substituting:

equation 37

$$\eta_c(1+\gamma\eta_d s_{dt})^{\phi+1} A_{c,t-1}^{-\phi} = \eta_d(1+\gamma\eta_c s_{ct})^{\phi+1} A_{d,t-1}^{-\phi},$$

together with feasibility:$s_{ct} + s_{dt} = 1$.This characterizes an interior solution. Because researchers face a linear allocation problem and innovation values depend asymmetrically on sectoral productivity and

[17] This exponent $\phi + 1$arises because profits depend on: productivity growth $(+1)$,market-size effects $(+\phi)$.

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spillovers, equilibrium R&D is:corner (clean or dirty) when one sector dominates in expected value,interior only when expected values are exactly equal.This proves the lemma ■.

Proposition 1

laissez-faire equilibrium always leads to environmental disaster

Proposition 2

When two inputs are strong substitutes $\varepsilon > \frac{1}{1-\alpha}$ and $\bar{S}$ is sufficiently high, a temporary clean subsidy will prevent environmental disaster

Proof of proposition 1 and 2: Under laissez-faire, the equilibrium path satisfies $\lim_{t\to\infty} S_t = -\infty$.Under laissez-faire, R&D is allocated to the sector with higher private innovation profitability:$\Pi_{jt}^{LF} = \eta_j A_{j,t-1}^{-\phi}$.Thus initially:

inequality 6

$$\Pi_{dt}^{LF} > \Pi_{ct}^{LF} \Rightarrow s_{dt} = 1$$

Dirty technology evolves as:$A_{dt} = (1+\gamma)^t A_{d0}$. With CES aggregation:$Y_{dt} \propto A_{dt}^{\frac{1}{1-\alpha}}$. Environmental law of motion states:$S_{t+1} = (1-\delta)S_t - \xi Y_{dt}$.Iterating forward:

equation 38

$$S_t = (1-\delta)^t S_0 - \xi \sum_{k=0}^{t-1} (1-\delta)^{t-k-1} Y_{dk}$$

Since Y_{dk}grows exponentially while $(1-\delta)^{t-k}$decays geometrically, $\sum_{k=0}^{t-1}(1-\delta)^{t-k-1}Y_{dk} \to \infty$.Thus:$\lim_{t\to\infty} S_t = -\infty$.Because environmental damage is unpriced, laissez-faire necessarily implies environmental disaster. ■ For the second proposition statement is given as: If:

inequality 7

$$\varepsilon > \frac{1}{1-\alpha} \text{ and } \bar{S} \text{ is sufficiently high,}$$

then a temporary subsidy to clean R&D suffices to permanently avoid environmental collapse. Final output is given as:$Y_t = \left(Y_{ct}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{dt}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$. Demand ratio:$\frac{Y_{ct}}{Y_{dt}} = \left(\frac{A_{ct}}{A_{dt}}\right)^{\varepsilon}$ .If $\varepsilon > \frac{1}{1-\alpha}$, then small clean productivity gains cause large market shifts.Now we introduce subsidy $\sigma_c > 0$for $t \le T$:

equation 39

$$\Pi_{ct}^{sub} = (1+\sigma_c)\eta_c A_{ct-1}^{-\phi}$$

Choose σ_c so that:$(1+\sigma_c)\eta_c A_{ct-1}^{-\phi} > \eta_d A_{dt-1}^{-\phi}$Thus:$s_{ct} = 1 \forall t \le T$. After Tperiods:$\frac{A_{cT}}{A_{dT}} = (1+\gamma)^T \frac{A_{c0}}{A_{d0}}$With strong substitutability:

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inequality 8

$$\frac{Y_{cT}}{Y_{dT}} \gg 1$$

Hence even without subsidy:

inequality 9

$$\Pi_{ct}^{LF} > \Pi_{dt}^{LF} \forall t > T$$

The economy endogenously locks into clean innovation. Dirty output converges to zero:$Y_{dt} \to 0$.Thus:

inequality 10

$$S_{t+1} = (1-\delta)S_t \Rightarrow S_t \to \bar{S} > 0$$

A temporary clean subsidy permanently redirects technical change and prevents environmental disaster when inputs are sufficiently substitutable.

■

Profits (up to proportionality):

equation 40

$$\Pi_c(x) = A_c(x)^{\frac{\alpha}{1-\alpha}},$$
$$\Pi_d(x) = \frac{A_d(x)^{\frac{\alpha}{1-\alpha}}}{(1+\tau)^{\frac{1}{1-\alpha}}}$$

we will rewrite before use diminishing returns: Use diminishing returns to research:

equation 41

$$\dot{A}_c = \gamma\eta_c x^{\phi} A_c \dot{A}_d = \gamma\eta_d (1-x)^{\phi} A_d \text{with } 0 < \phi < 1$$

Expected research returns:

equation 42

$$R_c(x) = \eta_c x^{\phi-1} A_c^{\frac{\alpha}{1-\alpha}}$$
$$R_d(x) = \eta_d (1-x)^{\phi-1} \frac{A_d^{\frac{\alpha}{1-\alpha}}}{(1+\tau)^{\frac{1}{1-\alpha}}}$$

The equilibrium share of scientists is where marginal social profitability of ideas is equalized across technologies[18]. At each time t expected returns in clean and dirty technologies are given as:

[18] The marginal social profitability of ideas refers to the net benefit to society from producing or utilizing one additional unit of an idea, innovation, or piece of information. Unlike physical goods, ideas are often non-rivalrous, meaning their use by one person does not prevent use by others, often leading to a significant gap between the private gains of the creator and the total benefit to society.

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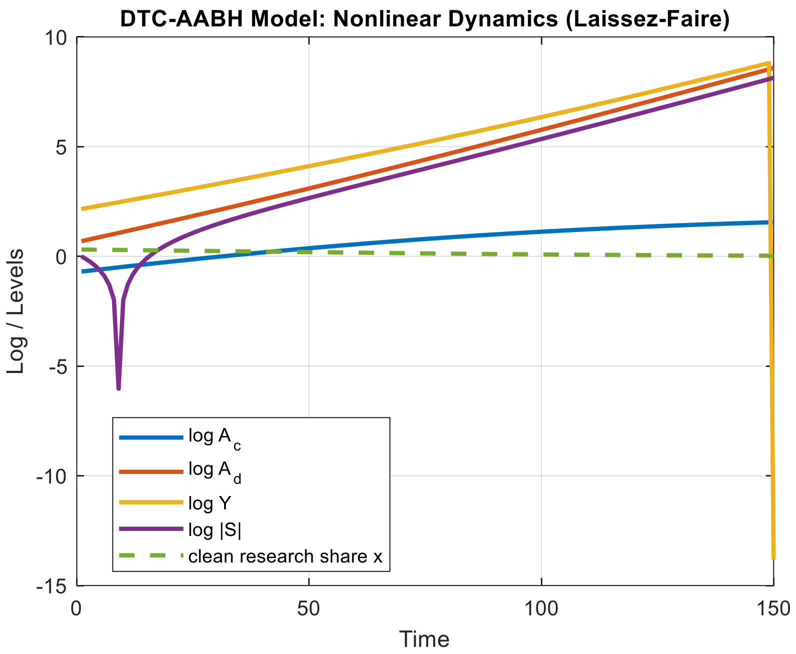
equation 43

$$R_{c,t}(x) = V_{c,t}(1 - e^{-\eta_c x})$$
$$R_{d,t}(x) = \frac{V_{d,t}}{(1+\tau_t)^{\frac{1}{1-\alpha}}}\left(1 - e^{-\eta_d(1-x)}\right)$$

Figure 3 DTC-AABH Model: Nonlinear Dynamics (Laissez-Faire)



Source: Author's calculations

4.Credit rationing model

This model is due to Stiglitz, J. E., & Weiss, A. (1981).Mean preserving spread [19]conditions are :

inequality 11

$$\int_0^\infty Rf(R,\theta_1)dR = \int_0^\infty Rf(R,\theta_2)dR\,, y \geq 0,$$
$$\int_0^y F(R,\theta_1)dR \geq \int_0^y F(R,\theta_2)dR$$

Where : $R \sim F(R,\theta), \theta = riskiness$.Borrower payoff and bank payoff are given as:

equation 44

$$\pi(R,\hat{r}) = \max(R - (1+\hat{r})B - C) - return\ of\ net\ borrower$$
$$\rho(R,\hat{r}) = \min\left(R + C; B(1+\hat{r})\right) - return\ to\ the\ bank$$

The value of $\hat{\theta}$ for which expected profits are zero satisfies:

[19] A random variable Y is a mean-preserving spread of X if has the same mean as X , but more probability weight is in the tails, see Rothschild, M., & Stiglitz, J. E. (1970)

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equation 45

$$\Pi(\hat{r},\hat{\theta}) \equiv \int_0^{\infty} \max[R-(\hat{r}+1)B;-C]dF(R,\hat{\theta})=0$$

Where C is collateral. Differentiating previous gives:

equation 46

$$\frac{d\hat{\theta}}{d\hat{r}} = \frac{B\int_{(1+\hat{r})B-C}^{\infty} dF(R,\hat{\theta})}{\frac{\partial\Pi}{\partial\hat{\theta}}}$$

Key result is: $\frac{d\hat{\theta}}{d\hat{r}} > 0$ Higher interest rates → only riskier firms borrow. Now, $\rho(\theta,r)$ - expected return to the bank $G(\hat{\theta})$ distribution of projects by riskiness, θ riskiness

equation 47

$$\tilde{\rho}(\hat{r}) = \frac{\int_{\tilde{\theta}_{\hat{r}}}^{\infty} \rho(\theta,\hat{r})dG(\theta)}{1-G(\hat{\theta})}$$

inequality 12

$$\frac{d\bar{\rho}(\hat{r})}{d\hat{r}} < 0$$

equation 48

$$\rho(\hat{\theta},\hat{r}) = \hat{\rho}$$

$$\frac{d\bar{\rho}}{d\hat{r}} = -\frac{g(\hat{\theta})}{[1-G(\hat{\theta})]}(\hat{\rho}-\bar{\rho})\frac{d\hat{\theta}}{d\hat{r}} + \frac{\int_{\hat{\theta}}^{\infty}[1-F((1+\hat{r})B-C,\theta)]dG(\theta)}{1-G(\hat{\theta})}$$

equation 49

$$p(R)R+[1-p(R)]D=T$$

Key result is $\frac{d\bar{\rho}}{d\hat{r}} < 0$,meaning: Banks do not increase interest rates beyond a point, because: Higher r⇒ worse borrower pool.

4.1 Credit rationing added to DTC-AABH model

Now we will replace :

equation 50

$$p_c = p + \psi$$

With endogenous function**:**

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equation 51

$$p_d(\hat{r}_d) = 1 - G(\hat{\theta}(\hat{r}_d))$$
$$p_c(\hat{r}_c) = 1 - G(\hat{\theta}(\hat{r}_c))$$

Table 1 Mapping Stiglitz-Weiss, DTC-AABH

Stiglitz–Weiss	**DTC-AABH**
(θ) (risk)	firm/project type
($\hat{r}$)	sectoral interest rate
credit rationing	(p_c, p_d)
adverse selection	$\Gamma(\theta)$ distortion

Source: Author's calculations

Value functions (fully microfounded)

equation 52

$$V_c = \frac{p_c(\hat{r}_c)\,\pi_c}{\hat{r}_c + \delta}$$
$$V_d = \frac{p_d(\hat{r}_d)\,\pi_d}{\hat{r}_d + \delta}$$

Modified research equation is given as:

equation 53

$$R_c = \frac{1}{2} + \frac{1}{2}\tanh(\Gamma(\theta) + \underbrace{\log\frac{p_c(\hat{r}_c)}{p_d(\hat{r}_d)}}_{\text{credit rationing}})$$

Interest rate affects innovation via: $\hat{r} \uparrow \Rightarrow \hat{\theta} \uparrow \Rightarrow p_c \downarrow \Rightarrow V_c \downarrow \Rightarrow R_c \downarrow \Rightarrow \dot{\theta} \downarrow$.

Theorem 1

In the presence of asymmetric information (Joseph Stiglitz–Andrew Weiss):

inequality 13

$$\frac{d\bar{\rho}}{dr} < 0 \;\Rightarrow\; \frac{dR_c}{dr} < 0$$

proof: In credit market expected bank return is : $\bar{\rho}(r) = \rho(r, \theta(r))$,where:$r$: interest rate ,$\theta(r)$: endogenous borrower risk (increasing in r).Clean research share is: $R_c = \frac{1}{2} + \frac{1}{2}\tanh(\Gamma(\theta) + \underbrace{\log\frac{p_c(\hat{r}_c)}{p_d(\hat{r}_d)}}_{\text{credit rationing}})$, where:$p_c(r)$ is probability clean firms to get credit,and $p_d(r)$ is probability of dirty firms to get credit. Credit rationing implies: From Stiglitz–Weiss:$\frac{d\bar{\rho}}{dr} < 0$.We differentiate:

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equation 54

$$\frac{d\bar{\rho}}{dr} = \underbrace{\frac{\partial \rho}{\partial r}}_{>0} + \underbrace{\frac{\partial \rho}{\partial \theta}}_{<0} \cdot \underbrace{\frac{d\theta}{dr}}_{>0}$$

Implication is that:$\frac{d\bar{\rho}}{dr} < 0 \;\; \Rightarrow$ composition effect dominates/Therefore:

- Risk increases strongly with r
- Repayment probability falls

inequality 14

$$\frac{dp_c}{dr} < 0$$

For effect on research allocation we differentiate R_c:

equation 55

$$\frac{dR_c}{dr} = \frac{1}{2}(1 - \tanh^2(\cdot)) \cdot \left(\frac{1}{p_c}\frac{dp_c}{dr} - \frac{1}{p_d}\frac{dp_d}{dr}\right)$$

Key assumption is following inequality :

inequality 15

$$\left|\frac{dp_c}{dr}\right| > \left|\frac{dp_d}{dr}\right|$$

Then $\frac{dR_c}{dr} < 0$. Or from credit rationing: $\frac{d\bar{\rho}}{dr} < 0 \Rightarrow \frac{d\theta}{dr} > 0$, Since:

equation 56

$$p_c(r) = 1 - F(R^*(r), \theta(r))$$

then[20]:

equation 57

$$\frac{dp_c}{dr} = -f(R^*)\frac{dR^*}{dr} - \frac{\partial F}{\partial \theta}\frac{d\theta}{dr}$$

Plug into:

equation 58

$$\frac{dR_c}{dr} \propto \frac{dp_c}{dr} - \frac{dp_d}{dr}$$

[20] Both terms are negative:$\frac{dp_c}{dr} < 0$,similarly :$\frac{dp_d}{dr} \leq 0$

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Now if: $\frac{dp_c}{dr} < \frac{dp_d}{dr}$ then $\frac{dR_c}{dr} < 0$■ .
reducing r_c by credit subsidy implies :

inequality 16

$$\frac{dR_c}{dr_c} < 0 \Rightarrow R_c \uparrow \Rightarrow \dot{\theta} \uparrow$$

Which means that credit policy directly affects clean technological change.By link to DTC-AABH model:

equation 59

$$\dot{\theta} = \theta[(\eta_c + \eta_d)R_c(r_c) - \eta_d]$$

Thus:

equation 60

$$\frac{d\dot{\theta}}{dr_c} = \theta(\eta_c + \eta_d)\frac{dR_c}{dr_c} < 0$$

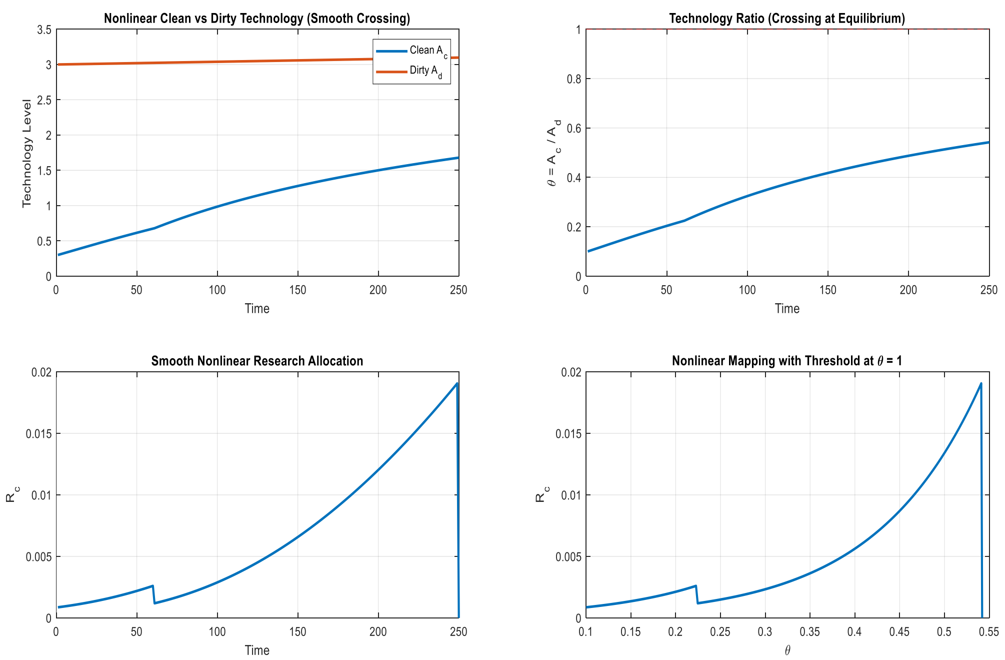
So out final result here :

inequality 17

$$\frac{d\bar{p}}{dr} < 0 \ \Rightarrow \ \frac{dR_c}{dr} < 0 \ \Rightarrow \ \frac{d\dot{\theta}}{dr} < 0$$

Financial frictions redirect innovation towards dirty technology

Figure 4 credit rationing in DTC-AABH model I



Source: Author's calculations

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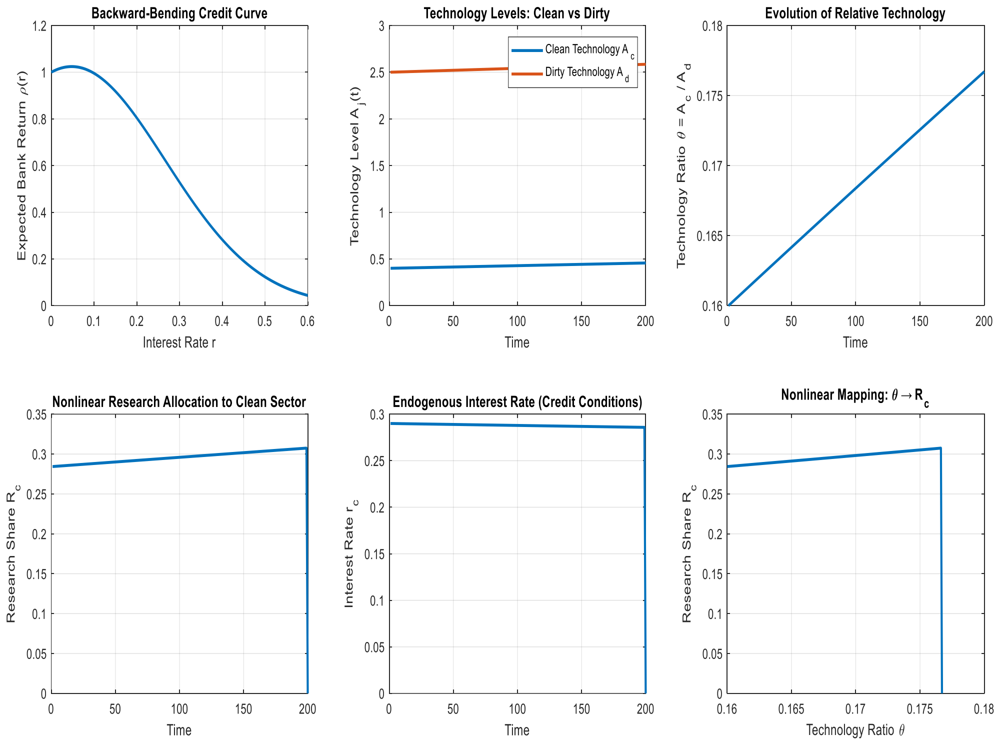
Figure 5 credit rationing in DTC-AABH model II



Source: Author's calculations

5.DTC-SBTC model integration

Here we are going to replace the simple labor L_j in DTC with Katz–Murphy CES aggregator:

equation 61

$$Y_{jt} = [(A_{ljt}L_{jt})^{\rho} + (A_{hjt}H_{jt})^{\rho}]^{\frac{1}{\rho}} \cdot \left(\int_0^1 A_{jit}^{1-\alpha}\, x_{jit}^{\alpha} di\right)$$

Where $\rho < 1$, $\sigma = \frac{1}{1-\rho}$ and A_{ljt}, A_{hjt} are skill-biased technologies[21] within sector. Marginal products are given as:$w_{H_j} = \frac{\partial Y_j}{\partial H_j}, w_{L_j} = \frac{\partial Y_j}{\partial L_j}$ and the skill premium for sector j and aggregate skill-premium is given as:

equation 62

$$\pi_j = \frac{w_{H_j}}{w_{L_j}} = \left(\frac{A_{hjt}}{A_{ljt}}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{H_{jt}}{L_{jt}}\right)^{-\frac{1}{\sigma}}, \pi_t = \frac{w_H}{w_L} = \sum_{j\in\{c,d\}} \omega_j \pi_j$$

[21] Skill biased-technologies refers to technology shifts that increase the productivity and demand for highly skilled workers while reducing the demand for low-skilled labor, see Card, D., & DiNardo, J. E. (2002).

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Skill direction may also be chosen by scientist:

equation 63

$$A_{h_{jt+1}} = A_{h_{jt}}\left(1+\gamma_h s_{h_{jt}}\right)$$
$$A_{l_{jt+1}} = A_{l_{jt}}\left(1+\gamma_l s_{l_{jt}}\ \right)$$

with:$s_{hjt} + s_{ljt} = 1$ is share of resources (researchers' effort). New, innovation direction now is : direction depends on[22]:

equation 64

$$\frac{\Pi_{Ct}}{\Pi_{Dt}} = \underbrace{\frac{\eta_c}{\eta_d}}_{\text{tech}} \cdot \underbrace{\left(\frac{p_C}{p_D}\right)^{\frac{1}{1-\alpha}}}_{\text{price}} \cdot \underbrace{\frac{Y_C}{Y_D}}_{\text{market size}} \cdot \underbrace{\frac{A_C}{A_D}}_{\text{direct}} \cdot \underbrace{\frac{\text{skill demand in } C}{\text{skill demand in } D}}_{\text{NEW SBTC channel}}$$

Proposition 3

Environmental policy affects income inequality through directed technical change.

Intuition here is that carbon tax lowers dirty sector. And dirty sector is typically more unskilled-intensive[23] ,and as demand for lower skilled lowers, skill premium rises. Here mathematical expression for wage premium is:

equation 65

$$\ln \pi_t = \frac{\sigma-1}{\sigma}\ln\left(\frac{A_h}{A_l}\right) - \frac{1}{\sigma}\ln\left(\frac{H}{L}\right)$$
$$\frac{A_h}{A_l} = \frac{A_{h_c} + A_{h_d}}{A_{l_c} + A_{l_d}}$$

Theorem 2

If elasticity[24] of clean and dirty inputs is: $\varepsilon > \frac{1}{1-\alpha}$ and $\sigma > 1$ and $\frac{\partial \ln\left(\frac{A_h}{A_l}\right)}{\partial \ln C} > 0$a temporary clean subsidy shifts innovation towards clean sector and increases skilled wage bias raises wage inequality permanently. So, a temporary subsidy implies: skill bias increase: $\frac{A_h}{A_l}\uparrow$, wage inequality rises: $\frac{w_h}{w_l}\uparrow$

Proof:

equation 66

$$\frac{\Pi_c}{\Pi_d} \propto \left(\frac{A_c}{A_d}\right)^{-\phi} \cdot \left(\frac{Y_c}{Y_d}\right)$$

---

22 $\frac{s_{h_{jt}}}{s_{l_{jt}}} = \frac{V_{hjt}}{V_{ljt}}$ this is skill-based innovation condition and $V_{h_{jt}} \propto A_{hjt}^{-\phi}$

23 Unskilled-intensive refers to production processes or industries that heavily rely on labor requiring minimal training, education, or specialized skills to perform tasks.

24 For econometric estimation with explicit goal of identifying a substitution parameter between clean and dirty inputs using a binary distinction see Papageorgiou, C., Saam, M., & Schulte, P. (2017).

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If we introduce subsidy $\sigma_c > 0$:

equation 67

$$\Pi_c^{sub} = (1 + \sigma_c)\Pi_c$$
$$\frac{\Pi_c^{sub}}{\Pi_d} = (1 + \sigma_c\ )\frac{\Pi_c}{\Pi_d}$$

For a sufficiently large $\sigma_c \gg 0 \Rightarrow s_c = 1$ all innovation goes clean. Technology evolves:

equation 68

$$A_{c,(t+1)} = (1 + \gamma)A_{c,t}$$
$$A_{d,t+1} = A_{d,t}$$

Demand ratio and locked in after subsidy removal:

equation 69

$$\frac{Y_c}{Y_d} = \left(\frac{A_c}{A_d}\right)^{\varepsilon}$$
$$\frac{\Pi_c}{\Pi_d} \propto \left(\frac{A_c}{A_d}\right)^{-\phi}, \varepsilon > \frac{1}{1-\alpha}, -\phi = \varepsilon - \frac{1}{1-\alpha}$$

From Katz–Murphy(1992).:

equation 70

$$\ln \pi = \frac{\sigma - 1}{\sigma} \ln\left(\frac{A_h}{A_l}\right) - \frac{1}{\sigma} \ln\left(\frac{H}{L}\right)$$

Holding $\frac{H}{L}$ fixed:

equation 71

$$\frac{\partial \ln \pi}{\partial \ln\left(\frac{A_h}{A_l}\right)} = \frac{\sigma - 1}{\sigma}$$

Now, since $\sigma_c \gg 0$, $\frac{A_h}{A_l} \uparrow \Rightarrow \pi \uparrow$.A temporary clean subsidy: permanently redirects innovation, increases skill bias, raises wage inequality■. State variables in our system are:

equation 72

$$X_t = \begin{pmatrix} A_c \\ A_d \\ A_h \\ A_l \end{pmatrix}$$

A_h, A_l are skill-biased technologies.Now, we introduce sinusoidal non-linearity:

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equation 73

$$\dot{A}_c = \gamma_c s_c^{\phi} A_c + \underset{skill - clean\ complementarity}{\theta_1 A_c \sin\left(\frac{A_h}{A_l}\right)} \quad \underset{crowding}{-k_1 A_c \frac{A_d}{1 + A_c}}$$

Dirty technology:

equation 74

$$\dot{A}_d = \gamma_d (1 - s_c)^{\phi} A_d + \theta_2 A_d \cos\left(\frac{A_l}{A_h}\right) - k_2 A_d \frac{A_c}{1 + A_d}$$

Skill-biased technology:

equation 75

$$\dot{A}_h = \gamma_h s_h^{\phi} A_h + \psi_1 A_1 \sin\left(\frac{A_c}{A_d}\right)$$

Unskilled technology is given as:

equation 76

$$\dot{A}_l = \gamma_l (1 - s_h)^{\phi} A_l + \psi_2 A_l \cos\left(\frac{A_d}{A_c}\right)$$

Sectoral R&D (from DTC profits):

equation 77

$$s_c = \frac{1}{1 + \exp(-\Omega_c)}$$
$$\Omega_c = \ln\left(\frac{A_c}{A_d}\right) + \beta \sin\left(\frac{A_h}{A_l}\right) - \tau$$

Where s_c is probability of choosing clean innovation based on relative skill, technology and policy. And Ω_c is payoff index of clean innovation (or the attractiveness of clean innovation). And τ is carbon tax. From previous we see there is smooth interior solution that avoids corner only equilibria, sinus terms applied: multiple equilibria, plus cycles plus bifurcations[25]. Steady state satisfies:$\dot{A}_c = \dot{A}_d = \dot{A}_h = \dot{A}_l = 0$.Example entries (1,1):

equation 78

$$\frac{\partial \dot{A}_c}{\partial A_c} = \gamma_c s_c^{\phi} + \theta_1 \sin\left(\frac{A_h}{A_l}\right) - k_1 \frac{A_d(1 + A_c) - A_c A_d}{(1 + A_c)^2} + s_c(1 - s_c) \cdot \frac{1}{A_c}$$

Where θ is the strength of interaction between skill and clean innovation, it is scaling parameter (it controls how strongly the oscillatory interaction term affects clean technology growth). If $\theta = 0$ there is no SBTC interaction, if $\theta > 0$ there is mild nonlinearity, and curved frontier, if $\theta \gg 0$ strong

[25] Bifurcation theory is the mathematical study of changes in the qualitative or topological structure of a given family of curves, such as the integral curves of a family of vector fields, and the solutions of a family of differential equations. Most commonly applied to the mathematical study of dynamical systems, a bifurcation occurs when a small smooth change made to the parameter values (the bifurcation parameters) of a system causes a sudden "qualitative" or topological change in its behavior, Blanchard, P.; Devaney, R. L.; Hall, G. R. (2006).

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oscillations and multiple equilibria with possibles cycles of instability are present. In short θ shows how strongly skills create potentially unstable clean innovation dynamics.

Lemma 2

DTC-SBTC non-linear system

equation 79

$$\dot{A}_c = F_c(A_c, A_d) = A_c\left[g + \gamma s_c^{\phi} + \theta \sin \Psi\left(\frac{A_h}{A_l}\right) - kA_d\right]$$
$$\dot{A}_d = F_d(A_c, A_d) = A_d\left[g + \gamma_d (1 - s_c)^{\phi} + \psi \sin(\Phi\left(\frac{A_h}{A_l}\right)) - kA_c\right]$$

with: $\phi \in (0,1)$ implies diminishing returns, $\Psi = \frac{A_h}{A_l}$, ψ is strength of nonlinear sinusoidal effect in dirty sector, and k is cross-sectional technology crowding i.e. negative interaction between clean and dirty technologies[26]. There exist at least three steady states: Dirty–unskilled equilibrium, Clean–skill equilibrium, Interior mixed equilibrium.

Proof: In steady state: $F_c(A_c, A_d) = F_d(A_c, A_d) = 0$.In dirty equilibrium $A_c = 0$ and $\dot{A}_d = A_d[g + \gamma_d - k \cdot 0 + \psi \sin(\cdot)], \exists A_d^* > 0$ such that:$\dot{A}_d = 0, (A_c, A_d) = \exists(0, A_d^*)$.In clean equilibrium $A_d = 0, (A_c, A_d) = \exists(A_c^*, 0)$,now we have two equilibria. Now, we define nullclines[27]. Clean nullcline:$F_C = 0 \Rightarrow g + \gamma_c s_C^{\phi} + \theta\sin(\cdot) = \kappa A_D$.Dirty nullcline:$F_D = 0 \Rightarrow g + \gamma_d(1 - s_C)^{\phi} + \psi\sin(\cdot) = \kappa A_C$.Next we define mapping:

equation 80

$$A_d = \frac{1}{k}\left[g + \gamma_c s_c^{\phi} + \theta \sin(\cdot)\right]$$
$$A_c = \frac{1}{k}\left[g + \gamma_d (1 - s_c)^{\phi} + \psi \sin(\cdot)\right]$$

Now, since in previous right-hand sides (RHS) are continuous, domain is compact bounded by sinus and diminishing returns, by Brouwer fixed point[28] $\exists(A_c^*, A_d^*) \in \mathbb{R}_+^2$,such that both equations hold simultaneously and interior equilibrium exists. ■There exist:

- one equilibrium on $A_C = 0$
- one on $A_D = 0$
- one interior

Next, on a plot we will show dirty and clean equilibrium:

[26] g is trend , κ is competition , θ, ψ represent nonlinearity , Ψ iswhere nonlinearity comes from.
[27] Nullclines are curves in a phase plane where the rate of change of one variable in a system of differential equations is zero: $\frac{dx}{dt} = 0, \frac{dy}{dt} = 0$.So nullcline is a set of points on plane so that $\frac{dx}{dt} = 0$. Geometrically, these are the points where the vectors are either straight up or straight down. Algebraically, we find the x-nullcline by solving $f(x, y) = 0$.
[28] Any continuous function $G: \mathbb{B} \rightarrow \mathbb{B}^n$ has a fixed point where: $\mathbb{B}^n = \{x \in \mathbb{R}^n: x_1^2, \dots, x_n^2 \leq 1\}$

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equation 81

$$A_c \approx 0, A_d^* = \frac{g + \gamma_d (1 - s_c)^{\phi}}{k}$$
$$A_c^* = \frac{g + \gamma_c s_c^{\phi}}{k}, A_d \approx 0$$

Interior equilibrium system is a solution to:

equation 82

$$kA_d = g + \gamma_c s_c^{\phi} + \theta \sin\left(\frac{A_c}{A_d}\right)$$
$$kA_c = g + \gamma_d (1 - s_c)^{\phi} + \psi \sin\left(\frac{A_c}{A_d}\right)$$

If $x = A_c, y = A_d, r = \frac{x}{y}$,system becomes:

equation 83

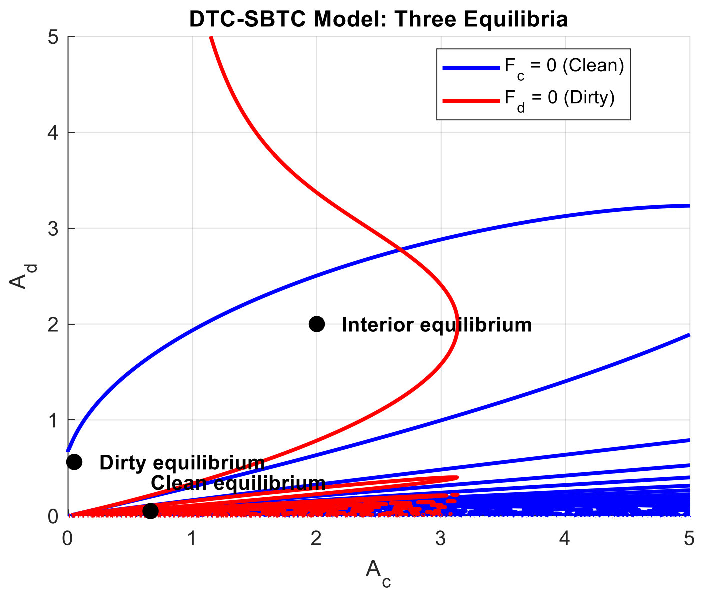
$$ky = a_c + \theta \sin(r)$$
$$kx = a_d + \psi \sin(r)$$

Where $a_c = g + \gamma_c s_c^{\phi}, a_d = g + \gamma_d (1 - s_c)^{\phi}$,and from equations: $x = \frac{a_d + \psi \sin(r)}{k}; y = \frac{a_c + \theta \sin(r)}{k}$,,

equation 84

$$r = \frac{a_d + \psi \sin(r)}{a_c + \theta \sin(r)}$$

Figure 6 DTC-SBTC model: Three equilibria



Source: Author's calculations

5.1 Ramsey-Mirrlees-Pigou in DTC-SBTC model

This section will loosely apply Ramsey (1927), Mirrlees(1971), and Pigou (1932).here planner maximizes:

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equation 85

$$\max_{\{c_t,\tau_t,s_{c_t}\}} \sum_{t=0}^{\infty} \beta^t u(C_t, S_t)$$

s.t.:Resource constraint:$C_t + I_t = Y_t(A_c, A_d, A_h, A_l)$,technology dynamics:

equation 86

$$\dot{A}_c = \gamma s_c^{\phi} A_c$$
$$\dot{A}_d = \gamma(1 - s_c)^{\phi} A_d$$

Environmental law is:$S_{t+1} = (1 - \delta)S_t - \xi Y_{Dt}$,and IC for Mirrlees (Agents differ by skill, planner cannot observe type directly).

1. Optimal Carbon (Pigouvian) Tax (internalizes environmental externality):

equation 87

$$\tau_t^{carbon} = \sum_{s=t}^{\infty} \beta^{s-t} \frac{\partial u / \partial S_s}{\partial u / \partial C_t} \cdot \xi \frac{\partial Y_{Ds}}{\partial Y_{Dt}}$$

2. Optimal skill tax from Mirrlees:

equation 88

$$\tau_h - \tau_l = \frac{1 - g(w)}{1 + \varepsilon_l}$$

Where :$g(w)$: social marginal welfare weight ,ε_L: labor supply elasticity
Innovation subsidy is given as:

equation 89

$$\sigma_C^* = \underbrace{\frac{\partial Y}{\partial A_C}}_{growth} + \underbrace{\lambda_S \frac{\partial S}{\partial A_C}}_{environment} + \underbrace{\lambda_\pi \frac{\partial \pi}{\partial A_C}}_{inequality}$$

for derivation of this part see Appendix 1.

Proposition 4

Optimal clean policy satisfies: $\sigma_C^* > 0$ even if $\tau^{carbon} > 0$, because:

- carbon tax fixes flow externality
- subsidy fixes direction of innovation

one paper by Wiskich, (2024), found that even though carbon tax is favored policy instrument, still this does not hold if optimal policy is eventually reached. We now simulate:

equation 90

$$\dot{A}_C = A_c \, [g + \sigma_C - \kappa A_d]$$
$$\dot{A}_D = A_d \, [g - \tau \; - \kappa A_c]$$

Skill bias:

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equation 91

$$\dot{A}_h = A_h[\eta + \chi \ln \left(\frac{A_c}{A_d}\right)]$$
$$\dot{A}_l = A_l \eta$$

full Lagrangian in previous is given as:

equation 92

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \, [u(C_t, S_t, \pi_t) + \lambda_t(Y_t - C_t) + \mu_t((1-\delta)S_t - \xi Y_{dt} - S_{t+1})]$$

Pigouvian tax FOC is w.r.t. Y_{dt} so : $\frac{\partial \mathcal{L}}{\partial Y_{dt}} = 0, \tau_t^{Pigou} = \mu_t \xi$ .Ramsey tax is FOC w.r.t σ_{ct}:

equation 93

$$\tau_t^R = \frac{\partial Y_t}{\partial A_c} \cdot \frac{\partial A_c}{\partial \sigma_{ct}}$$

i.e. $\tau_t^R = \lambda_t \gamma A_c \ln \left(\frac{A_c}{A_d}\right)$.This tax: directs innovation toward cleaner sector and depends on relative technological gap.And Mirrlees tax : From incentive constraint:

equation 94

$$\tau_t^M = \frac{1 - g(w_t)}{1 + \varepsilon_L}$$

This tax means: trade-off between redistribution and efficiency and depends on skill elasticity.Where then multipliers in the Lagrangian are defined as: Define multipliers:λ_t: resource constraint ,μ_t: environment ,ν_t: innovation distortion ,η_t: inequality constraint.This will be simulated in following table and graphically depicted afterwards.

Table 2 DTC-SBTC and subsidies results (falling skill premium)

Time	A_c	A_d	A_h	A_l	SkillPremium	Ac_{norm}	Ad_{norm}	π_{norm}
1	1	5	2	3	0.7736	0.9658	1	1
2	1	4.9925	1.9901	3.0045	0.77194	0.9658	0.9985	0.99785
3	1	4.985	1.9803	3.009	0.77028	0.96581	0.997	0.99571
4	1	4.9775	1.9706	3.0135	0.76863	0.96583	0.99551	0.99358
5	1	4.9701	1.9609	3.018	0.76698	0.96585	0.99401	0.99145
6	1.0001	4.9626	1.9512	3.0226	0.76534	0.96588	0.99252	0.98933
7	1.0001	4.9552	1.9417	3.0271	0.76371	0.96591	0.99103	0.98722
8	1.0002	4.9477	1.9321	3.0316	0.76208	0.96596	0.98955	0.98511
9	1.0002	4.9403	1.9227	3.0362	0.76045	0.96601	0.98806	0.98301
10	1.0003	4.9329	1.9133	3.0407	0.75883	0.96606	0.98658	0.98091
11	1.0003	4.9255	1.9039	3.0453	0.75722	0.96613	0.9851	0.97882
12	1.0004	4.9181	1.8947	3.0499	0.75561	0.9662	0.98362	0.97674
13	1.0005	4.9107	1.8854	3.0544	0.754	0.96628	0.98215	0.97467
14	1.0006	4.9034	1.8763	3.059	0.7524	0.96637	0.98067	0.9726
15	1.0007	4.896	1.8671	3.0636	0.7508	0.96646	0.9792	0.97053

Source: Author's calculations

Where $\pi_{norm,t} = \frac{\pi_t}{\max\, x_t \pi_t}$.And:

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equation 95

$$A_{C,\text{norm},t} = \frac{A_{C,t}}{\max_t A_C}$$
$$A_{D,\text{norm},t} = \frac{A_{D,t}}{\max_t A_D}$$

In previous we did encounter problem so that skill premium declined because:

equation 96

$$\frac{d}{dt}\ln\left(\frac{A_h}{A_l}\right) = \chi \ln\left(\frac{A_c}{A_d}\right)$$

To solve the problem we define:

equation 97

$$\frac{d}{dt}\ln\left(\frac{A_h}{A_l}\right) = \chi \cdot \frac{A_c}{A_c + A_d}$$

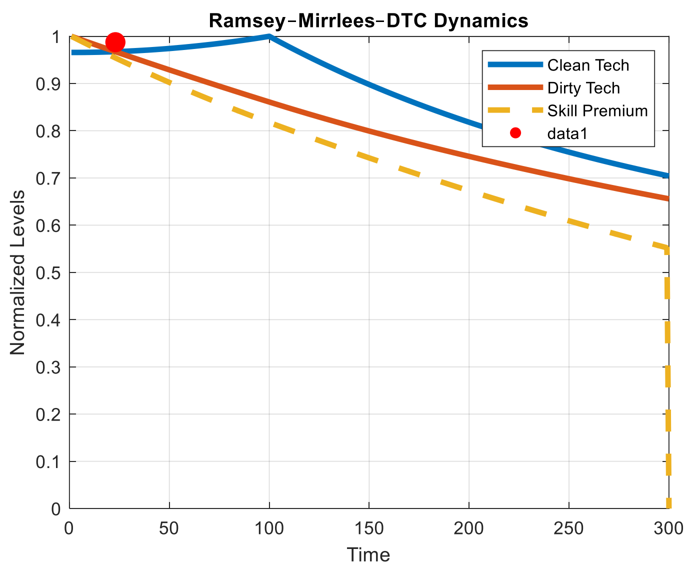
Since: $\frac{A_c}{A_c+A_d} \in (0,1)$ leads always: $\ln\left(\dot{\frac{A_h}{A_l}}\right) > 0$

Table 3 Regime comparison in DTC-SBTC and subsidies results (falling skill premium)

Ac_{Sub}	Ac_{NoSub}	π_{Sub}	π_{NoSub}
1.0116	0.85871	0.69993	0.52149

Source: Author's calculations

Figure 7 DTC-SBTC and subsidies results (falling skill premium)



Source: Author's calculations

UDK: 331.225.3:336.226.4]:330.831
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336.77:330.831
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Table 4 DTC-SBTC and subsidies results (rising skill premium)

Time	A_c	A_d	A_h	A_l	SkillPremium	Ac_{norm}	Ad_{norm}	π_{norm}
1	1	5	2	3	0	0.9658	1	0
2	1	4.9925	2.0043	3.004	0.77381	0.9658	0.9985	0.91291
3	1	4.985	2.0087	3.008	0.77403	0.96581	0.997	0.91317
4	1	4.9775	2.013	3.012	0.77424	0.96583	0.99551	0.91342
5	1	4.9701	2.0174	3.016	0.77446	0.96585	0.99401	0.91368
6	1.0001	4.9626	2.0218	3.02	0.77467	0.96588	0.99252	0.91393
7	1.0001	4.9552	2.0262	3.0241	0.77489	0.96591	0.99103	0.91419
8	1.0002	4.9477	2.0306	3.0281	0.77511	0.96596	0.98955	0.91444
9	1.0002	4.9403	2.035	3.0321	0.77532	0.96601	0.98806	0.9147
10	1.0003	4.9329	2.0394	3.0362	0.77554	0.96606	0.98658	0.91495
11	1.0003	4.9255	2.0438	3.0402	0.77576	0.96613	0.9851	0.91521
12	1.0004	4.9181	2.0483	3.0443	0.77598	0.9662	0.98362	0.91547
13	1.0005	4.9107	2.0527	3.0483	0.77619	0.96628	0.98215	0.91573
14	1.0006	4.9034	2.0572	3.0524	0.77641	0.96637	0.98067	0.91598
15	1.0007	4.896	2.0617	3.0564	0.77663	0.96646	0.9792	0.91624

Source: Author’s calculations

Table 5 Regime comparison in DTC-SBTC and subsidies results (rising skill premium)

Ac_{Sub}	Ac_{NoSub}	π_{Sub}	π_{NoSub}
1.0116	0.85871	0.77689	0.82219

Source: Author’s calculations

UDK: 331.225.3:336.226.4]:330.831
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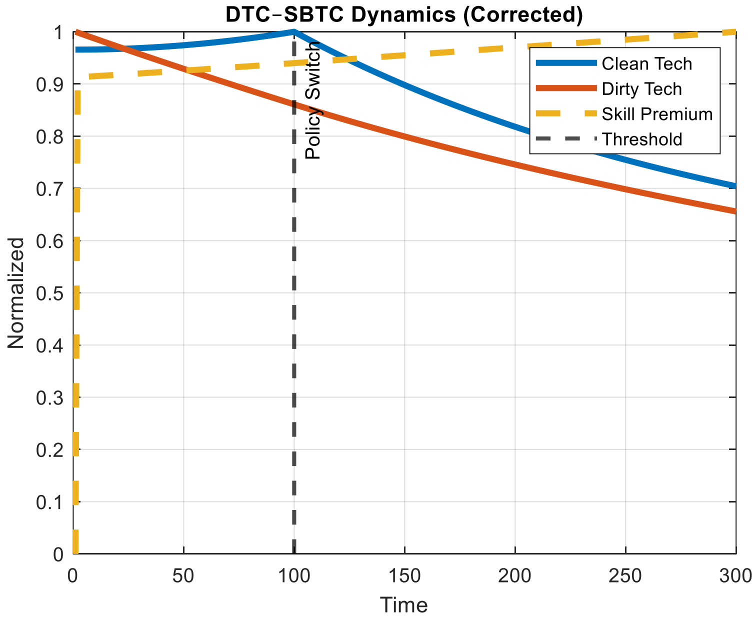
Figure 8 DTC-SBTC and subsidies results (rising skill premium)



Source: Author's calculations

No crossings detected (single regime equilibrium).

Lemma 3

Monotone skill premium

If $\sigma > 1, \frac{d}{dt}\ln\left(\frac{A_h}{A_l}\right) = \chi f(A_c, A_d) \Rightarrow \dot{\pi} > 0, \forall t$

Proof:

equation 98

$$ln\dot{\pi} = \frac{\sigma - 1}{\sigma} \cdot \chi f(A_c, A_d)$$

Since $\frac{\sigma-1}{\sigma} > 0 \Rightarrow \sigma \ > 1, \chi > 0, f > 0, ln\dot{\pi} > 0$ or π is strictly increasing∎.

5.2 Optimal trade-off between carbon taxes and clean innovation subsidies

Let social welfare with inequality be like :

equation 99

$$W = \sum_{t=0}^{\infty} \beta^t [u(C_t) - D(S_t) - \lambda \cdot \ln \pi_t]$$

Where $D(S)$ is environmental damage, $\lambda > 0$ is inequality aversion, and $\pi = w_h/w_l$. Environment moves as:

equation 100

$$S_{t+1} = (1 - \delta)S_t - \xi Y_{dt}$$

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Technology dynamics is as defined previously: $\dot{A}_C = A_c\,[g + \sigma_C - \kappa A_d]$; $\dot{A}_D = A_d\,[g - \tau - \kappa A_c]$.Corrected form for rising skill premium is also applied here:$\frac{d}{dt}\ln\left(\frac{A_h}{A_l}\right) = \chi \cdot \frac{A_c}{A_c+A_d}$.And wage inequality is:

equation 101

$$\ln\pi = \frac{\sigma-1}{\sigma}\cdot\ln\left(\frac{A_h}{A_l}\right) - \frac{1}{\sigma}\ln\left(\frac{H}{L}\right)$$

Hamiltionian is given as:

equation 102

$$\mathcal{H} = u(C) - D(S) - \lambda\ln\pi + \mu_S[(1-\delta)S - \xi Y_D] + \mu_C\dot{A}_C + \mu_D\dot{A}_D$$

FOC for carbon tax:

equation 103

$$\tau^* = \frac{D'(S)\cdot\xi}{u'(C)} + \lambda\cdot\frac{\partial\ln\pi}{\partial\tau}$$

$\frac{D'(S)\cdot\xi}{u'(C)}$ this is Pigouvian term, while this $\lambda\cdot\frac{\partial\ln\pi}{\partial\tau}$ is inequality correction.FOC for clean subsidy gives:

equation 104

$$\sigma_c^* = \mu_c A_c + \frac{D'(S)\cdot\xi}{u'(C)}\cdot\frac{\partial Y_d}{\partial A_c} - \lambda\cdot\frac{\partial\ln\pi}{\partial\sigma_c}$$

Where $\lambda\cdot\frac{\partial\ln\pi}{\partial\sigma_c}$ are inequality costs, $\frac{D'(S)\cdot\xi}{u'(C)}\cdot\frac{\partial Y_d}{\partial A_c}$ is the environment, $\mu_c A_c$ is growth. Since:

equation 105

$$\ln\pi = \frac{\sigma-1}{\sigma}\ln\left(\frac{A_h}{A_l}\right)$$
$$\frac{d}{dt}\ln\left(\frac{A_h}{A_l}\right) = \chi\cdot\frac{A_c}{A_c+A_d}$$
$$\frac{\partial ln\pi}{\partial A_c} = \frac{\sigma-1}{\sigma}\cdot\chi\cdot\frac{A_c}{(A_c+A_d)\text{^}2}$$

Then :

equation 106

$$\frac{\partial\ln\pi}{\partial\sigma_c} = \frac{\sigma-1}{\sigma}\cdot\chi\cdot\frac{A_d}{(A_c+A_d)^2}\cdot\frac{\partial A_c}{\partial\sigma_c}$$

So clean subsidy will be:

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equation 107

$$\sigma_c^* = \mu_c A_c + \frac{D'(S)\cdot\xi}{u'(C)} \cdot \frac{\partial Y_d}{\partial A_c} - \lambda \cdot \frac{\sigma-1}{\sigma} \cdot \chi \cdot \frac{A_d}{(A_c+A_d)^2} \cdot \frac{\partial A_c}{\partial \sigma_c}$$

Carbon tax is : $\tau^* = \frac{D'(S)\cdot\xi}{u'(C)} + \lambda \cdot \frac{\partial \ln \pi}{\partial \tau}$ .In previous μ= environmental shadow value, λ= inequality weight

Theorem 3

Environment -inequality trade-off theorem: We assume that:

inequality 18

$$\varepsilon > \frac{1}{1-\alpha}, \sigma > 1, \chi > 0$$

Then : clean subsidies increase inequality, carbon taxes have an ambiguous effect on inequality, and optimal clean subsidy satisfies:

inequality 19

$$\frac{\partial \sigma_c^*}{\partial \tau} < 0\,, \frac{\partial \sigma_c^*}{\partial \mu} < 0, \frac{\partial \sigma_c^*}{\partial \lambda} \mp$$

Proof: Inequality transmission mechanism is : $\ln \pi = \frac{\sigma-1}{\sigma} \cdot \ln\left(\frac{A_h}{A_l}\right) - \frac{1}{\sigma}\ln\left(\frac{H}{L}\right)$, from $\ln \pi = \frac{\sigma-1}{\sigma}\ln\left(\frac{A_h}{A_l}\right)$, $(\partial \ln \pi)/\partial \ln\left(\frac{A_h}{A_l}\right) = \frac{\sigma-1}{\sigma} > 0$, inequality increases with skill-biased technology. Since,$\dot{A}_h = A_h\left(\eta + \chi \cdot \frac{A_c}{A_c+A_d}\right)$, where $\frac{A_c}{A_c+A_d}$ is clean share. Where, χ= *skill-bias intensity of clean technology*. Where,$\frac{\partial\left(\frac{A_h}{A_l}\right)}{\partial A_c} > 0$, more clean technology means more skill bias. Clean subsidy enters in : $\dot{A}_C = A_c\,[g + \sigma_C - \kappa A_d]$, so $\frac{\partial\left(\frac{A_c}{\partial A_d}\right)}{\partial \sigma_c} > 0$,since $\varepsilon > \frac{1}{1-\alpha}$, so $\sigma_c \uparrow \Rightarrow \frac{A_c}{A_d} \uparrow \Rightarrow \frac{A_c}{A_c+A_d} \uparrow$,also : $\sigma_c \uparrow \Rightarrow \frac{A_c}{A_d} \uparrow \Rightarrow \frac{A_h}{A_l} \uparrow \Rightarrow \pi \uparrow$.Carbon tax reduces dirty output $\tau \uparrow \Rightarrow Y_d \downarrow$, but it increases $\frac{A_c}{A_d} \uparrow \Rightarrow \pi \uparrow$.In terms of labor market: Lower dirty production: may reduce demand for low-skill labor → inequality ↑ or reduce complementarity → inequality ↓ .This depends on substitution elasticity ε.So, $\frac{\partial \pi}{\partial \tau} \mp$ is ambiguous here we use this sign $\mp$ for ambiguous. Planner maximizes: $\max_{\{c_t,\tau_t,s_{c_t}\}} \sum_{t=0}^{\infty} \beta^t u(C_t, S_t)$,and subsidy is:

equation 108

$$\sigma_C^* = \underbrace{\frac{\partial Y}{\partial A_C}}_{\text{growth}} + \underbrace{\lambda_S \frac{\partial S}{\partial A_C}}_{\text{environment}} + \underbrace{\lambda_\pi \frac{\partial \pi}{\partial A_C}}_{\text{inequality}}$$

And $\frac{\partial \sigma_c^*}{\partial \tau} < 0\,, \frac{\partial \sigma_c^*}{\partial \mu} < 0, \frac{\partial \sigma_c^*}{\partial \lambda} \mp$ and $\frac{\partial \pi}{\partial A_c} < 0$ so if inequality is costly , reduce subsidy, but if growth/environment dominate, increase subsidy. So clean subsidy increase inequality, effect of carbon tax is ambiguous,and optimal subsidy : $\frac{\partial \sigma_c^*}{\partial \tau} < 0\,, \frac{\partial \sigma_c^*}{\partial \mu} < 0, \frac{\partial \sigma_c^*}{\partial \lambda} \mp$.

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6. Clean wage premium and credit rationing asymmetry in DTC-AABH model

In the baseline DTC model final output,sectoral outputs and technology evolution are given as:

equation 109

$$Y = \left[\alpha Y_c^{\frac{\sigma-1}{\sigma}} + (1-\alpha) Y_d^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma-1}{\sigma}}$$
$$Y_c = A_c K_c^{\beta} L^{1-\beta}\ ; Y_d = A_d K_d^{\beta} L^{1-\beta}$$
$$\dot{A} = \eta_c R_c A_c ; \dot{A_d} = \eta_d R_d A_d$$

Clean wage premium is given as:

equation 110

$$w_c = w \cdot \left(1 + \phi(A_c, A_d\,)\right)$$
$$w_d = w$$
$$\phi(A_c, A_d\,) = \phi_0 + \phi_1 \left(\frac{A_c}{A_d}\right)$$

When clean tech is more advanced leads to higher marginal productivity and to localized wage premium. Researchers choose sector with higher expected payoff:

equation 111

$$\Pi_c^R = \eta_c A_c V_c - w_c$$
$$\Pi_d^R = \eta_d A_d V_d - w_d$$

Interior equilibrium:

equation 112

$$\eta_c A_c V_c - w(1+\phi) = \eta_d A_d V_d - w$$
$$\eta_c A_c V_c - \eta_d A_d V_d = w\phi(A_c, A_d)$$

Here we introduce financial frictions credit rationing :

equation 113

$$r_c = r - \Delta_r$$
$$r_d = r + \Delta_r$$

Where , $\Delta_r > 0$ means green finance advantage. Credit access probability is :$p_c = p + \psi, p_d = p$, where $\psi > 0$. Firm value with credit frictions is given as:

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equation 114

$$V_j = \frac{p_j \cdot \pi_j}{r_j + \delta - g_j} \quad j \in \{c, d\}$$
$$V_c = \frac{(p + \psi)\pi_c}{(r - \Delta_r) + \delta - g_c}$$
$$V_d = \frac{p\pi_d}{(r + \Delta_r) + \delta - g_d}$$

Now with previous modified research arbitrage condition[29] becomes:

equation 115

$$\eta_c A_c \frac{(p + \psi)\pi_c}{(r - \Delta_r) + \delta - g_c} - \eta_d A_d \frac{p\pi_d}{(r + \Delta_r) + \delta - g_d} = w\phi(A_c, A_d)$$

Definition 2

equation 116

$$\theta = \frac{A_c}{A_d}$$
$$\frac{\dot{\theta}}{\theta} = \eta_c R_c - \eta_d R_d$$
$$R_c + R_d = 1; R_c = \frac{1}{1 + \exp\big(-\Gamma(\theta)\big)}$$

where:

equation 117

$$\Gamma(\theta) = \underbrace{\log\left(\frac{\eta_c (p + \psi)\pi_c}{\eta_d p\pi_d}\right)}_{CREDIT+PROFIT\ ADVANTAGE} + \underbrace{\log\left(\frac{r + \Delta_r + \delta - g_d}{r - \Delta_r + \delta - g_c}\right)}_{INTEREST\ RATE\ GAP} - \underbrace{\frac{w\phi(\theta)}{\eta_d A_d V_d}}_{WAGE\ DISTORTION}$$

Technology dynamics and environment are :

equation 118

$$\dot{\theta} = \theta[\eta_c R_c(\theta) - \eta_d (1 - R_c(\theta))]$$
$$\dot{S} = \xi Y_d - \rho S$$

Table 6 Clean sector triple advantage

Channel	Effect
Wage bias	attracts researchers

[29] A modified research arbitrage condition generally refers to adapting the classic "no-risk profit" assumption to reflect real-world market imperfections, such as transaction costs, financing constraints, limited liquidity, or heterogeneous agent beliefs,see Suh, S., & Kim, Y. J. (2016)

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Channel	Effect
Lower interest rate	raises firm value
Higher credit probability	increases expected profits

Source: Author's calculations

Steady state condition is:

equation 119

$$\eta_c R_c(\theta^*) = \eta_d(1 - R_c(\theta^*)$$
$$R_c(\theta^*) = \frac{\eta_d}{\eta_c + \eta_d}$$
$$\Gamma(\theta^*) = 0$$

nonlinear research allocation and distortion function:

equation 120

$$R_c(\theta) = \frac{1}{2} + \frac{1}{2}\tanh(\Gamma(\theta))$$
$$\Gamma(\theta) = \underbrace{a_0}_{baseline} + \underbrace{a_1\theta}_{creditadvantage} - \underbrace{a_2\theta}_{wagecost} + \underbrace{a_3\sin(\omega\theta)}_{\text{NONLINEAR interaction}}$$

Where: a_1 : credit + interest advantage (clean) , a_2: wage bias (cost) , $a_3\sin(\omega\theta)$: creates multiple equilibria. Law of motion is:

equation 121

$$\dot{\theta} = \theta[\eta_c R_c(\theta^*) - \eta_d(1 - R_c(\theta)]$$
$$\dot{\theta} = \theta[(\eta_c + \eta_d)R_c(\theta) - \eta_d]$$

Steady states are[30]:

$$\dot{\theta} = 0 \Rightarrow R_c(\theta^*) = \frac{\eta_d}{\eta_c + \eta_d}$$
$$tanh(\Gamma(\theta^*)) = \frac{2\eta_d}{\eta_c + \eta_d} - 1$$
$$\left(\Gamma(\theta^*)\right) = \tanh^{-1}\left(\frac{2\eta_d}{\eta_c + \eta_d} - 1\right)$$

for the Jacobian we define: $F(\theta) = \dot{\theta}$; $J = \frac{dF}{d\theta}$

[30] The tanh (hyperbolic tangent) function is a mathematical function that maps any real-valued number to an output between -1 and 1, creating an S-shaped curve.Defined as:$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$,see Abramowitz, M., & Stegun, I. A. (Eds.). (1972).

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equation 122

$$\frac{dR_c}{d\theta} = \frac{1}{2}(1 - \tanh^2(\Gamma)) \cdot \Gamma'(\theta)$$
$$\Gamma'(\theta) = a_1 - a_2 + a_3\omega\cos(\omega\theta)$$
$$J = (\eta_c + \eta_d)\left[R_c(\theta) + \theta\frac{dR_c}{d\theta}\right] - \eta_d$$

now we substitute:

equation 123

$$J = (\eta_c + \eta_d)\left[R_c + (\theta) \cdot \frac{1}{2}(1 - \tanh^2(\Gamma) \cdot (a_1 - a_2 + a_3\omega\cos(\omega\theta))\right] - \eta_d$$

stable is $J(\theta^*) < 0$ unstable is $J(\theta^*) > 0$. Stability depends on:$a_3\omega\cos(\omega\theta)$

Theorem 4

Multiple equilibria theorem: If:$a_3\omega$sufficiently large and credit advantage $a_1 > a_2$ locally then the system admits:$\exists\ \theta_1^*, \theta_2^*, \theta_3^*$,such that:

- θ_1^*: stable (dirty trap)
- θ_2^*: unstable (threshold)
- θ_3^*: stable (clean takeoff)

Proof: Technology evolves $\dot{\theta} = \theta[(\eta_c + \eta_d)R_c(\theta) - \eta_d]$. Equilibria satisfy: $R_c(\theta^*) = \frac{\eta_d}{\eta_c+\eta_d}$. $R_c(\theta)$is nonlinear and S-shaped . Strong nonlinearity ($a_3\omega$ large) ⇒ non-monotonic curvature. Credit advantage ($a_1 > a_2$) shifts $R_c$upward for intermediate θ.We define $F(\theta) = R_c(\theta) - \bar{R}, \bar{R} = \frac{\eta_d}{\eta_c+\eta_d}$.$F(\theta)$ changes sign three times. There $\exists\theta_1^*, \theta_2^*, \theta_3^*$ at least three roots. And :

equation 124

$$\frac{d\dot{\theta}}{d\theta} = \theta(\eta_c + \eta_d)R_c'(\theta)$$

- If $R_c'(\theta^*) < 0$⇒ stable
- If $R_c'(\theta^*) > 0$⇒ unstable

So $\exists\theta_1^*, \theta_2^*, \theta_3^*$ with stability pattern (stable, unstable, stable)■.In economic terms:
Nonlinear innovation plus credit frictions generate traps and takeoff equilibria.Jacobian is now given:

equation 125

$$J = \begin{bmatrix} F'(\theta) & 0 \\ \xi Y_d'(\theta) & -\rho \end{bmatrix}$$

Eigenvalues:$\lambda_1 = F'(\theta), \lambda_2 = -\rho$.Always one negative eingevalue[31]

[31] A negative eigenvalue indicates that the corresponding eigenvector is reversed (flipped 180 degrees) when a linear transformation is applied, rather than just being stretched or compressed. It signifies direction reversal and can indicate system instability, such as in buckling, or a non-convexity in optimization.

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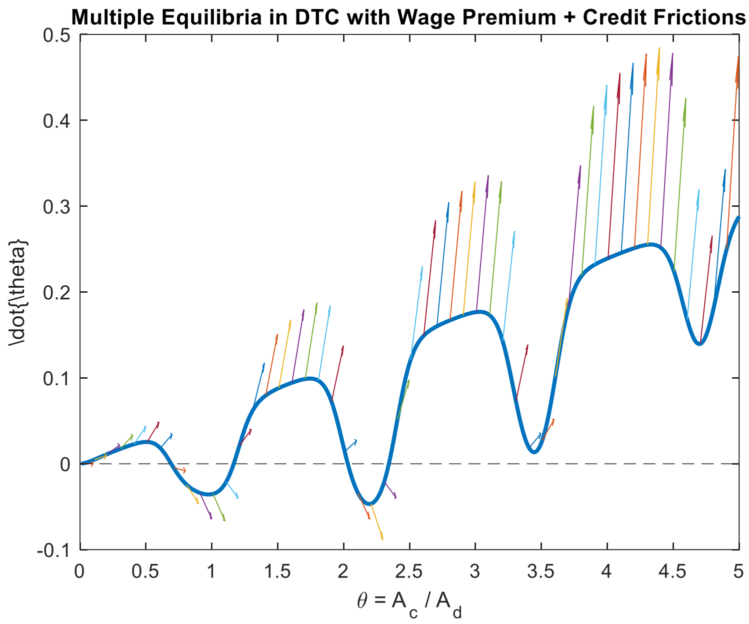
Figure 9 Multiple Equilibria in DTC with Wage Premium + Credit Frictions



Source: Author's calculations

We can think of $\eta_j productivity\ of\ research\ (innovation\ efficiency)$:

equation 126

$$\eta_j = \underbrace{\text{knowledge spillovers}}_{\text{learning}} \times \underbrace{\text{research efficiency}}_{\text{institutions}} \times \underbrace{\text{technology maturity}}_{\text{frontier distance}}$$

Credit policy in our system is distorted:

equation 127

$$V_c = \frac{(p+\psi)\pi_c}{r-\Delta_r} \neq \frac{p\pi_d}{r+\Delta_r}$$

For subsidy Private condition and planner condition are:

equation 128

$$\eta_c A_c V_c - w_c + s_c = \eta_d A_d V_d - w_d$$
$$\lambda_\theta \theta(\eta_c + \eta_d) = \kappa R_c$$

So, by matching clean subsidy is: $s_c = \lambda_\theta \theta(\eta_c + \eta_d) - (\eta_c A_c V_c - \eta_d A_d V_d)$.Complete decentralized equilibrium is

equation 129

$$\tau_d = D'(S)$$
$$s_c = \lambda_\theta\, \theta(\eta_c + \eta_d) - \text{"}private\ wedge\text{"}$$
$$\frac{p+\psi}{r-\Delta_r} = \frac{p}{r+\Delta_r}$$

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Theorem 5

Backward bending expected return in Stiglitz -Weiss implemented in DTC model.At $r = \hat{r}$, where firms switch from safe to risky projects: $\frac{d\bar{\rho}(r)}{dr} < 0$. Where, $\hat{r}$ =loan interest rate charged by the bank.

Proof:Aggregate return:

equation 130

$$\bar{\rho}(r) = \begin{cases} \rho(r,k), & r < \hat{r} \\ \rho(r,j), & r \geq \hat{r} \end{cases}$$
$$\frac{d\bar{\rho}}{dr} = \frac{\partial\rho}{\partial r} + \underbrace{\frac{\partial\rho}{\partial\theta} \cdot \frac{d\theta}{dr}}_{\text{composition effect}}$$

first term is positive second negative $\frac{d\bar{\rho}}{dr} < 0$ ■

Corollary 1

equation 131

$$\exists r^* = argmax\bar{\rho}(r)$$

such that:Banks do not raise rates above r^* so excess demand persists and this is Credit rationing equilibrium.Now we will embed this into DTC: Endogenous credit access:

equation 132

$$p(r) = 1 - F(R^*(r), \theta(r))$$

Innovation value is:

equation 133

$$V_c = \frac{p_c(r_c)\pi_c}{r_c + \delta}; V_d = \frac{p_d(r_d)\pi_d}{r_d + \delta}$$

Research allocation

equation 134

$$R_c = \frac{1}{2} + \frac{1}{2}\tanh(\Gamma(\theta) + \log\frac{p_c(r_c)}{p_d(r_d)})$$

research dynamics ais as previous:

equation 135

$$\dot{\theta} = \theta[\eta_c R_c(\theta^*) - \eta_d(1 - R_c(\theta)]$$
$$\dot{\theta} = \theta[(\eta_c + \eta_d)R_c(\theta) - \eta_d]$$

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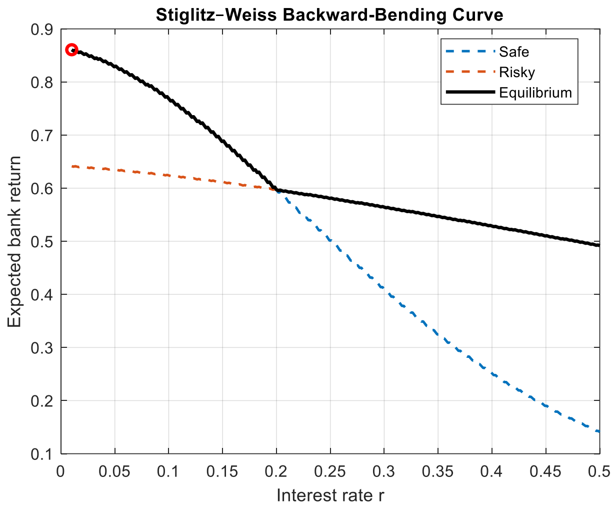
Figure 10 Stiglitz–Weiss Backward-Bending Curve embedded in DTC model



Source: Author's calculations

Next, we will code and plot skill demand in DTC model i.e. skill premium vs skill ratio:

equation 136

$$\frac{w_H}{w_L} = D\left(\frac{H}{L}\right) = A + B\sin\left(\omega \cdot \frac{H}{L}\right)$$

Supply of skills

equation 137

$$\frac{w_H}{w_L} = S\left(\frac{H}{L}\right) = s_0 + s_1\frac{H}{L}$$

Equilibria

equation 138

$$D\left(\frac{H}{L}\right) = S\left(\frac{H}{L}\right)$$

Multiple intersections leads to multiple equilibria.

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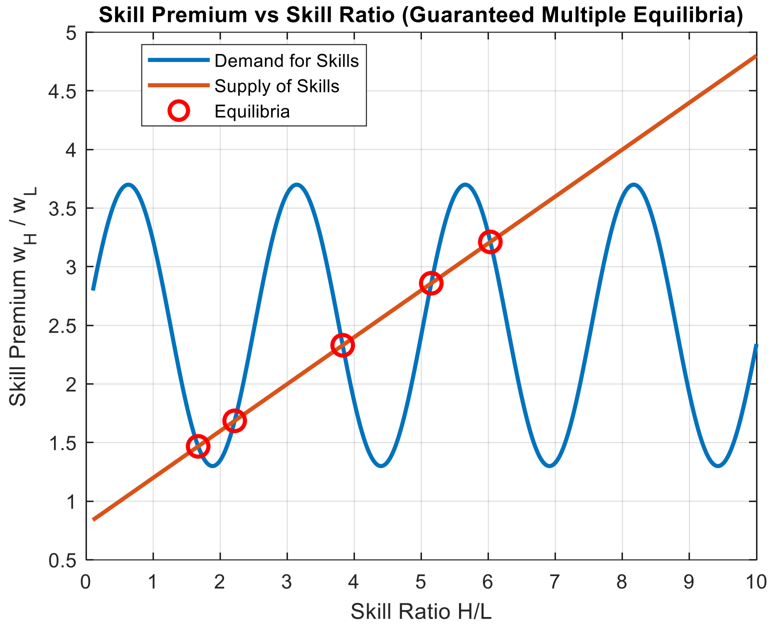
Figure 11 Skill Premium vs Skill Ratio (Guaranteed Multiple Equilibria)



Source: Author's calculations

Table 7 Equilibria in demand-supply skills model

Equilibria (H/L):				
1.6706	2.2151	3.8266	5.1482	6.0292

Source: Author's calculations

7.Conclusions

In the KORRV Model: Nonlinear General Equilibrium Dynamics skill premium ω can accelerate or flatten. If driven by technology it rises, if it is supply response, it moves downwards, if it is capital complementary it moves up. Capital-skill complementarity is an economic concept where capital equipment (like computers or machinery) is more closely linked with and increases the productivity of skilled labor rather than unskilled labor, driving wage inequality. More on Capital-skill complementarity see in Parro, (2013).Relative supply H/L is endogenous and curved in our model and responds strongly when $\theta > 1$. Where θ is : $\frac{\partial \ln\left(\frac{H}{L}\right)}{\partial ln\omega} = \theta$,so θ is the elasticity of relative skill supply with respect to the skill premium. In Nonlinear Dynamics: CES Skill Premium Model: $ln\left(\frac{H}{L}\right)$ is S-shaped curve (education expansion slows down). While $\ln\left(\frac{A_h}{A_l}\right)$ is curved upward indicating endogenous skill-biased tech. $\ln(\pi)$ skill premium is non-monotonic, curved and can: first fall (supply dominates), then rise (technology dominates). In DTC-AABH Model: Nonlinear Dynamics (Laissez-Faire): dirty technology A_d dominates innovation as in theory and initially moves up very quickly. This model indicates that A_c moves very slowly and may never catch-up without policy. Environment S_t declines continuously and may collapse. Research allocation x_t converges toward dirty sector $x \to 0$, it is path-dependent and non-linear. If we add subsidy to clean (negative tax), $x_t \to 1$ i.e. clean innovation dominates, $Y_d \to 0$, $S_t \to \bar{S}$.So, temporary policy permanently redirects innovation. In Credit rationing in DTC-AABH model I showed as that: green transition occurs when the economy crosses a technology threshold. A temporary intervention can permanently redirect innovation. In credit rationing in DTC-AABH model II: main conclusion is that non-linear credit frictions plus directed technological change (DTC) generate

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tipping dynamics in green transition. Here θ, R_c show non-linear equilibrium structure.That is directional technological change $\theta = A_c/A_d$ with R_c scientist allocation. In DTC-SBTC model we exactly pinpointed that there exist three equilibria exactly: dirty, clean and interior equilibrium. With Ramsey–Mirrlees–DTC Dynamics one can see regime comparison:clean technology twice as that without subsidies, and wage premium with subsidies is larger than that without subsidies. Subsidy regime is from zero to period 100, afterwards followed by no subsidy regime.In the first example subsidy declined due to: $\frac{d}{dt}\ln\left(\frac{A_h}{A_l}\right) = \chi\ln\left(\frac{A_c}{A_d}\right)$.We changed that into$\frac{d}{dt}\ln\left(\frac{A_h}{A_l}\right) = \chi \cdot \frac{A_c}{A_c+A_d}$. In Multiple Equilibria in DTC with Wage Premium + Credit Frictions we found that: market imperfections (like credit issues) can cause an economy to settle into several different long-term outcomes rather than just one,i.e. multiple equilibria exists. Credit rationing and wage bias jointly determine direction of innovation. Skill ratio, skill premium and demand for skill show that: An economy can "get stuck" in a low-skill/low-wage equilibrium or transition to a high-skill/high-wage equilibrium depending on initial conditions or external shocks. And about stability, typically, intersections where demand is steeper than supply (or crossing from above to below) are stable, while others might be unstable "tipping points.

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Appendix 1

Lagrangian here is:

inequality 20

$$\mathcal{L} \supset \sum_{t=0}^{\infty} \beta^t \left[u(C_t, S_t) + \mu_t((1-\delta)S_t - \xi Y_{dt} - S_{t+1})\right]$$

FOC w.r.t. dirty output Y_{dt}:$\frac{\partial \mathcal{L}}{\partial Y_{dt}} = 0$ gives:

equation 139

$$-\beta^t \mu_t \xi + \sum_{s=t}^{\infty} \beta^s \frac{\partial u}{\partial S_s} \frac{\partial S_s}{\partial Y_{Dt}} = 0$$

propagate effect forward

Since:

equation 140

$$S_s = (1-\delta)^{s-t} S_t - \xi \sum Y_{Dt}$$

we get:

equation 141

$$\frac{\partial S_s}{\partial Y_{Dt}} = -\xi(1-\delta)^{s-t}$$

if we divide by marginal utility :$\frac{\partial u}{\partial C_t}$ we get :

equation 142

$$\tau_t^{carbon} = \sum_{s=t}^{\infty} \beta^{s-t} \frac{\partial u / \partial S_s}{\partial u / \partial C_t} \cdot \xi \frac{\partial Y_{Ds}}{\partial Y_{Dt}}$$

This is just present discounted marginal environmental damage.For Mirrlees tax : Workers differ by type $\theta \in \{h, l\}$.Planner cannot observe type → incentive constraint:

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inequality 21

$$u(c_h) - u(c_l) \geq w_h - w_l$$

Mirrlees tax is inequality tax = equity vs efficiency tradeoff.Planner internalizes three channels:

$$\frac{\partial Y}{\partial A_c} - output$$

$$\frac{\partial S}{\partial A_c} = -\xi \frac{\partial Y_d}{\partial A_c} - environment$$

$$\frac{\partial \pi}{\partial A_c} - SBTC\ link\ ineqyuality$$

Lagrangian FOC for A_c; $\frac{\partial \mathcal{L}}{\partial A_C} = 0$

gives:

equation 143

$$\lambda_Y \frac{\partial Y}{\partial A_c} + \lambda_S \frac{\partial S}{\partial A_c} + \lambda_\pi \frac{\partial \pi}{\partial A_c} = 0$$

final result:

equation 144

$$\sigma_c^* = \underbrace{\frac{\partial Y}{\partial A_c}}_{\text{growth}} + \underbrace{\lambda_S \frac{\partial S}{\partial A_c}}_{\text{environment}} + \underbrace{\lambda_\pi \frac{\partial \pi}{\partial A_c}}_{\text{inequality}}$$