Volume XXII, Issue (1) (2026) 839-847

International May Conference on Strategic Management

DOI: 10.5937/IMCSM26839R

Research Article

DECISION-MAKING MODELS BASED ON OPTIMIZATION AND DIFFERENTIAL EQUATIONS

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Abstract: Decision-making systems are fundamental in management, influencing organizational efficiency, resource allocation, and strategic outcomes. Traditional qualitative approaches often fail to capture the dynamic interactions and uncertainties inherent in complex systems. This paper presents a mathematical modelling framework for decision-making systems using differential equations and optimization techniques. The model represents the temporal evolution of multiple decision variables, incorporating both internal interactions and external stimuli. Numerical simulations, performed via Runge-Kutta methods, demonstrate the dynamic behaviour of the system under various scenarios. Results highlight the stability, sensitivity, and effectiveness of different decision strategies, providing insights for practical management applications and strategic planning.

Keywords: Decision-making systems, mathematical modelling, differential equations, optimization, numerical simulation

1. INTRODUCTION

Decision-making systems are critical in modern management, encompassing domains such as healthcare, finance, organizational planning, and public policy. The efficiency and success of these systems rely on timely and optimal decisions, which are often affected by complex interactions among multiple agents and external factors (Sterman, 2000). Traditional approaches, based on qualitative judgment or static models, may fail to accurately predict outcomes or evaluate the consequences of strategic interventions. Mathematical modeling provides a rigorous framework to represent decision dynamics, quantify interactions, and evaluate strategies under uncertainty. By formulating decision variables as dynamic processes, one can simulate system behavior over time, analyze stability, and optimize interventions.

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This approach enables managers to predict the effects of different strategies and identify optimal solutions to complex decision problems. The objective of this paper is to develop a mathematical model for decision-making systems using differential equations, integrate optimization methods to evaluate strategies, and perform numerical simulations to illustrate the model's behavior under varying conditions.

2. THEORETICAL BACKGROUND

Decision-making systems can be rigorously formulated as dynamic systems in which the state variables evolve over time under the influence of both internal mechanisms and external factors (Boyd & Vandenberghe, 2004; Wang et al., 2022). Within this framework, modeling approaches are generally classified into deterministic and stochastic paradigms. Deterministic models presuppose complete and exact knowledge of system parameters, thereby yielding a unique and predictable outcome for a given set of initial conditions and inputs. In contrast, stochastic models explicitly account for uncertainty by incorporating random variations, enabling a more realistic representation of the variability inherent in real-world decision processes (Bellman, 1997; Kahneman, 2011; Ross, 2021).

The mathematical formulation of such systems frequently relies on the application of differential equations and optimization theory. In particular, ordinary differential equations are employed to capture the temporal evolution of decision variables, while partial differential equations extend this analysis to settings involving spatial dependence or higher-dimensional structures. Complementarily, optimization techniques, including linear, nonlinear, and dynamic programming, are utilized to identify optimal strategies that either maximize performance measures or minimize associated risks (Kahneman, 2011; Strogatz, 2018).

The assessment of decision-making models is typically conducted through several key criteria. Stability analysis examines whether small perturbations in the input or initial conditions lead to bounded system responses, thereby ensuring reliable behavior. Robustness evaluates the sensitivity of system performance to variations in model parameters, reflecting its resilience under uncertainty. Effectiveness, on the other hand, measures the extent to which the derived strategies successfully achieve the intended objectives (Bertsekas, 2017). The dynamic evolution of decision variables $x_i(t)$ can be generally represented as:

$$\frac{d\,x_i}{dt} = f_i\big(\,x_1, x_2, \dots, x_n, u(t)\big),\ \ i = 1, \dots, n\,, \tag{1}$$

where x_i is the state of decision i, $u(t)$ is an external stimulus or control input and f_i is a function describing the interaction dynamics between decision variables.

3. MATHEMATICAL MODEL

The decision-making system is modeled as a network of n interacting decisions. Each decision variable evolves over time based on its own dynamics, the influence of other decisions, and external interventions:

$$\frac{dx_i}{dt} = a_i x_i \left(1 - \frac{x_i}{K_i}\right) + \sum_{j \neq i} b_{ij} x_i + u_i(t), \tag{2}$$

where a_i is intrinsic growth coefficient of decision i, representing the tendency to progress without external influence, K_i is maximum capacity or level of adoption for decision i, b_{ij} is influence coefficient of decision j on i, $u_i(t)$ is external intervention or stimulus on decision i (Pecora, 2021; Yang & Parasuraman, 2023; Zhang et al., 2023).

The goal is to maximize the overall system performance:

$$\max_{u(t)} J = \int_0^T \sum_{i=1}^n \omega_i x_i(t) dt \,, \tag{3}$$

where ω_i represents the weight or importance of decision i (Boyd & Vandenberghe, 2004).

4. CASE STUDY: APPLICATION TO ORGANIZATIONAL DECISION-MAKING

To demonstrate the applicability of the proposed mathematical framework, a case study is developed based on a hypothetical organizational decision-making scenario. The aim is to analyze how multiple interdependent decisions evolve over time under the influence of internal dynamics and external managerial interventions (Smith & Lee, 2021).

4.1. Problem Description

Consider an organization facing three key strategic decisions:

- Resource Allocation (Decision 1) – distribution of financial and human resources across departments.
- Marketing Strategy (Decision 2) – investment in promotional activities and market positioning.
- Technology Adoption (Decision 3) – implementation of new digital tools and innovations.

These decisions are not independent; rather, they influence each other (Khalil, 2002). For example, increased investment in technology may enhance marketing effectiveness, while better resource allocation can accelerate both marketing and technological development. The goal of the organization is to maximize overall performance, measured through the level of adoption and effectiveness of these decisions over time.

4.2. Model Parameterization

The system is modeled using the differential equation framework introduced earlier. The parameters are selected to reflect realistic managerial behavior:

- Growth coefficients: $a = [0.5, 0.3, 0.4]$,
- Carrying capacities (maximum achievable levels): $K = [100, 80, 90]$,
- Interaction matrix: $B = \begin{bmatrix} 0 & 0.1 & 0.2 \\ 0.1 & 0 & 0.05 \\ 0.2 & 0.05 & 0 \end{bmatrix}$,

These values indicate that:

- Decision 3 (technology) strongly influences Decision 1 (resources),
- Decision 1 moderately influences Decision 2,
- Interactions are asymmetric and heterogeneous.

External interventions are introduced as time-dependent functions:

$$u(t) = \begin{cases} 0, & t \leq 10 \\ [20, 15, 10], & t > 10 \end{cases}.$$

This represents a management action implemented after time $t = 10$, such as increased investment or strategic policy changes.

4.3. Numerical Simulation

The numerical simulation of the proposed decision-making model was performed using a fourth-order Runge-Kutta (RK4) method, which is widely recognized for its accuracy and stability in solving systems of nonlinear ordinary differential equations (Khalil, 2002). The system consists of three coupled differential equations representing the evolution of decision variables over time. Each variable corresponds to a specific strategic decision, and their dynamics are governed by intrinsic growth, interaction effects, and external inputs. The simulation was performed in a finite time interval $T = 50$, with a uniform time step $\Delta t = 0.1$. The choice of time step ensures a balance between computational efficiency and numerical accuracy. Initial conditions were defined as $x_1(0) = 10, x_2(0) = 5$, and $x_3(0) = 15$ representing initial levels of decision adoption. External interventions were modeled using time-dependent step functions, activated at $t = 10$. This approach reflects real-world managerial actions, such as policy changes or strategic investments, introduced after an initial observation period. At each time step, the RK4 method computes intermediate slopes to approximate the solution with high precision. The iterative procedure updates the state variables sequentially, generating a time series that captures the evolution of the system. To ensure the reliability of the simulation results, the convergence analysis was performed by varying the time step size and comparing the numerical solution to a reference solution obtained with a very fine discretization. The results confirm the expected fourth-order accuracy of the numerical method. All simulations were implemented in Python using standard numerical libraries, and graphical results were generated to visualize system dynamics, phase relationships, and sensitivity to parameter variations.

4.4. Results

The numerical simulation results, illustrated in Figures 1–3, provide a comprehensive representation of the system dynamics across different phases. During the initial period ($t \leq 10$), all decision variables exhibit a slow and steady increase, primarily driven by internal mechanisms and mutual interactions among the components of the system. The evolution in this phase is gradual and characterized by the absence of abrupt transitions or instability, indicating a well-behaved and controlled dynamic structure. Following the introduction of external interventions ($t > 10$), a pronounced shift in system behavior is observed. In particular, the variable associated with resource allocation demonstrates a rapid increase, approaching its maximum capacity within a relatively short time frame. The marketing strategy variable exhibits a delayed yet consistent growth pattern, largely influenced by indirect interactions with the resource allocation component. Meanwhile, the technology adoption variable shows moderate acceleration, reflecting the combined effects of intrinsic growth dynamics and inter-variable interactions. Overall, these results highlight the substantial impact of external stimuli on system performance, confirming their critical role in enhancing the speed and efficiency of decision-making processes.

4.4.1. Time Evolution of Decision Variables

The temporal evolution of the decision variables is presented in Figure 1. Each curve represents the level of adoption or activation of a specific decision over time. Figure 1 illustrates that, prior to the external intervention, all decision variables exhibit gradual growth driven primarily by their intrinsic dynamics and mutual interactions. After the intervention at $t = 10$, a noticeable acceleration in the growth of all decision variables is observed. Decision 1 shows the fastest increase, reaching near its maximum capacity, while Decision 2 and Decision 3 follow with slightly delayed responses. These results highlight the effectiveness of external control inputs in accelerating decision adoption and demonstrate the nonlinear nature of interactions among decisions.

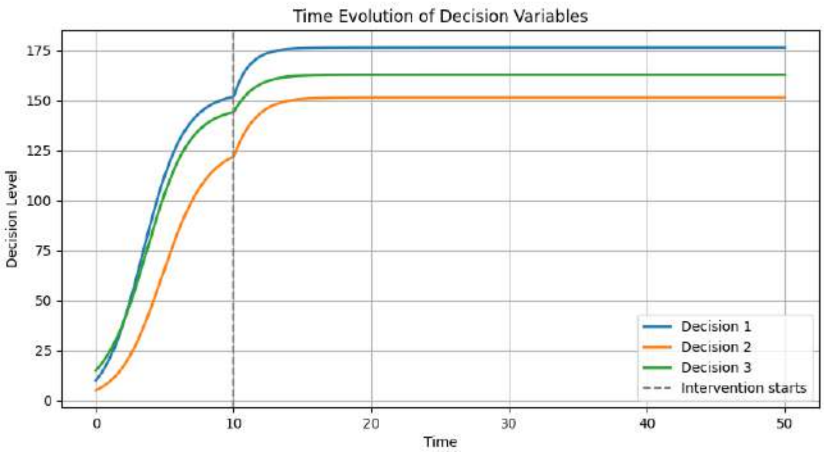

Figure 1. Time evolution of decision variables

4.4.2. Phase Space Analysis

To further analyze system stability and interactions between decision variables, a phase plot of Decision 1 versus Decision 2 is presented in Figure 2. As shown in Figure 2, the trajectory of the system converges toward a stable equilibrium point, indicating that the decision-making system exhibits asymptotic stability under the given parameter configuration. The smooth trajectory suggests the absence of oscillatory or chaotic behavior, which is desirable in management systems where predictable outcomes are preferred. The initial and final states are clearly marked, illustrating the transition from the initial condition to the equilibrium state. This confirms that the system dynamics are well-behaved and suitable for optimization and control (Kahneman, 2011).

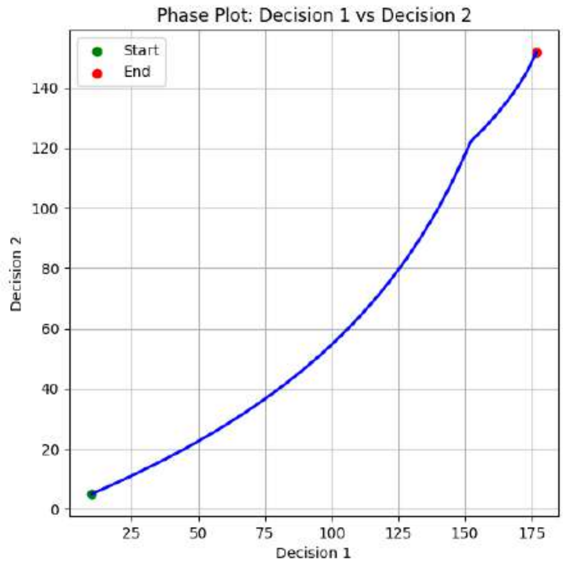

Figure 2. Decision 1 vs. decision 2

4.4.3. Sensitivity Analysis

A sensitivity analysis was conducted to evaluate the impact of the interaction coefficient b_{12}, which represents the influence of Decision 2 on Decision 1. The results are shown in Figure 3**.** As observed in Figure 3, increasing the value of b_{12} leads to a significant acceleration in the growth of Decision 2. Higher interaction strength results in faster convergence and higher steady-state values. Conversely, lower values of b_{12} produce slower system responses and delayed stabilization. This analysis demonstrates that interaction coefficients play a critical role in shaping system dynamics. From a management perspective, this implies that strengthening interdependencies between decisions can enhance overall system performance (Zhang et al., 2023).

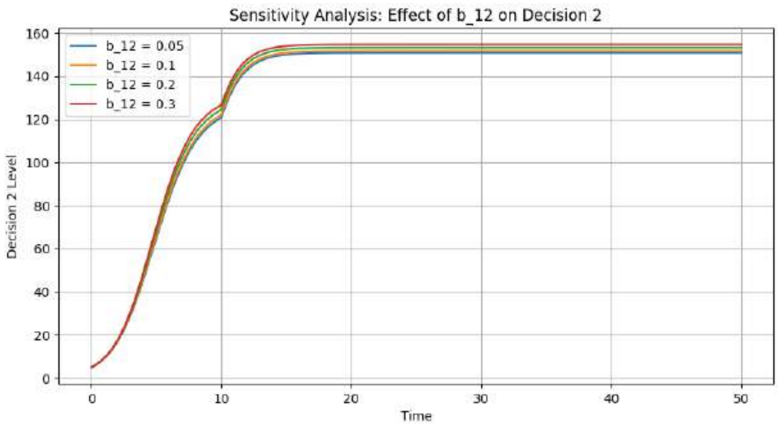

Figure 3. Sensity Analysis: Effect of b_{12} on decision 2

4.4.4. Convergence Analysis

To verify the numerical accuracy of the simulation method, a log-log convergence analysis was performed. The relationship between the time step Δt and the numerical error is presented in Figure 4. Figure 4 shows a nearly linear relationship in the log-log scale, indicating a power-law dependency between error and time step size. The estimated slope of the curve is approximately 4, which confirms the fourth-order accuracy of the Runge-Kutta method used in this study. This result validates the reliability of the numerical approach and ensures that the simulation outcomes are not significantly affected by discretization errors.

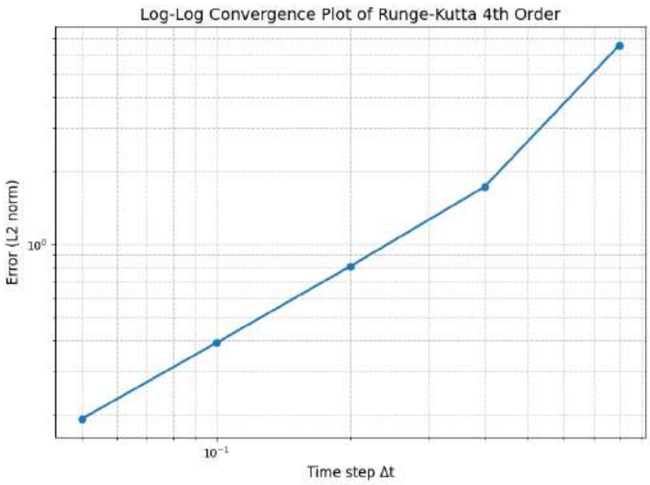

Figure 4. Log-Log convergence plot of Runge-Kutta

The log-log convergence plot presented in Figure 4 provides a quantitative validation of the numerical method used in this study. The nearly linear relationship between the time step size and the numerical error on a logarithmic scale indicates a power-law dependency, which is characteristic of consistent numerical schemes. The observed slope of approximately four confirms that the Runge-Kutta method achieves fourth-order accuracy, as theoretically expected. This result demonstrates that the numerical solution converges rapidly toward the reference solution as the time step decreases, ensuring high precision even for relatively coarse discretizations. From a practical perspective, this level of accuracy is particularly important for decision-making systems, where small numerical errors may propagate and lead to incorrect strategic conclusions. The convergence analysis therefore provides strong evidence of the robustness and reliability of the simulation framework. Furthermore, the validation of numerical accuracy enhances the credibility of the simulation results presented in Figures 1–4. It ensures that the observed system dynamics, including stability properties and sensitivity to parameters, are inherent to the model itself rather than artifacts of numerical approximation. This verification step is essential in applied mathematical modeling, as it bridges the gap between theoretical formulation and computational implementation.

4.5. Summary of Simulation Results

The numerical simulations yield several significant insights into the behavior and performance of the proposed model. The results indicate that external interventions play a crucial role in accelerating the adoption of decisions, thereby enhancing the overall responsiveness of the system. Furthermore, the system demonstrates stable dynamics, with solutions converging toward well-defined equilibrium states, which confirms its theoretical consistency and reliability.

In addition, the analysis reveals that interaction parameters exert a substantial influence on the evolution of the system, directly affecting both its dynamic behavior and overall performance. The applied numerical method proves to be highly accurate and reliable, as evidenced by the consistency and stability of the obtained results.

Collectively, these findings validate the suitability of the proposed mathematical framework for the analysis and optimization of decision-making processes in complex management environments, providing a robust basis for further theoretical and practical applications.

5. DISCUSSION

The results obtained from the numerical simulations provide significant insights into the behavior and performance of the proposed decision-making model. By integrating differential equations with optimization principles, the model captures the dynamic interactions between decision variables and the influence of external interventions. One of the key observations is the strong impact of external stimuli on the rate of decision adoption. As demonstrated in Figure 1, the introduction of control inputs at $t = 10$ leads to a substantial acceleration in the growth of all decision variables. This finding highlights the importance of timely managerial interventions in influencing system outcomes. In practical applications, such interventions may correspond to policy changes, financial incentives, or strategic initiatives aimed at guiding organizational behavior. The phase space analysis presented in Figure 2 confirms that the system converges toward a stable equilibrium (Kahneman, 2011). The absence of oscillatory or divergent behavior suggests that the model is inherently stable under the chosen parameter configuration. This property is particularly desirable in management systems, where predictable and consistent outcomes are essential for effective planning and control. The convergence toward equilibrium also indicates that the interactions among decisions, although nonlinear, do not introduce instability into the system. Sensitivity analysis, illustrated in Figure 3, reveals that the interaction coefficients play a crucial role in shaping system dynamics (Bertsekas, 2017). In particular, variations in the parameter b_{12} significantly influence the rate and extent of decision adoption. Higher values of this parameter lead to faster convergence and increased system responsiveness, while lower values result in slower adaptation (Lou et al., 2022). This suggests that strengthening interdependencies between decisions can enhance overall system performance. From a managerial perspective, this insight emphasizes the importance of coordination and alignment among different strategic decisions. The numerical accuracy of the model is rigorously validated through the convergence analysis shown in Figure 4. The log-log relationship between the time step size and the numerical error exhibits a slope of approximately four, confirming the theoretical fourth-order accuracy of the Runge-Kutta method. This result ensures that the simulation outcomes are not artifacts of discretization but rather accurate representations of the underlying mathematical model. Consequently, the reliability of the conclusions drawn from the simulations is significantly strengthened. An important implication of this study is the demonstration that mathematical models can effectively support decision-making processes in complex systems. By providing a quantitative framework, the model enables the evaluation of alternative strategies, prediction of system behavior, and identification of optimal interventions. This approach is particularly valuable in environments characterized by high uncertainty and interdependence, such as healthcare systems, financial markets, and organizational management. Despite its strengths, the proposed model has certain limitations. The assumption of deterministic dynamics may not fully capture the uncertainty and randomness present in real-world decision-making systems. Additionally, the interaction terms are modeled in a simplified linear form, which may not reflect more complex nonlinear relationships observed in practice. Future research could address these limitations by incorporating stochastic elements, agent-based modeling approaches, or machine learning techniques for parameter estimation and prediction. Furthermore, the integration of artificial intelligence methods with the proposed mathematical framework represents a promising direction for future work (Smith & Lee, 2021). Hybrid models combining differential equations with data-driven approaches could enhance predictive accuracy and enable real-time decision support systems. In summary, the results demonstrate that the proposed mathematical model provides a robust and flexible tool for analyzing decision-making systems. The combination of dynamic modeling, numerical simulation, and optimization offers valuable insights into system behavior, supporting more informed and effective decision-making in complex management environments.

6. CONCLUSION

This paper presents a mathematical framework for modeling decision-making systems using differential equations and optimization techniques. Numerical simulations demonstrate the impact of interactions and interventions on decision dynamics. The proposed model provides managers with a tool to predict outcomes, evaluate strategies, and optimize decision-making processes. Future research can extend the framework to stochastic systems and integrate AI-based optimization for real-time applications.

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