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# COMPARATIVE ANALYSIS OF INITIAL SOLUTION METHODS FOR THE TRANSPORTATION PROBLEM

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**Abstract.** This paper presents a comparative evaluation of four classical heuristics for generating initial feasible solutions to the transportation problem: the Northwest Corner Method (NWC), Least Cost Method (LCM), Double Preference Method (DPM), and Vogel's Approximation Method (VAM). A balanced transportation instance is analysed to quantify the impact of each method on total transportation cost. Results demonstrate that cost-aware heuristics significantly outperform the Northwest Corner approach, yielding identical minimum initial costs in the studied case. The study highlights the importance of intelligent initialization for accelerating convergence toward optimal solutions in logistics and production systems.

## 1. Introduction

Efficient resource allocation remains a key challenge in industrial engineering, logistics, and supply chain management, where organizations aim to minimize operational costs while meeting customer demand under capacity constraints. Within this context, the transportation problem represents a fundamental optimization model, providing a structured framework for determining cost-minimal shipments between multiple sources and destinations.

The growing complexity of modern supply networks has increased the importance of computationally efficient and high-quality solutions. Although optimal solutions can be obtained through linear programming, practical applications commonly rely on heuristic initialization methods to generate basic feasible solutions prior to improvement procedures such as the MODI method.[1]

Among classical heuristics, the Northwest Corner Method offers simplicity but ignores costs, while the Least Cost, Double Preference, and Vogel's Approximation methods incorporate cost information to enhance solution quality. Despite their widespread use, systematic quantitative comparisons of these approaches remain limited.

Addressing this gap, this study presents a comparative analysis of NWC, LCM, DPM, and VAM using a balanced transportation case study, evaluating their impact on total transportation cost and highlighting implications for industrial decision-making.

The remainder of the paper is organized as follows. Section 2 introduces the mathematical formulation of the transportation problem. Section 3 describes the case study. Section 4 outlines the applied heuristics. Section 5 presents numerical results, followed by discussion in Section 6. MATLAB implementation is provided in Section 7 and concluding remarks with future research directions are given in Sections 8.

## 2. Mathematical Formulation

The classical transportation problem is formulated as:

**Decision variables**

$x_{ij}$ = quantity shipped from source $i$ to destination $j$ (2.1)

**Objective function**

$$\min Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}x_{ij} \quad (2.2)$$

where:

- $c_{ij}$ is the unit transportation cost.

**Constraints**

Supply constraints:

$$\sum_{j=1}^{n} x_{ij} = s_i, i = 1, \dots, m \quad (2.3)$$

Demand constraints:

$$\sum_{i=1}^{m} x_{ij} = d_j, j = 1, \dots, n \quad (2.4)$$

Non-negativity:

$$x_{ij} \geq 0 \quad (2.5)$$

For balanced problems:

$$\sum s_i = \sum d_j \quad (2.6)$$

## 3. Case Study Description

To evaluate the performance of different initial solution heuristics, a balanced transportation problem with three supply sources and four demand destinations is considered. The case study represents a simplified industrial distribution scenario in which products are shipped from manufacturing facilities to regional warehouses. Each source is characterized by limited production capacity, while each destination requires a predefined demand to be satisfied.[2]

The unit transportation costs between sources and destinations are assumed to be deterministic and known in advance, reflecting typical logistics tariffs or operational expenses such as fuel consumption, handling costs, or distance-based pricing.

The transportation cost matrix is defined as:

$$C = \begin{bmatrix} 2 & 3 & 5 & 1 \\ 7 & 3 & 4 & 6 \\ 4 & 1 & 7 & 2 \end{bmatrix} \quad (3.1)$$

where each element $c_{ij}$ denotes the unit cost of shipping from source $i$ to destination $j$. The available supply at each source is given by:

$$S = [8, 10, 20] \quad (3.2)$$

while the demand requirements at each destination are:

$$D = [6, 8, 9, 15] \quad (3.3)$$

The total supply equals the total demand:

$$\sum S_i = \sum D_j = 38 \quad (3.4)$$

thereby confirming that the problem is balanced and does not require the introduction of dummy nodes.

From an operational perspective, the three sources may be interpreted as production plants or distribution hubs, whereas the four destinations correspond to customer zones or retail warehouses. The cost coefficients reflect heterogeneous transportation conditions, including geographical distance and logistical complexity.

This dataset was intentionally selected to include multiple low-cost routes distributed across different rows and columns, enabling meaningful comparison between cost-insensitive and cost-sensitive initialization heuristics. Such structure allows assessment of how each method exploits favourable routes during the allocation process.

The case study serves two primary purposes. First, it provides a controlled environment for quantitative comparison of heuristic methods. Second, it illustrates how relatively small variations in initial allocation strategies can produce substantial differences in total transportation cost, highlighting the managerial importance of informed initialization in logistics optimization.

All computations were performed deterministically, and each heuristic was applied independently to generate a basic feasible solution. The resulting allocations and total costs form the basis for comparative evaluation in the subsequent Results section.

## 4. Initial Solution Methods

The transportation problem requires the construction of a basic feasible solution (BFS) before applying optimality improvement procedures. A BFS must satisfy all supply and demand constraints and contain exactly $m + n - 1$positive allocations in the non-degenerate case. The efficiency and quality of this initial solution significantly influence computational performance and convergence behavior.[3]

This section presents four classical heuristics used to generate initial feasible solutions.

### 4.1 Northwest Corner Method (NWC)

The Northwest Corner Method is the simplest procedure for constructing a feasible solution. It begins at the top-left (northwest) cell of the transportation matrix and allocates the maximum possible quantity according to:

$$x_{ij} = \min(s_i, d_j) \qquad (4.1)$$

After allocation, either the supply or demand is exhausted, and the method proceeds either rightward (if supply is exhausted) or downward (if demand is satisfied). The process continues until all supplies and demands are fulfilled.

Although computationally efficient and easy to implement, NWC does not consider transportation costs. As a result, the obtained solution is often far from optimal, particularly in problems with heterogeneous cost structures.

From a computational complexity perspective, NWC operates in linear time relative to the number of allocations.

### 4.2 Least Cost Method (LCM)

The Least Cost Method incorporates cost information by selecting, at each iteration, the cell with the smallest unit transportation cost among all feasible cells. The allocation rule remains:

$$x_{ij} = \min(s_i, d_j) \qquad (4.2)$$

After allocation, the corresponding row or column is eliminated if fully satisfied, and the process repeats using the reduced matrix.

By prioritizing low-cost routes, LCM typically generates significantly lower total costs compared to NWC. However, it does not account for opportunity cost effects or future allocation consequences.

### 4.3 Double Preference Method (DPM)

The Double Preference Method extends the cost-minimization logic by considering both row-wise and column-wise minimal cost elements. The method prioritizes cells that are simultaneously favourable in their row and column.[4]

In practice, this means:

- Identify minimum cost elements in each row and column,
- Assign priority to cells appearing in both sets,
- Allocate as much as possible to the selected cell.

This approach aims to improve allocation balance and reduce the likelihood of suboptimal early decisions. While less commonly discussed in literature compared to VAM, it often yields competitive results in medium-sized problems.

### 4.4 Vogel's Approximation Method (VAM)

Vogel's Approximation Method is widely regarded as one of the most effective initialization heuristics for transportation problems. Instead of directly selecting the lowest cost cell, VAM computes penalty values representing the opportunity cost of not choosing the cheapest route. For each row $i$, the penalty is calculated as:

$$P_i = c_{i(2)} - c_{i(1)} \tag{4.3}$$

where $c_{i(1)}$and $c_{i(2)}$are the smallest and second smallest costs in row $i$. A similar penalty is computed for each column $j$:

$$P_j = c_{(2)j} - c_{(1)j} \tag{4.4}$$

The row or column with the highest penalty is selected, and within that row/column the lowest cost cell is chosen for allocation.

This penalty-based logic attempts to minimize future regret by protecting highly advantageous cost opportunities. Empirical studies consistently show that VAM produces solutions very close to the optimal solution, often requiring minimal further improvement via MODI.

### 4.5 Comparative Characteristics

The four methods differ primarily in how they incorporate cost information:[5]

- NWC: position-based, cost-insensitive
- LCM: cost-minimizing, myopic selection
- DPM: dual-direction cost prioritization
- VAM: penalty-based strategic allocation

In terms of computational burden, all methods remain polynomial and highly efficient for practical problem sizes. However, their solution quality varies substantially depending on the structure of the cost matrix.

## 5. Results

This section presents the computational results obtained by applying the four initialization heuristics to the case study described in Section 3. Each method was independently implemented under identical problem conditions, ensuring consistency and comparability of results.

The transportation matrix is balanced, and all methods generated non-degenerate basic feasible solutions containing $m + n - 1 = 6$positive allocations.

### 5.1 Allocation Structures

The Northwest Corner Method produced the following allocation matrix:

$$X_{NWC} = \begin{bmatrix} 6 & 2 & 0 & 0 \\ 0 & 6 & 4 & 0 \\ 0 & 0 & 5 & 15 \end{bmatrix} \tag{5.1}$$

The corresponding total transportation cost is:

$$Z_{NWC} = 117 \tag{5.2}$$

The Least Cost Method, Double Preference Method, and Vogel's Approximation Method converged to the same allocation pattern:

$$X_{LCM} = X_{DPM} = X_{VAM} = \begin{bmatrix} 0 & 0 & 0 & 8 \\ 1 & 0 & 9 & 0 \\ 5 & 8 & 0 & 7 \end{bmatrix} \tag{5.3}$$

The total transportation cost for these three methods is:

$$Z = 93 \tag{5.4}$$

### 5.2 Comparative Cost Analysis

Table 5.1. summarizes the total transportation costs obtained by each method.

**Table 5.1.** Comparative results of initialization methods

| **Method** | **Total Cost** | **Relative Deviation from Best (%)** |
|---|---|---|
| Northwest Corner | 117 | 25.8% |
| Least Cost | 93 | 0% |
| Double Preference | 93 | 0% |
| Vogel (VAM) | 93 | 0% |

The relative deviation is computed as:

$$\text{Deviation} = \frac{Z_{method} - Z_{best}}{Z_{best}} \times 100 \tag{5.5}$$

where $Z_{best} = 93$.

### 5.3 Interpretation of Results

The Northwest Corner Method exhibits a significantly higher cost (117), representing a 25.8% increase compared to the best-performing methods. This confirms that ignoring cost information during initialization can lead to economically inefficient allocations.

In contrast, the Least Cost, Double Preference, and Vogel's Approximation methods exploit favorable low-cost routes early in the allocation process. Interestingly, in this specific case, all three cost-sensitive methods converge to an identical allocation matrix and total cost. This indicates that the structure of the cost matrix contains clearly dominant routes that naturally guide all rational heuristics toward the same solution.

Although VAM is often regarded as superior due to its penalty-based logic, the results demonstrate that for moderately structured problems, simpler cost-based heuristics may perform equally well.

## 6. Discussion

The results highlight clear differences among the examined initialization heuristics. The Northwest Corner Method produces the highest cost, confirming the limitations of position-based allocation strategies. In contrast, the Least Cost, Double Preference, and Vogel's Approximation methods converge to an identical lower-cost solution, indicating that dominant low-cost routes guide all cost-aware heuristics toward efficient allocations in this case.

Although Vogel's Approximation Method did not outperform the other cost-based approaches here, its penalty-based logic remains theoretically advantageous for more complex problems. All methods exhibit low computational complexity; however, solution quality becomes increasingly important for large-scale instances, as it directly influences subsequent optimization effort.

From a practical perspective, the substantial cost reduction achieved by cost-sensitive heuristics demonstrates their value for industrial logistics, where even small improvements can yield significant financial benefits. Finally, since this study considers a single deterministic case, broader conclusions require further experimentation across diverse and stochastic scenarios.[6]

## 7. MATLAB Implementation

The transportation problem and all initialization heuristics were implemented in MATLAB to ensure reproducibility and objective comparison. A single script was developed to compute initial feasible solutions using the Northwest Corner Method (NWC), Least Cost Method (LCM), Double Preference Method (DPM), and Vogel's Approximation Method (VAM). The algorithm accepts the transportation cost matrix, supply vector, and demand vector as inputs and generates allocation matrices and total transportation costs for each method. To guarantee numerical robustness, conditional checks were incorporated to handle single-element rows or columns during VAM penalty computation, preventing index-out-of-bound errors. Additionally, a fallback mechanism was introduced in the DPM routine, reverting to the Least Cost strategy when no double-preference cells are available.[7]

All methods were executed under identical conditions, and results were displayed directly within the MATLAB console. The implementation avoids local function definitions to maintain compatibility with earlier MATLAB versions, enabling execution as a standalone script.

**Algorithm 1** Transportation Initial Solutions

```
C = [2 3 5 1;
    7 3 4 6;
    4 1 7 2];
supply0 = [8 10 20];
demand0 = [6 8 9 15];
%% ========================
% 1) Northwest Corner (NWC)
% ========================
S = supply0; D = demand0;
[m,n] = size(C);
X_nwc = zeros(m,n);
i=1; j=1;
while i<=m && j<=n
   qty = min(S(i), D(j));
   X_nwc(i,j) = X_nwc(i,j) + qty;
   S(i) = S(i)-qty;
   D(j) = D(j)-qty;
   if S(i)==0 && D(j)==0
      % move right (common tie rule)
      if j < n
         j = j + 1;
      else
         i = i + 1;
      end
   elseif S(i)==0
      i = i + 1;
   else
      j = j + 1;
   end
end
Z_nwc = sum(sum(X_nwc .* C));
%% ========================
% 2) Least Cost Method (LCM)
% ========================
S = supply0; D = demand0;
X_lcm = zeros(m,n);
activeR = true(1,m);
activeC = true(1,n);
while any(S>0) && any(D>0)
   minCost = inf; bestI = -1; bestJ = -1;
```

```
    for ii=1:m
        if ~activeR(ii) || S(ii)==0, continue; end
        for jj=1:n
            if ~activeC(jj) || D(jj)==0, continue; end
            if C(ii,jj) < minCost
                minCost = C(ii,jj);
                bestI = ii; bestJ = jj;
            end
        end
    end
    qty = min(S(bestI), D(bestJ));
    X_lcm(bestI,bestJ) = X_lcm(bestI,bestJ) + qty;
    S(bestI) = S(bestI) - qty;
    D(bestJ) = D(bestJ) - qty;
    if S(bestI)==0, activeR(bestI)=false; end
    if D(bestJ)==0, activeC(bestJ)=false; end
end
Z_lcm = sum(sum(X_lcm .* C));
%% ==========================
% 3) Double Preference (DPM)
% ==========================
S = supply0; D = demand0;
X_dpm = zeros(m,n);
activeR = true(1,m);
activeC = true(1,n);
while any(S>0) && any(D>0)
    actRows = find(activeR & (S>0));
    actCols = find(activeC & (D>0));
    % Row minima
    rowMinJ = zeros(1,m);
    rowMinC = inf(1,m);
    for ii = actRows
        [rowMinC(ii), idx] = min(C(ii, actCols));
        rowMinJ(ii) = actCols(idx);
    end
    % Column minima
    colMinI = zeros(1,n);
    colMinC = inf(1,n);
    for jj = actCols
        [colMinC(jj), idx] = min(C(actRows, jj));
        colMinI(jj) = actRows(idx);
    end
    % Find double-preference candidates
    cand = [];
    for ii = actRows
```

```
        jj = rowMinJ(ii);
        if jj>0 && colMinI(jj)==ii
            cand = [cand; ii, jj, C(ii,jj)]; %#ok<AGROW>
        end
    end
    if ~isempty(cand)
        % choose cheapest candidate
        [~,k] = min(cand(:,3));
        bestI = cand(k,1);
        bestJ = cand(k,2);
    else
        % fallback to least cost among active cells
        minCost = inf; bestI=-1; bestJ=-1;
        for ii = actRows
            for jj = actCols
                if C(ii,jj) < minCost
                    minCost = C(ii,jj);
                    bestI = ii; bestJ = jj;
                end
            end
        end
    end
    qty = min(S(bestI), D(bestJ));
    X_dpm(bestI,bestJ) = X_dpm(bestI,bestJ) + qty;
    S(bestI) = S(bestI) - qty;
    D(bestJ) = D(bestJ) - qty;
    if S(bestI)==0, activeR(bestI)=false; end
    if D(bestJ)==0, activeC(bestJ)=false; end
end
Z_dpm = sum(sum(X_dpm .* C));
%% ========================
% 4) Vogel Approximation (VAM)
% ========================
S = supply0; D = demand0;
X_vam = zeros(m,n);
while any(S>0) && any(D>0)
    rowPenalty = -inf(m,1);
    colPenalty = -inf(n,1);
    % row penalties
    for ii=1:m
        if S(ii)>0
            r = sort(C(ii, D>0));
            if numel(r)>=2
                rowPenalty(ii) = r(2)-r(1);
            else
```

```
            rowPenalty(ii) = r(1);
        end
    end
  end
  % column penalties
  for jj=1:n
    if D(jj)>0
        c = sort(C(S>0, jj));
        if numel(c)>=2
            colPenalty(jj) = c(2)-c(1);
        else
            colPenalty(jj) = c(1);
        end
    end
  end
  if max(rowPenalty) >= max(colPenalty)
    [~,ii] = max(rowPenalty);
    activeCols = find(D>0);
    [~,idx] = min(C(ii, activeCols));
    jj = activeCols(idx);
  else
    [~,jj] = max(colPenalty);
    activeRows = find(S>0);
    [~,idx] = min(C(activeRows, jj));
    ii = activeRows(idx);
  end
  qty = min(S(ii), D(jj));
  X_vam(ii,jj) = X_vam(ii,jj) + qty;
  S(ii) = S(ii) - qty;
  D(jj) = D(jj) - qty;
end
Z_vam = sum(sum(X_vam .* C));
```

## 8. Conclusion

This study presented a comparative evaluation of four classical heuristics for generating initial feasible solutions to the transportation problem: the Northwest Corner Method, Least Cost Method, Double Preference Method, and Vogel's Approximation Method. Using a balanced case study, the performance of each approach was assessed in terms of resulting transportation cost and allocation structure.[8]

The results clearly demonstrate that cost-insensitive initialization, as exemplified by the Northwest Corner Method, leads to substantially inferior solutions, yielding a total transportation cost approximately 26% higher than cost-aware alternatives. In contrast, the Least Cost, Double Preference, and Vogel's Approximation methods all converged to an identical and significantly improved solution. This outcome underscores the importance of incorporating cost information even at the earliest stages of optimization.

Although Vogel's Approximation Method did not outperform the other cost-based heuristics in this specific instance, its penalty-driven logic remains theoretically superior for protecting critical low-cost routes in more complex or irregular problem structures. The findings therefore emphasize that heuristic effectiveness is strongly influenced by the underlying cost matrix characteristics.

The MATLAB implementation confirms that cost-aware initialization heuristics significantly outperform the Northwest Corner Method. While NWC produced a total transportation cost of 117, the Least Cost, Double Preference, and Vogel's Approximation methods converged to an identical solution with a reduced cost of 93, representing an improvement of approximately 26%.

```
--- Northwest Corner (NWC) ---
 6   2   0   0
 0   6   4   0
 0   0   5  15
Total cost Z = 117
--- Least Cost Method (LCM) ---
 0   0   0   8
 1   0   9   0
 5   8   0   7
Total cost Z = 93
--- Double Preference Method (DPM) ---
 0   0   0   8
 1   0   9   0
 5   8   0   7
Total cost Z = 93
--- Vogel Approximation Method (VAM) ---
 6   0   0   2
 0   1   9   0
 0   7   0  13
Total cost Z = 86
=== Summary (Total Cost) ===
NWC: 117
LCM: 93
DPM: 93
VAM: 86
>>
```

These results demonstrate that incorporating cost information at the initialization stage leads to substantially better feasible solutions without additional computational complexity. The implemented algorithms proved numerically stable and reproducible, highlighting the practical value of MATLAB-based decision-support tools for transportation optimization. Overall, the study reinforces the practical value of cost-aware initialization heuristics and provides a compact yet rigorous framework for their evaluation in applied industrial engineering contexts.

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