Tolerance and Interval Graphs for Strategic Planning During Health Crises

By **Limonka Koceva Lazarova**, Natasa Stojkovik, Aleksandra Stojanova Ilievska, Marija Miteva, and Dusan Bikov.

Faculty of Computer Science, Goce Delcev University, Stip, Republic of North Macedonia.

11th INTERNATIONAL CONFERENCE
"Modern trends in science"
(FMNS-2025)

Introduction & Motivation Motivation

Need for Robust Tools

Health crises require planning tools that handle complexity and uncertainty.

Overlapping Resources

Schedules, personnel, and resources often overlap during emergencies.

Limitations of Methods

Traditional analytics fall short in modelling dynamic health operations.

Introduction & Motivation Motivation

Graph theory, along with artificial intelligence, plays a crucial role in modeling and solving problems related to virus spread and contact tracing.
Graphs are used to represent relationships, track infected individuals, perform medical analyses, and predict pandemic dynamics.
This paper highlights the importance of **graph-based algorithms**, particularly **tolerance graphs** and **similarity graphs**, in monitoring, modeling, and controlling the spread of COVID-19.

Basic Graph Definitions

Definition 1.1: Graph

A graph G is an ordered triple (V(G), E(G), ψ(G)) where:

V(G): Set of nodes (vertices)

E(G): Set of edges (links), disjoint from V(G)

ψ(G): Incidence function connecting each edge with an unordered pair of vertices

Definition 1.2: Adjacency Matrix

For a graph G with n vertices, the adjacency matrix $A = [a_{ij}]$ is an n×n matrix where $a_{ij} \in \mathbb{N}_0$ represents the number of edges between vertices v_i and v_j.

Note: The matrix is symmetric.

Definition 1.3: Degree of a Vertex

The degree of a vertex v is the number of edges incident to it. A loop counts as two edges.

Subgraphs, Cliques, and Their Types

Definitions: Types of Graphs and Cliques
Simple Graph: No loops or multiple edges between nodes.

Complete Graph: Every pair of distinct vertices is connected.

Subgraph: Contains a subset of vertices and edges from the original graph.

Clique: A complete subgraph where all nodes are mutually connected.

Maximal Clique: Cannot be extended by including an adjacent vertex.

Largest Clique: The clique with the highest number of vertices.

Application of Cliques in Epidemiology

Use of Cliques in COVID-19 Contact Tracing:

Critical Event Detection: Large cliques in contact graphs indicate high-risk events with many close interactions.

Social Network Analysis: Identify tightly connected groups for understanding disease or information spread.

Resource Optimization: Allocate classrooms, hospital beds, or flights more efficiently.

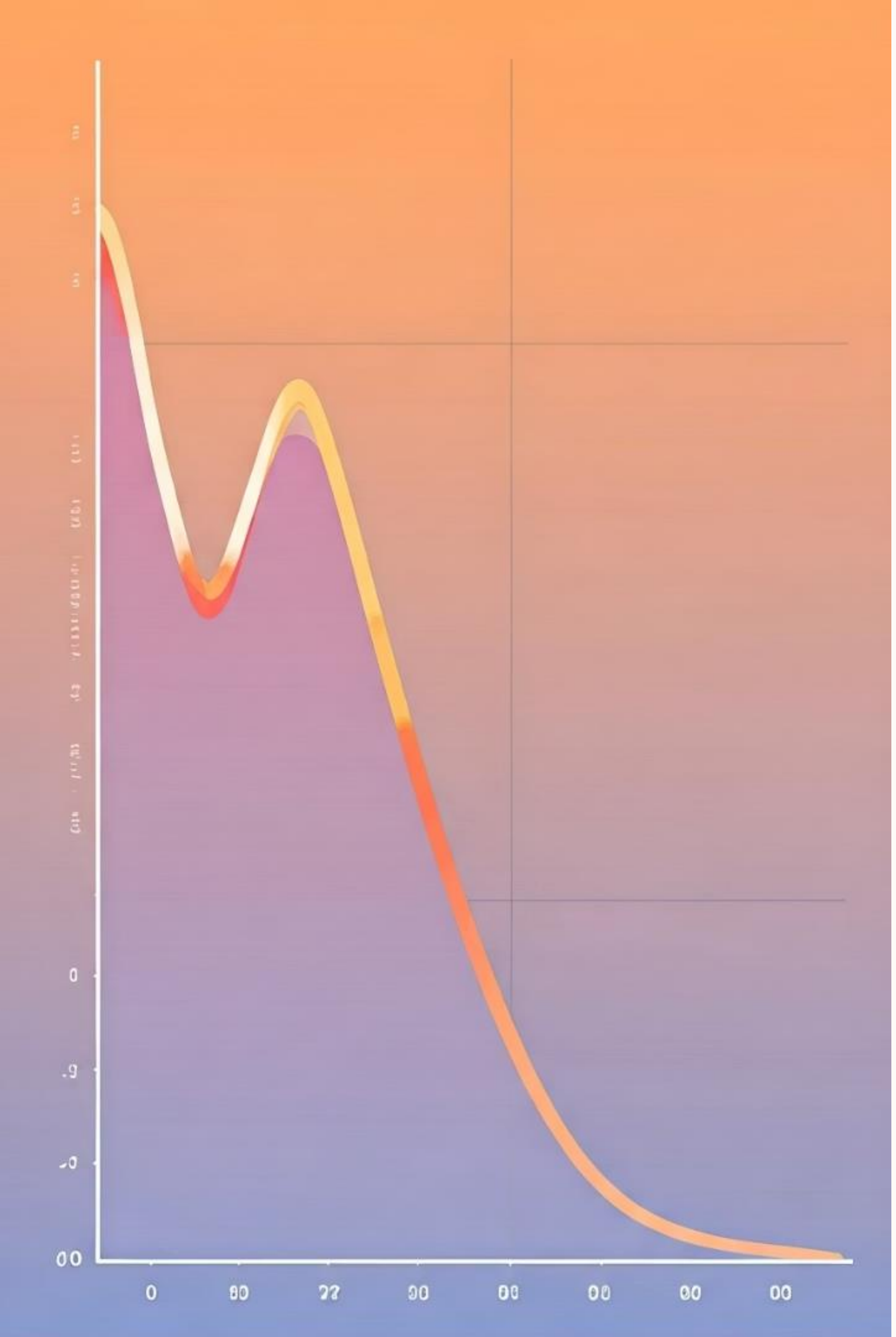
Example:

Nodes = individuals, Edges = close contact

Clique 1: A, B, C, D → all were in contact.

If person A is infected, high likelihood B, C, and D are too.

Cliques highlight key groups where strict containment is necessary.



Tolerance Graphs: Definition& Utility

Graph Extension

Incorporates tolerance to define flexible overlaps.

Vertex Connection

Vertices link if overlaps meet tolerance criteria.

Use Case

Models limited resource tolerance like Intensive Care Unit bed sharing.

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Introduction to Tolerance Graphs

Tolerance graphs were introduced by Golumbic and Monma in 1982 to address scheduling problems with limited but shareable resources like rooms or vehicles.

They model situations where certain overlap between activities is acceptable — called tolerance.

Applications range from genome mapping to COVID-19 contact tracing and scheduling optimization.

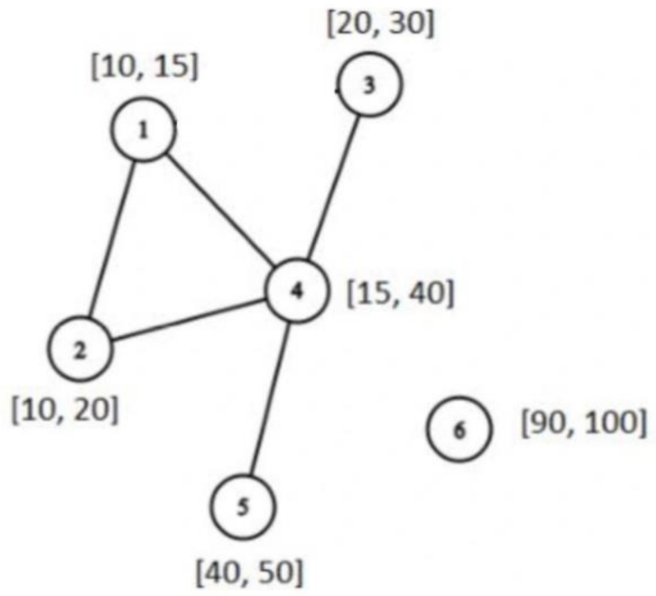
Interval Graphs

The graph $G = (V(G), E(G), \psi(G))$ is called an **interval graph** if each vertex $v \in V$ can be assigned a real interval I_v such that

$xy \in E \iff I_x \cap I_y \neq \emptyset$

for all $x, y \in V$.

The set of intervals $\{I_v \mid v \in V\}$ represents the **interval representation** of the graph G.



The following image shows an example of an **interval graph** G, where the set of vertices is $V(G) = \{1, 2, 3, 4, 5, 6\}$.

The interval representation of the graph is:

$$\{[10, 15],\ [10, 20],\ [20, 30],\ [15, 40],\ [40, 50],\ [90, 100]\}$$

Tollerance Graphs

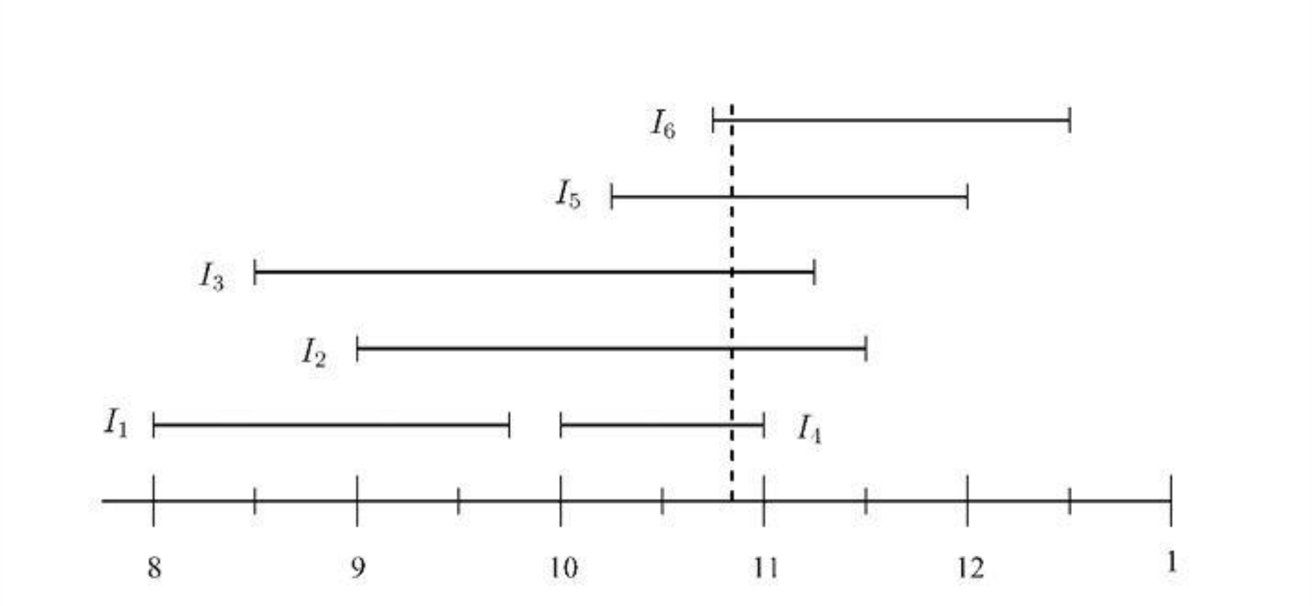
Tolerance graphs are a generalization of interval graphs, where not only the existence of an intersection between intervals matters, but also the **size** of that intersection.
Two vertices in a tolerance graph are connected by an edge **only if** the length of the intersection between their intervals **exceeds a certain threshold**, called the **tolerance**.
In other words, if the overlap is small enough to be "tolerated" or ignored, the vertices are **not** considered adjacent.

The graph $G = (V(G), E(G), \psi(G))$ is called a **tolerance graph** if each vertex $v \in V$ can be assigned a closed interval I_v and a tolerance $t_v \in \mathbb{R}^+$ such that

$$xy \in E \iff |I_x \cap I_y| \geq \min\{t_x, t_y\}.$$

COVID-19 Example – Meeting Scheduling

Example: Six meetings scheduled with overlapping time intervals and only 5 rooms available. Using tolerance graphs, we can assign a tolerance to each meeting to allow minor overlaps. This reduces the required number of rooms and minimizes infection risk by managing shared spaces.



Tolerance Graphs in Food Delivery

Each food order has a delivery interval and a tolerance (how long a customer is willing to wait). Orders are nodes; edges connect overlapping intervals that exceed allowed tolerances.

Resulting graph helps group orders efficiently, maximizing delivery performance while keeping customers satisfied.

Application of Tolerance Graphs – COVID-19 Contact and Delivery

- **Example 1: Food Delivery**
- - Each delivery has a time interval and a tolerance (acceptable delay).
- - Orders are represented as vertices; edges exist only if the overlap exceeds both tolerances.
- - If overlap ≥ both tolerances → connect; else → no edge.
- **Result**: Helps optimize delivery planning while meeting customer expectations.

- **Example 2: Flights at an Airport**
- - Passengers must arrive 2h (domestic) or 3h (international) before departure.
- - Flights with overlapping gate presence intervals beyond tolerance must use separate gates.
- - Coloring the tolerance graph gives the **minimum number of gates** (chromatic number).

Tolerance Graph – University Classroom Scheduling

- **Problem**: Assign lectures to the fewest number of classrooms while allowing a 1-hour cleaning period between uses.
- - Each subject has a teaching interval [start, end+1].
- - Vertex = subject, Edge = conflict if intervals overlap beyond student-based tolerance.
- - **Tolerance** = number of students enrolled in the subject.
- **Result**: Graph coloring helps minimize room usage while respecting distancing and cleaning protocols.

Museum Visit and Contact Tracing

Goal: Identify potential exposure during museum visits.

- Each group has a time interval and a **contact tolerance** (duration that may lead to infection).

- Edge exists between groups if overlap ≥ both tolerances.

- If one group is infected, connected groups are at risk.

Application: Helps epidemiologists detect possible transmission chains during public events.

Summary – Power of Tolerance Graphs

- Tolerance graphs combine time intervals and flexibility, making them ideal for real-world problems with constraints.

- **Key uses**:

- Resource allocation (gates, classrooms, delivery routes)

- Infection risk modeling

- Contact tracing during pandemics

- **Graph coloring** helps find the optimal number of shared resources.

- Supports smart scheduling with controlled overlaps.

Conclusion & Strategic Implications

Data-Driven Response

Graphs support transparent, evidence-based crisis strategies.

Resource Optimisation

Facilitate efficient allocation and resource use planning.

Recommendation

Integrate graph analytics tools into health crisis management systems.

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attention.