Adaptive Finite Element Analysis Using Dynamic Bubble System

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Abstract— **In this paper, a new adaptive method for finite element analysis using dynamic bubble system is proposed. The error estimation is performed according to the Zienkiewicz-Zhu method and using improved solution of the problem instead of the unknown true values which is both straightforward and computationally cheap. Mesh density is easily controlled by changing the radius of the nodes that describe the outline of the analysis region and according to the previously performed error estimation. The desired accuracy of the solution is defined by the user and can be achieved only with one refinement step enabling very fast computation. The effectiveness of the proposed adaptive method is investigated for two-dimensional magnetostatic field computations for a test model with known theoretical values. The obtained results show that the proposed method is suitable and very promising for adaptive electromagnetic field using the finite element method.**

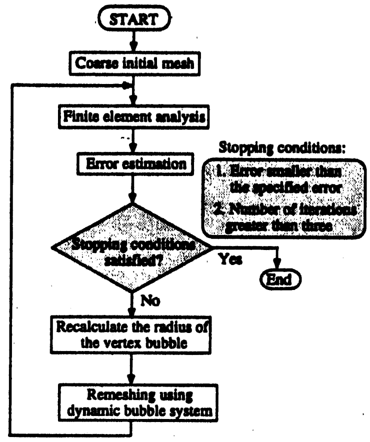

Fig. 1. Simple outline of the proposed adaptive method.

I. Introduction

Controlling the mesh density in the areas where the physical quantity of interest is changing rapidly is very advantageous in finite element analysis in order to reduce the computation time and to improve the accuracy of the results. For this purpose, various adaptive methods for mesh improvements has already been investigated [1], [2]. Recently, various posterior error estimators in connection with adaptive methods for mesh generation and mesh improvements have been proposed. Some of them are based on the continuity of the tangential component of the electric field or the normal component of the magnetic flux density, and some are based on the uniformity of magnetic stored energy. However, two main points must be addressed: First, the error estimation must be independent of the type and class of problems solved and must operate uniformly over materials with various electromagnetic properties. Second, to obtain accurate values for a physical quantity which is of primary importance for analysis. For example, for magnetic field problems it is desirable to estimate the error norm using directly magnetic flux density values, not magnetic vector potential.

In this paper, we applied the Zienkiewicz-Zhu error norm estimation [1] using directly magnetic flux density values to improve the quality of the finite element solution. The proposed method is based on the previously proposed dynamic bubble system [3] incorporated into a fast global two-dimensional h-adaptive method for magnetic field computation. The usefulness of the proposed approach is verified using numerical data for a simple test model.

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II. Outlined of the proposed algorithm

The simple outline of the proposed adaptive method is presented in Fig. 1. First, the analysis area is divided into a coarse 2-D mesh using the Dalauney triangulation method over the set of nodes that describe the outline of the analysis model. Then, the finite element magnetic field analysis using that coarse finite mesh is performed. Next, the error estimation is carried out and the relative error of the solution is investigated by comparing it with the prescribed value. Recalculation of the radius of several vertex bubbles is carried out according to the finite element results and the error estimation, and utilizing the dynamic bubble system a set of finite element nodes is generated. Finally, performing Dalauney triangulation over this set of nodes results in the desired finite element mesh for which the obtained results should have relative error less or equal than the user prescribed relative error.

III. Error Norm Estimation and Refinement Procedure

An local error norm $||E||_e$ for arbitrary finite element e is calculated by the following equation

$$||E||_e = \sqrt{\int_{\Omega_e} (\bar{\mathbf{B}} - \mathbf{B}) \cdot \nu \, (\bar{\mathbf{B}} - \mathbf{B}) \, d\Omega} \qquad (1)$$

Where $\bar{\mathbf{B}}$ is the exact value of magnetic flux density, $\mathbf{B}$ is the approximated value of magnetic flux density calculated directly from the finite element solution $\mathbf{B}=\nabla\times\mathbf{A}$, ν is the reluctivity and Ω_e is the area of element e.

Since the exact value of magnetic flux density $\bar{\mathbf{B}}$ is unknown, we replace the exact value with approximated value of magnetic flux density. Initially, an average value of magnetic flux density $\hat{\mathbf{B}}$ is computed at each node i according to the following equation

$$\hat{\mathbf{B}} = \frac{1}{L}\sum_{j=1}^{L} B_{ej} \qquad (2)$$

where $\mathbf{B}_{ej}$ is the magnetic flux density of element j and L is the total number of elements that has node i in their list of nodes. Next, the exact value of magnetic flux density $\bar{\mathbf{B}}$ is approximated using the following expression

$$\bar{\mathbf{B}} \approx \sum_{i=1}^{m} N_i\hat{\mathbf{B}}_i \qquad (3)$$

where m is the total number of nodes per element and N_i is the nodal shape function for node i.

The relative error for the entire analysis area is defined as follows

$$\eta_{all} = \frac{\sum_{e=1}^{NE} ||E||_e}{||Q||} \qquad (4)$$

$$||Q|| = \sum_{e=1}^{NE}\int_{\Omega_e} \bar{\mathbf{B}}^e\cdot\nu\bar{\mathbf{B}}^e d\Omega \qquad (5)$$

where NE is the total number of finite elements. Whenever the relative error η_{all} is greater than the user desired relative error η_s, mesh refinement method is performed using dynamic bubble system and according to the flowchart presented in Fig. 1.

According to [1], if the current size of the element is h_e^n, the element size h_e^{n+1} which satisfies the specified relative error η_s is calculated by

$$h_e^{n+1} = h_e^n/\xi^{1/p} \qquad (6)$$

$$\xi = ||E||_e/||\bar{E}|| \qquad (7)$$

$$||\bar{E}|| = \eta_s||Q|| \qquad (8)$$

where p is the order of test function.

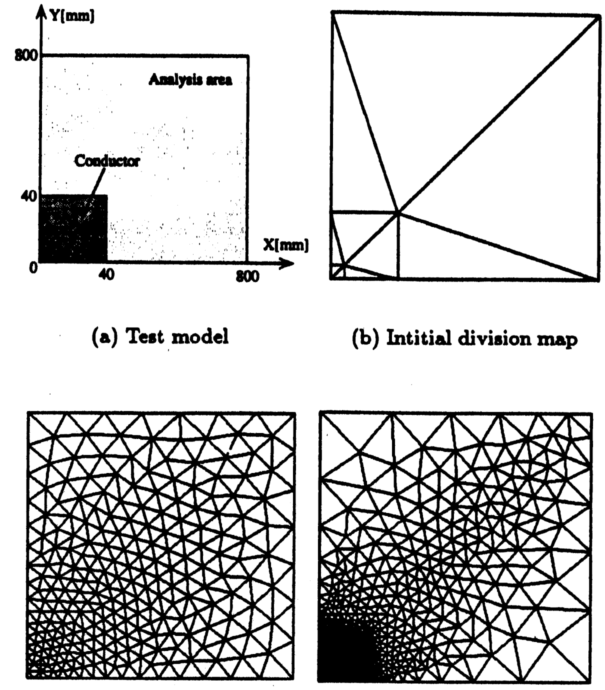
Thus, the definition of the radius of the vertex bubble is computed as

$$r_i = 0.5\cdot min(h^1, h^2, \cdots, h^M) \qquad (9)$$

where h^1, ..., h^M is the longest edge of element shared the vertex i.

IV. Results

A simple model shown in Fig. 2(a) was chosen as a test model. The inital coarse division map is given in Fig. 2(b), while the final division maps satisfing relative error 10% is shown in Fig. 2(c), and that satisfying relative error 5% is shown in Fig. 2(d). From the results two things are apparent: First, the division map which result in desired relative error can be obtained only in one step, and second, the generated finite elements are almost equilateral triangles with graded density of the entire division map. Generation of an optimal finite element mesh from the number of elements and computational time point of view can be easily achieved with gradualy decresing the prescribed error, e.g. first generation of a 10% error mesh, then using this mesh as an inital, generation of a 5% error mesh, etc.



(a) Test model

(b) Intitial division map

(c) Division map satifying relative error 10%

(d) Division map satifying relative error 5%

Fig. 2. Test model with the initial and the obtained division maps.

References

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