Obtaining an Analytical Solution of First-Order Differential Equations with ChatGPT

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ChatGPT is a part of the artificial intelligence developed by OpenAI, which was officially launched in 2022. In this paper, we analyze ChatGPT's capability to solve first-order ordinary differential equations, focusing on obtaining accurate analytical solutions. Our analysis will include the following types of equations: equations with separable variables, homogeneous differential equation, linear and Bernoulli differential equations, as well as nonlinear differential equations: exact differential equations, differential equations with integrating factor, Riccati differential equation, Clairaut's differential equation, Lagrange differential equation, and other differential equations not reduced after the derivative.

Example 1. The Clairaut's differential equation y=xy'+1/y' has the general solution y=Cx+1/C and the particular solution $y^2=4x$.

ChatGPT gives us:

☑ **Final Answer:**

The full general solution is the union of:

1. The curved solutions:

$$y = \pm 2\sqrt{x} + C$$

2. The linear solutions:

$$y = ax + \frac{1}{a}, \quad a \in \mathbb{R} \setminus \{0\}$$

Example 2. ChatGPT, the differential equation $(x+x^2y^2)$ y' + y = 0 does not recognize that it can use a transformation, which leads to the Bernoulli differential equation x'+ (1/y) x = -y x^2. It uses the substitution $u=y^2$, and it is solving as a differential equation with separable variables, where it made a mistake again.

☑ **Step 4: Separate variables**

$$\frac{1 + xu}{u} \, du = -\frac{2}{x}dx \Rightarrow \left(\frac{1}{u} + x\right) du = -\frac{2}{x}dx$$

Now integrate both sides.

Left-hand side:

$$\int \left(\frac{1}{u} + x\right) du = \ln|u| + xu$$

Example 3. For the differential equation $(y')^{2/5} + y^{2/5} = a^{2/5}$, the way of solving is wrong. Therefore, it obtains the wrong solution:

☑ **Final Answer (Implicit Form):**

The solution is given implicitly by:

$$x + C = \int \frac{dy}{\left(a^{2/5} - y^{2/5}\right)^{5/2}}$$

Advantages are:

1. ChatGPT solves the problems in detail;

2. ChatGPT writes down all the formulas it uses;

Conclusions:

1. ChatGPT easily obtains a particular solution from an already obtained general solution of a differential equation;

2. ChatGPT easily finds a general solution for more elementary separable differential equations;

3. If the linear and Bernoulli differential equations are presented in the form of a direct application of the solution scheme, ChatGPT recognizes them and has no problem finding a general solution. However, suppose it needs to transform the differential equation into a linear or Bernoulli one. In that case, ChatGPT sometimes has a problem recognizing the obtaining of such a differential equation. This results in an incorrect solution;

4. ChatGPT also has a problem obtaining a solution for more complex nonlinear differential equations;

5. ChatGPT almost always finds only a general solution to the differential equation, which it considers to be its ultimate solution. For differential equations that have singular solutions, it does not find them, and thus does not even treat them as solutions to a differential equation.