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DEVELOPMENT OF MATHEMATICAL MODELS FOR THE DESIGN OF COMPOSITE STRUCTURES

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Abstract - This article presents the development and improvement of mathematical models for fiber paths using filament winding technology (FW) for composite parts. The accuracy of these models was verified through simulations and experimental testing of manufactured composite symmetrical samples. The fiber winding angle was considered an influential factor for precise model creation. Symmetrical composite samples were produced using these models, with different winding angles (10°, 60°) and combinations of angles. All samples were laboratory tested for hoop tensile strength to meet quality requirements for composite pipes. The analysis of the results provided conclusions about the behavior of composite samples according to the predicted models. This research has significant applications in various industries, including space, aerospace, automotive, military, and marine.

Keywords - Mathematical models, Simulations, Filament Winding Technology, Winding angle, Symmetrical Composite.

I. INTRODUCTION

Composite materials are a class of engineering materials whose applications have been widely used in various industries in recent years. [1] The design and production of composites require specialized knowledge and expertise. The development of mathematical models for the design of symmetric composite parts is the basis for optimizing their performance, it offers a prediction of their behavior and, as such, represents a guide in the composites design process. Their lightweight, high strength, durability, and flexibility in design make composites ideal for a wide range of applications.

II. FILAMENT WINDING TECHNOLOGY

The production of composite structures, particularly those with cylindrical or axisymmetric shapes, widely employs the manufacturing technique of filament winding. [2] This technology involves the precise and controlled placement of continuous reinforcing fibers (filaments) on a rotating mandrel, which serves as a mold for the desired shape. A matrix material, usually a resin, impregnates the fibers as they wind onto the mandrel (Figure.1.). The threads (fibers) materials can be glass, carbon, aramid, or their combinations. The impregnation bath uses thermosetting resins, most commonly polyester, vinyl ester, epoxy, or phenolic resins, as the matrix for the threads. A tensioner maintains and regulates the tension of each thread, which then passes through the impregnation bath. This bath regulates the impregnation of the fibers through a controlled process of heat and tension. Lastly, an automated machine's head mounts a guiding eye with two to six degrees of freedom to wind the fibers onto a rotating core.

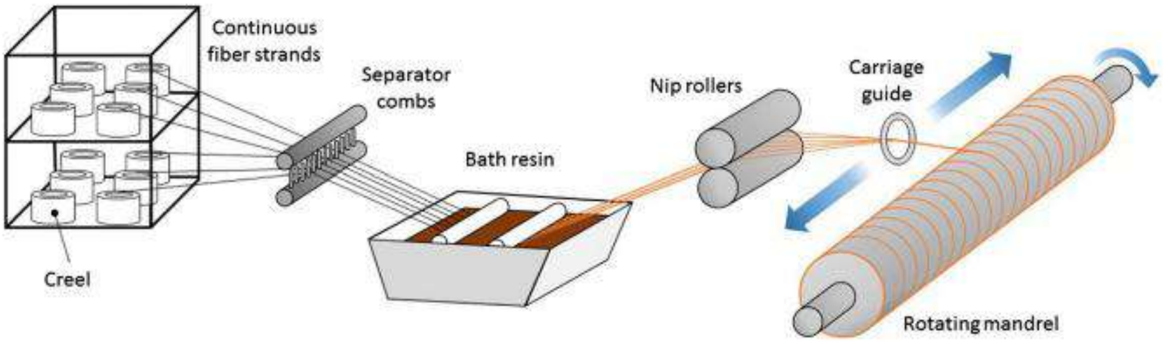


Figure 1.: Basic concept of filament winding technology [5]

III. DEVELOPMENT OF MATHEMATICAL MODELS FOR COMPOSITE PARTS

Mathematical models enable engineers to make informed decisions, predict outcomes, and optimize designs, improving product performance and competitiveness [3]. Robotic configurations to produce composite products have expanded in recent years along with the growth of the industry itself, which has wide application in all rapidly growing industries worldwide, such as transportation, aviation, space, military, and medical. These configurations incorporate additional robots and machines to accommodate various technologies. [4,6]. While robots are mechanical systems, the main research task is developing and implementing mathematical models for automated machines. That means the development of models that will represent the basic geometric aspects of robotic manipulation [3]. The connection between robotization in industry and production itself takes place through software solutions. The modern approach concentrates on computer programming, which means that the challenges in programming robots encompass all the issues encountered in general computer programming—and even more [4]. A significant benefit of mathematical models is their ability to optimize the design of composite components. Engineers can explore various design parameters through numerical simulations and optimization algorithms, such as fiber orientation, play stacking sequence, and thickness, to achieve their desired performance characteristics. By iteratively refining the design based on the model's predictions, engineers can maximize strength, stiffness, weight reduction, and economy [8]. Mathematical models facilitate an optimization process that significantly reduces the development time and costs associated with designing composite parts by minimizing the need for expensive physical prototypes and iterations.

IV. SYMMETRICAL COMPOSITE PARTS

Several analytical and numerical models exist to predict fiber paths and optimize winding parameters for symmetrical composite parts. These models consider the symmetry of the structure and consider factors such as winding angle, fiber tension, curvature effects, and geometric constraints specific to symmetrical parts [7,9]. By incorporating symmetrical-based assumptions and algorithms, these models enable a more efficient and streamlined fiber path design, resulting in improved structural integrity and performance of symmetrical composite parts.

Fiber-winding technology is the first to produce composite products, enabling high-performance structures with exceptional mechanical properties. The success of this technology is attributed to winding angles, a crucial parameter governing the orientation and arrangement of reinforcing fibers, which significantly influence the mechanical behavior and performance of composite parts.

Figure 1 presents the three types of winding angles (layers):

Radial refers to a radial winding layer with a ~90° winding angle, which is practically less than 90° and depends directly on the width of the fiber strip and the geometry of the mandrel.

Polar winding refers to a polar winding layer that represents winding from one pole to the other on the mandrel, with a small angle of approximately 10°, which is determined by the mandrel's geometry.

In practice, helical winding aligns with the rotational axis, although it's not a strict rule; for intricate geometric designs, one can select a surface curve to guide the orientation of the fibers.

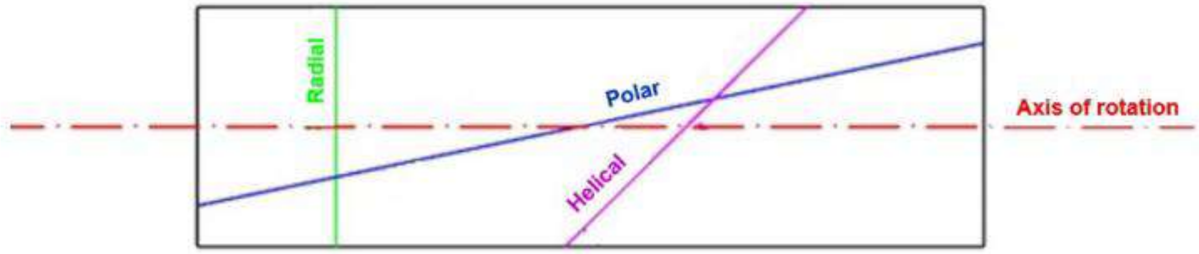


Figure 2: Filament winding angles

In addition to this key parameter for fiber path design, the concept of winding patterns dictates the arrangement of oriented fibers during the winding process. Continuous fiber winding relies on this arrangement pattern to distribute the composite material evenly and cover it completely.

If w is the width of the strip and α is the winding angle of a rotating surface with a cross-sectional length L, the following equations (1-2) calculate the corrected winding angle. Equation (1) predicts a certain overlap of the fibers if the given angle remains uncorrected.

$$\alpha_{\mathrm{corr}} = \mathrm{acos}\left(\frac{\mathrm{Nw}}{\mathrm{L}}\right) \qquad \mathrm{N} = \left\lfloor \frac{\mathrm{L}\cos\,\alpha}{\mathrm{w}} \right\rceil \tag{1}$$

$$\mathrm{cov}[\%] = \frac{\mathrm{Nw}}{\mathrm{L}\cos\,\alpha} \tag{2}$$

The following Figure. 3. presents the geometries of several symmetrical models that were analyzed in MATLAB and simulated in Rhinoceros 3D, in which the transition parts of the shapes are shown in purple.

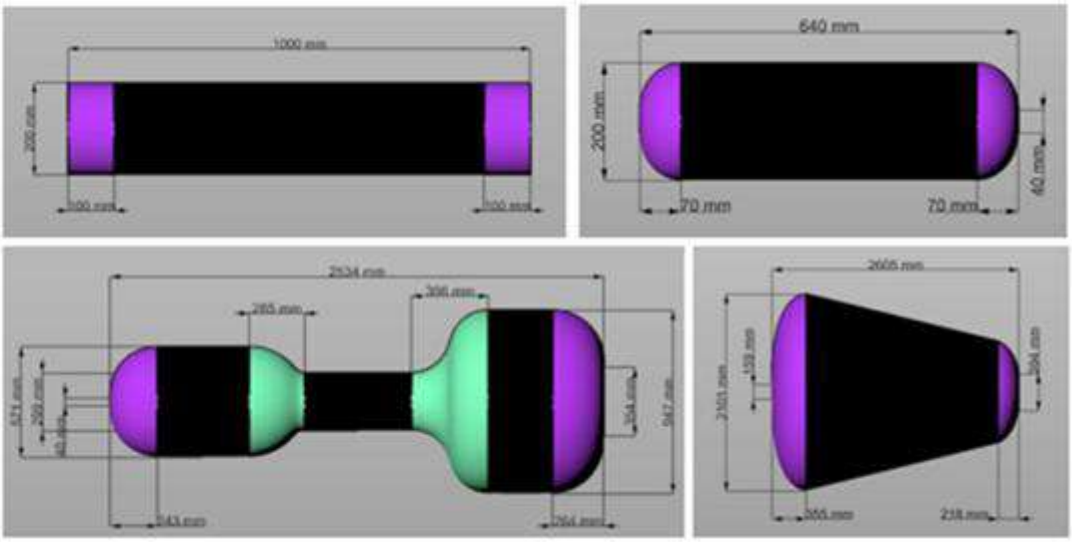


Figure 3: Geometries of symmetric models: tube (left-up), vessel (right-up), double-sided tank (left-down) and conic capsule (right-down)

Transitional parts (purple parts of the surfaces) are those parts in which the fiber path does not maintain the defined angle but gradually approaches 90° so that it can continue continuously to the next cycle. These are the parts where there is the most accumulation of material due to the transition itself, in the case of pipes (where in practice this part is cut off after processing) or in the domes of closed vessels (the remaining three measurements in Figure. 3.) where the accumulation is due primarily to the geometry, but also to the fact of the fiber transition. Figure. 4. shows a simulation of the thickness of a single-layer tank in MATLAB, in black is the upper contour of the tank, and in purple is the final thickness of the composite after the first layer.

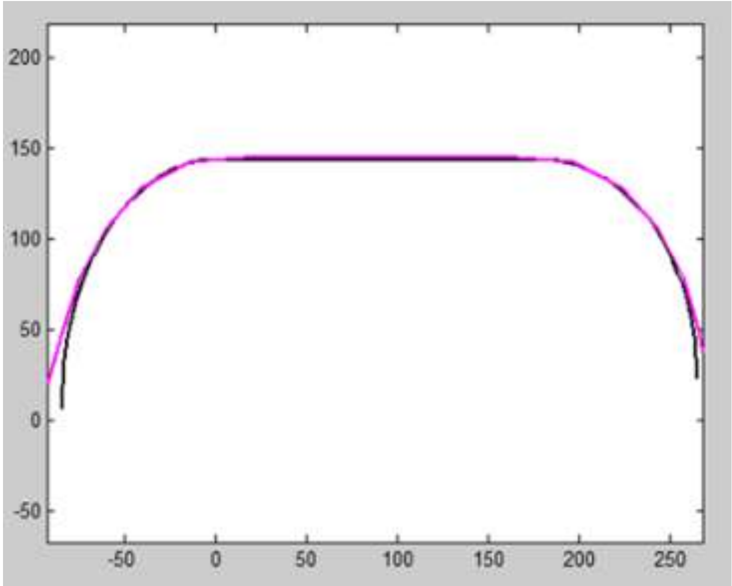


Figure 4: Composite layer contour (violet) over mandrel contour (black)

In Figure. 5. of the double-sided vessel model (bottom left), the parts of the surface marked with light blue are not transition parts, there is no change in angle, but due to the change in geometry, it can be seen that although the path moves programmed geodesically, on the larger surface there is a greater difference between the paths than on the surface where it narrows. Similarly, the geodesic paths maintain a closer connection with the cylindrical part that separates the vessels. This is because the programmed angle and geodesic path are based on covering the largest surface therefore, as the surface of the rotation model narrows or decreases, the paths will also converge, which can be best illustrated precisely in this model. The conical capsule, located at the bottom right of thesame figure, presents a similar scenario.

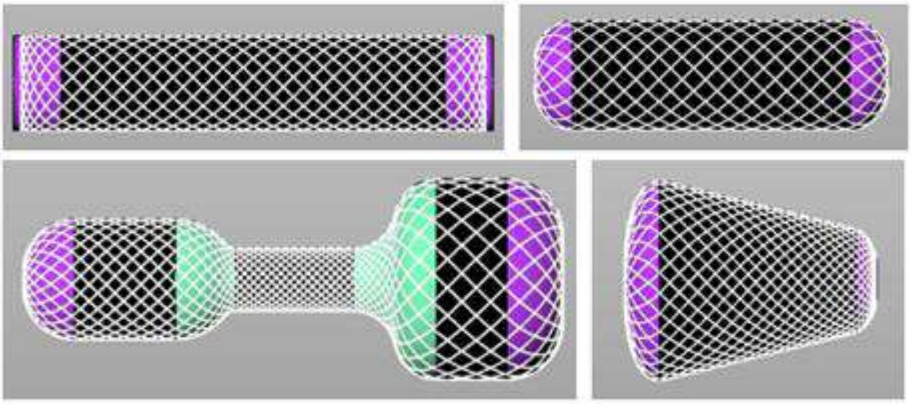


Figure 5: Simulation of 45° fiber paths on symmetric models: tube (left-up), vessel (right-up), double-sided tank (left-down) and conic capsule (right-down)

V. DESIGN OF COMPOSITE STRUCTURES

Designing a fiber path algorithm for various geometric shapes is a complex and iterative process that requires a deep understanding of composite materials, mechanical behavior, and shape geometry. By systematically optimizing the fiber path, the algorithm ensures that the resulting composite structure meets the desired mechanical requirements while adhering to the geometry constraints. The developed algorithm serves as a powerful tool in the field of fillament winding technology, offering a means for designing lightweight and high-strength composite structures for a wide range of engineering applications. Figure. 6. illustrates simulations within the Rhinoceros 3D software. To achieve this is developed a plug-in software in C++, utilizing the Rhino SDK geometry libraries for object geometric representation, geometric tests, and functionalities, along with the standard libraries of the C++ programming language. The graphical simulation shows four layers of a tube with fiber orientations at 10°, 45°, 60°, and 90°.

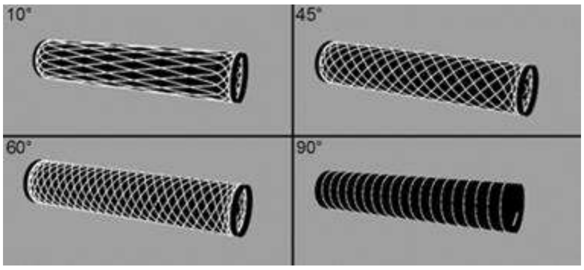


Figure 3: Graphical view of tube simulation on four different oriented layers

The following four images present analyses performed in the MATLAB software package for each layer individually. To perform the analyses, algorithms have been implemented with which an appropriate simulation has been performed in this software package (and language). The simulation of the fiber path is a necessity for the analysis of the fiber path itself. Two programming languages, C++ and MATLAB, have developed algorithms related to mathematical models for a specific purpose. Simulations in Rhinoceros 3D are used to visualize the results of the fiber path design in a 3D CAD environment in which the paths, patterns, overlaps, and other key elements of the winding technology are traced. MATLAB simulations graphically analyze the dependency diagrams between the participants in the path design.

The results of four simulation diagrams of a tube with a winding angle of 10° are displayed in Figure.7. The first view (top left) presents a corresponding simulation of the tube's path, using the same dimensions as in Figure 3. The transition sections, located 100 mm from the tube's ends, are marked in purple. The second graph in the figure (top right) shows the dependence of the winding angle along the position of the rotation axis. The tube outside the transition sections maintains a constant 10° angle at each axis position, with the angle increasing towards 90° in the transition areas, a prerequisite for continuous winding. The second graph (bottom left) illustrates the smoothness of the mandrel's rotation, specifically the radial parameter of the curve, with the position of the rotation axis. In the part where the fiber is wound at a constant angle (geodesic on a tube), the change in the radial parameter of the curve behaves smoothly, while in the transition parts where the angle increases in each step, this parameter increases. The third graph (bottom right) shows the dependence of the winding angle on the radial parameter of the curve (mandrel rotation). It is noted that when the radial parameter is moved constantly, then the angle does not change, it is wound constantly, while its change in the transition parts causes an increase in the winding angle. The two graphs on the right of the figure clearly show that the change in the angle and the change in the radial parameter take place exactly in the transition areas determined by the designer.

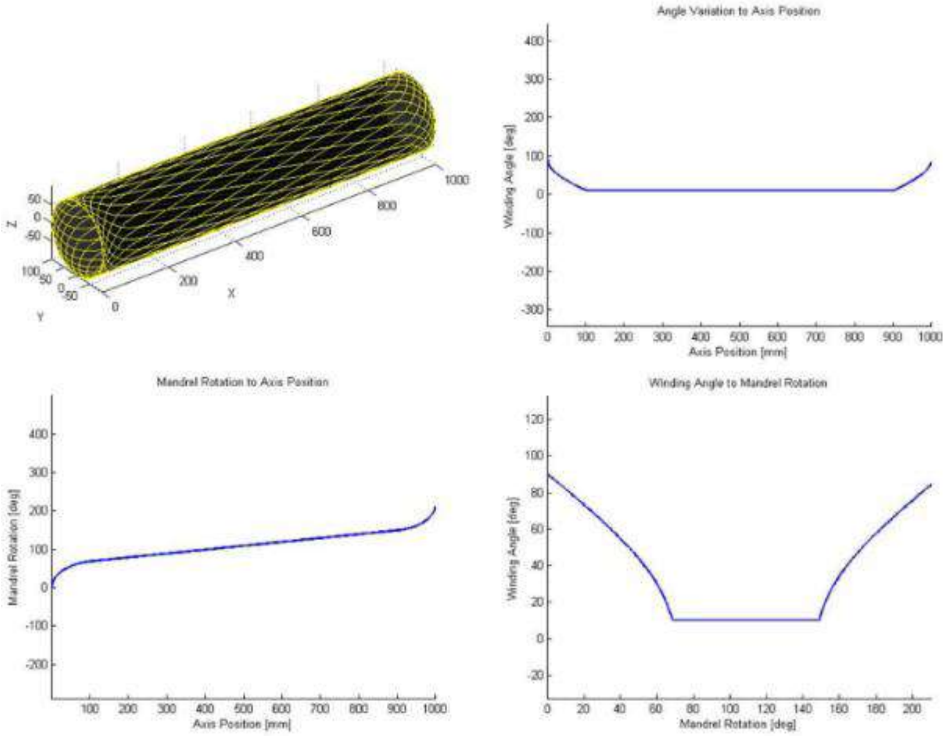


Figure 7: Diagrams of MATLAB Tube simulation on Polar 10° oriented layer and correlation graphs

Figure. 8. displays the simulation results and diagrams of a pipe with a layer under 60°. This example illustrates the changes that occur when the layer's orientation changes by 15°. First, it is observed that the change in the angle in the transition sections relative to the position of the rotation axis is insignificant, precisely because the range of angle change is increased by only 15°. In practice, the transition is smoother and faster, but insignificant in the process. Second, the radial parameter maintained its rotation for approximately two-thirds of a rotation, not the full cycle. Finally, in this example, the 45° layer needed 19 cycles, while the 60° layer needed 13 cycles, the total accumulated value of the radial parameter is equal in both cases. Similar to the first case, the third graph (bottom right) shows that the angle transition occurs within a significantly longer, negligible radial range.

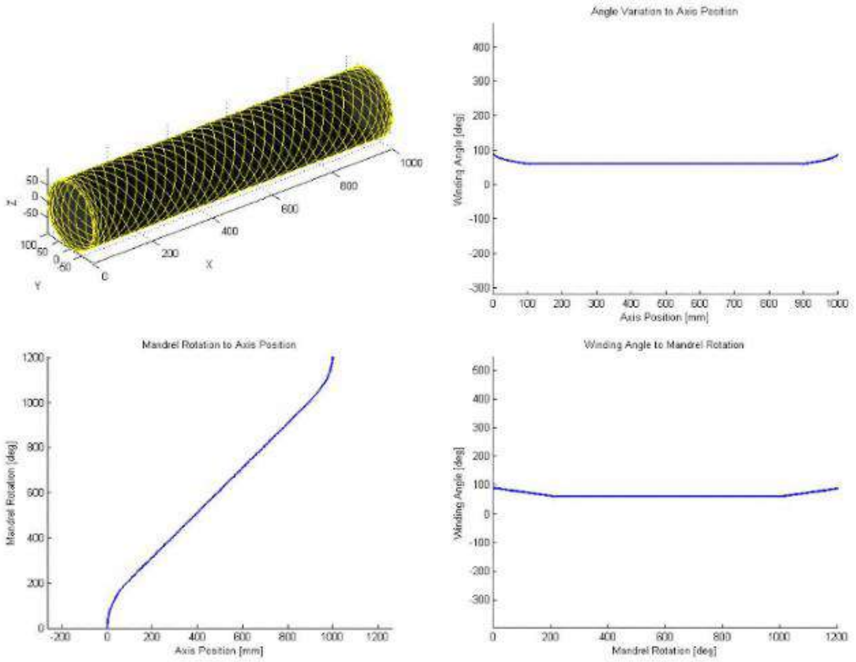
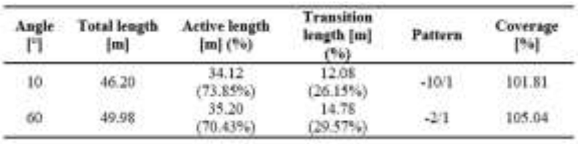


Figure 8: Diagrams of MATLAB Tube simulation on Polar 60° oriented layer and correlation graphs

The following table 1. presents data for all four layers of the tube under mechanical analysis, detailing the material utilization along its length and the percentage of total material that enters the transition sections. The part between the transitions where the angle is constant will be called the "active part," while the rest is the "transition" (in practice, this part is either cut off or processed; in any case, it is a matter of material loss). The table also shows the design scheme and mandrel coverage.



Angle [°]	Total length [m]	Active length [m] (%)	Transition length [m] (%)	Pattern	Coverage [%]
10	46.20	34.12 (73.85%)	12.08 (26.15%)	-10/1	101.81
60	49.98	35.20 (70.43%)	14.78 (29.57%)	-2/1	105.04

Table 1: Active and transition fiber path length, pattern, coverage, and dwell on tube

Table 1., which does not include the radial layer, shows that when winding at an angle, a significant portion of the total thread path, approximately 25% to 30%, rests on the transition part, which in this example accounts for 20% of the tube's entire surface. This means that between 5 and 10 percent of the material depends on how much the angle changes for a continuous fiber connection and how much pure radial winding can be kept at the ends of the tube to fit the pattern, which is partly normal to the continuous connection of the paths. Additionally, the mandrel's layer coverage ranges from 101% to 105%, indicating an excess of 1% to 5% per layer. This can be optimized by changing the angles with a correction according to equation (1), which changes the design of the composite from the original, and by experimentally confirming the transition parts (for example, for larger angles, 10% on both sides is an unnecessarily large transition). For instance, by making corrections in layers with orientations of 44.26° and 61.48°, we can save slightly more than 3m of material per thread (3x4=12m per strip) for both layers.

VI. EXPERIMENTAL TESTING OF MANUFACTURED COMPOSITE SYMMETRICAL SAMPLES

Five composite pipes (GFRP—Glass Fiber Reinforced Pipes) with different fiber winding angles were manufactured in the laboratory of the company LaminatiPrilep, to demonstrate the influence of the laying angle on the mechanical properties of composite pipes. These tubes had the following design:

(1) A1- Glass fibers/epoxy resin tube only with a winding angle of 60º,
(2) A2- Glass fibers/epoxy resin tube only with a winding angle of 10º, and
(3) A3- Glass fibers/epoxy resin tube combined from the previous three angles 45º/60º/90º

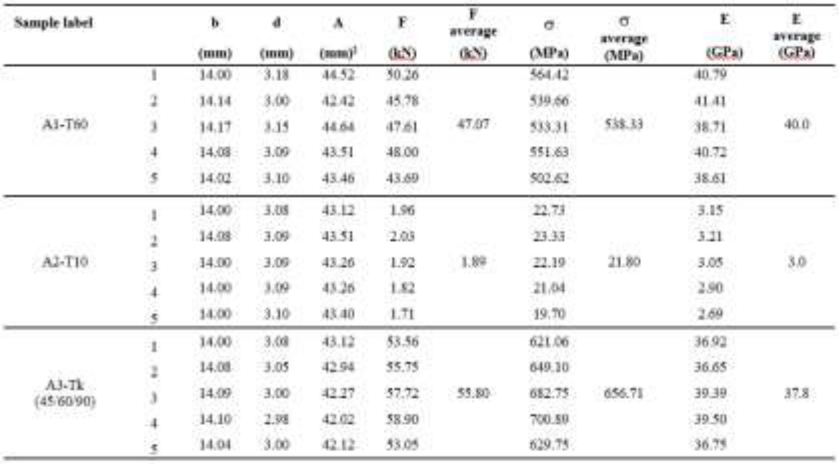
Using the split-disk test method (ASTM D2290 standard), the hoop tensile strength of the produced composite tubes (A1, A2, and A3) was tested, i.e., five test pieces—rings from each tube (A1 to A3)—were tested. A total of 15 tests were performed.

Five samples of the pipes with 10°

Damage for samples of pipe with 10°

Figure. 9: Damages made during tensile tests of the split-disk specimens with 10^{o}winding angles of the glass fibers

Table 2. shows the designations of the manufactured composite pipes, the basic characteristics of the ring test specimens (width, thickness, and winding angle), and the results of the split-disk method tests for the ultimate tensile strength of the ring test specimens.



Sample label		b (mm)	d (mm)	A (mm)²	F (kN)	F average (kN)	σ (MPa)	σ average (MPa)	E (GPa)	E average (GPa)
A1-T60	1	14.00	3.18	44.52	50.26	47.07	564.42	538.33	40.79	40.0
	2	14.14	3.00	42.42	45.78		539.66		41.41	
	3	14.17	3.15	44.64	47.61		533.31		38.71	
	4	14.08	3.09	43.51	48.00		551.63		40.72	
	5	14.02	3.10	43.46	43.69		502.62		38.61	
A2-T10	1	14.00	3.08	43.12	1.96	1.89	22.73	21.80	3.15	3.0
	2	14.08	3.09	43.51	2.03		23.33		3.21	
	3	14.00	3.09	43.26	1.92		22.19		3.05	
	4	14.00	3.09	43.26	1.82		21.04		2.90	
	5	14.00	3.10	43.40	1.71		19.70		2.69	
A3-Tk (45/60/90)	1	14.00	3.08	43.12	53.56	55.80	621.06	656.71	36.92	37.8
	2	14.08	3.05	42.94	55.75		649.10		36.65	
	3	14.09	3.00	42.27	57.72		682.75		39.39	
	4	14.10	2.98	42.02	58.90		700.89		39.50	
	5	14.04	3.00	42.12	53.05		629.75		36.75	

Table 2: Hoop tensile strength results of split-disk tests

There were different fiber orientation angles tested, and the samples combined from the three angles, A3-Tk (45/60/90), had the highest tensile strength at 656.71 MPa. The tensile strengths of samples A1-T60 and A4-T10 are 537.34 MPa and 21.8 MPa, respectively. The highest tensile modulus had values of 40 GPa for samples A1-T60. The tensile modules of A1-T60 and A3-T45/60/90 are 2.0 GPa and 37.8 GPa, respectively. All samples displayed matrix cracking and delamination defects. The results obtained show that a higher orientation angle has a positive effect on the mechanical behavior of the samples. This is because the fibers become stronger when the tensile force acts in their direction. The different modes of failure during tensile testing at different winding angle configurations were expected since composite materials exhibit anisotropic behaviour under different stress modes. Being highly anisotropic when loaded in the fiber direction, composite materials exhibit the best mechanical properties. The higher radial resistance, dominant during the lower axial resistance as the winding angle increases, explains the expected increase in the strength to failure (rupture) of the tested tubes with higher winding angle configurations. Once the strength to rupture of the rings reaches its maximum value, we observe a significant decrease, which we attribute to the dominant decrease in axial mechanical resistance during the increase in radial resistance.

VII. CONCLUSION

The development and application of mathematical models for fiber path design in symmetrical composite parts has ushered in a new era of innovation and precision in the field of fiber winding technology. These mathematical models are powerful tools, allowing engineers to achieve precise control over fiber orientation and tailor the mechanical properties of composite structures to meet specific design requirements. The ability to tailor fiber paths and winding parameters not only improves the mechanical properties of composite parts but also streamlines the manufacturing process. Through these developed algorithms and simulations, engineers can efficiently explore a wide range of fiber winding configurations, reducing the need for costly trial-and-error iterations. The insights gained from mechanical testing are the starting point used to design lightweight, high-strength composite tubes that improve the overall performance and efficiency of structures and systems. Optimized fiber winding parameters contribute to an efficient and economical manufacturing process, leading to reliable composite structures with desired mechanical characteristics.

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