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SIX SIGMA AND MATHEMATICAL MODELLING FOR SYMMETRIC COMPOSITE PARTS DESIGN

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Abstract - This paper applies the Six Sigma DMAIC methodology to improve the production of symmetric composite parts, in our case composite pipes, focusing on their quality requirement of high interior pressure resistance. The filament winding process has been utilized to enhance the industrial process by developing mathematical models of fiber movement paths, resulting in the creation of symmetrical composite parts like pipes and tanks. It analyzed the importance of the fiber winding angle while maintaining the other process factors that were taken to be constant. Based on the mathematical models created for fiber path and pipe design, symmetrical composite samples, or composite pipes, were made with different winding angles (45° and 90°). For all manufactured composite samples, laboratory tests were conducted on the hoop tensile strength to ensure they met the quality requirements for composite pipes, specifically their resistance to high internal pressure. The predicted behavior of symmetrical composite parts (composite pipes) was found to align with the results of the tests conducted on the produced samples.

Keywords - Six Sigma, Mathematical models, Path design, Symmetric composites, Filament winding process, Composite pipes, Hoop tensile strength

I. INTRODUCTION

Solving a multitude of problems in industrial engineering and advancing or improving them is often associated with performing complex and costly experiments. Therefore, the existence of process improvement methodologies is crucial, as they offer a variety of ways and methods for optimal experiment planning. These methods can significantly reduce the time and material costs associated with conducting research.

For a considerable amount of time, the researcher's personal experience and intuition dictated the order in which experiments were conducted. The process of continuous improvement requires a team of experts who, together with the company's management, will actively use methodologies and tools in their activities to improve the industrial process.

There are many methodologies and tools available for process improvement, but choosing the most appropriate one is not always an effortless task. Knowing how, when, and which tools to use to solve problems or improve processes is crucial. Six Sigma methodology is an organization-wide approach used to determine exactly how a management team should set and achieve goals. It demonstrates the need for significant improvements to achieve the desired results [1,2].

II. SIX SIGMA METODOLOGY

Six Sigma is a set of techniques and tools for process improvement. Engineer Bill Smith introduced Six Sigma while working at Motorola in 1986. Later, in 1995, engineer Jack Welch made the introduction of Six Sigma central to his business strategy at General Electric. Many industrial sectors use Six Sigma today [1]. Six Sigma aims to improve the quality of the production process by identifying and eliminating the causes of defects and minimizing variability in the field of production and business processes. It uses a set of quality management methods, mainly empiricaland statistical methods, and creates a special infrastructure of people within the organization who are experts in these methods.

The application of the Six Sigma methodology within an organization follows a specific sequence of steps and aims to achieve specific goals, such as reducing the cycle time of the process, reducing pollution, reducing costs, increasing customer satisfaction, increasing profits, and so on [1, 3].

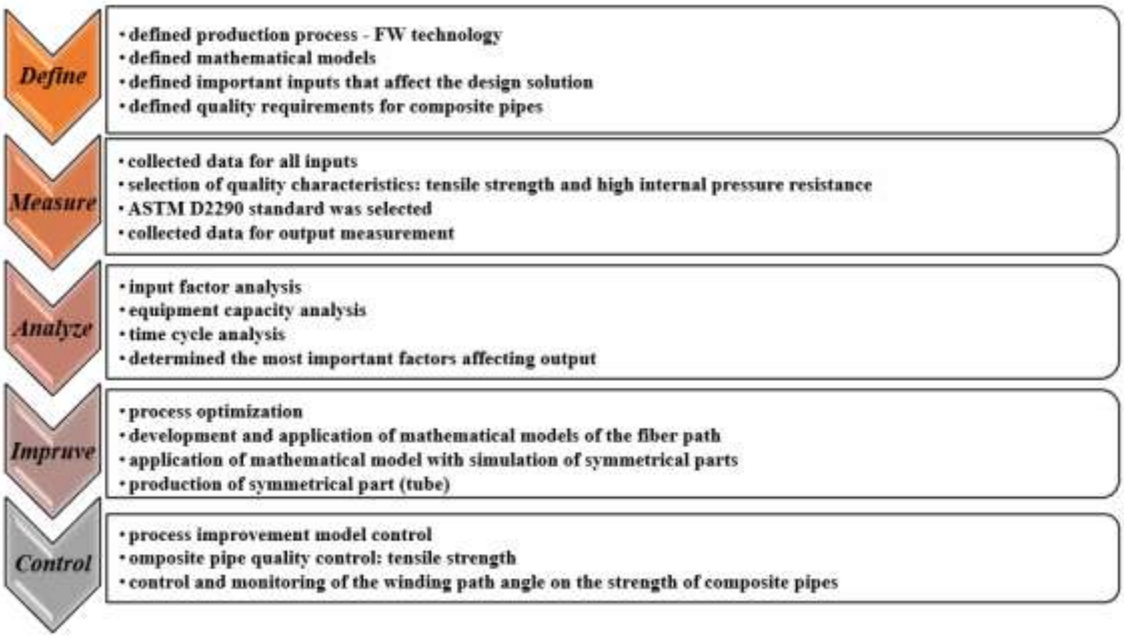
Six Sigma methodology can be defined as the systematic application of business and statistical concepts and techniques to reduce process variation and improve it. Various types of production can use the Six Sigma methodology. It would be illogical to apply Six Sigma to all organizational processes because they are not equally important. Six Sigma methodology should direct its focus toward the most critical areas. The criticality of processes should be driven by the needs and requirements of customers [1, 4, 5].

Six Sigma has two key methods: DMAIC and DMADV. DMAIC focuses on defining project goals based on customer needs and wants, measuring the current process, analyzing the current process to understand problems, improving the process by identifying and piloting solutions, and controlling the improved process with standardization and ongoing monitoring. These phases can be realized using different tools and techniques, with some tools being used in multiple processes. The DMAIC method is best for problems with unknown solutions in existing products, processes, or services.

Design for Six Sigma (DFSS) is a developing method that aims to create defect-free products, processes, or services that meet customer expectations in the eyes of the customer.

III. EXPERIMENTAL

Within the frames of this paper, the six-sigma DMAIC methodology was applied to improve the production of symmetric composite parts, in our case composite pipes. The quality needs of the composite pipes were defined: as high interior pressure resistance. The proposed model for improving the production process of composite pipes is presented in Figure. 1.



Define

- defined production process - FW technology
- defined mathematical models
- defined important inputs that affect the design solution
- defined quality requirements for composite pipes

Measure

- collected data for all inputs
- selection of quality characteristics: tensile strength and high internal pressure resistance
- ASTM D2290 standard was selected
- collected data for output measurement

Analyze

- input factor analysis
- equipment capacity analysis
- time cycle analysis
- determined the most important factors affecting output

Impruve

- process optimization
- development and application of mathematical models of the fiber path
- application of mathematical model with simulation of symmetrical parts
- production of symmetrical part (tube)

Control

- process improvement model control
- omposite pipe quality control: tensile strength
- control and monitoring of the winding path angle on the strength of composite pipes

Figure 1: Six Sigma DMAIC proposed model for improving the production process of symmetric composite pipes

Composite pipes were created using glass fibers (E-glass fiber roving) as reinforcement, with a fineness of 600 tex, manufactured by the company SHANGDONG from China. A polymer solution of epoxy resin as a matrix impregnates the glass fibers. An epoxy system from the company Hexion from the USA was used with the following composition: epoxy resin BisfenolEpicote 828, hardener Methyltetrahydrophthalic Anhydride (MTHPA) Epikure 866, and accelerator Hexion EPC 101. The process involved impregnating the fibers winding them into a tool (mandrel) and hardening the wound structure. In the laboratory of the company LaminatiPrilep, were produced the composite pipes for the research. The laboratory fiber winding machine produced samples of composite pipes with different designs. The basis of this process includes the winding of resin-impregnated fibers into a tool (mandrel) and the hardening of the wound structure. After the winding of the fibers, the composite pipes were fully cured at temperatures of 80ºC and 140ºC within 240 minutes at both temperatures. The glass/epoxy composite pipes were chosen due to their cost-effectiveness compared to high-performance fiber composite pipes. Factors such as resin viscosity, speed of impregnation, winding angle, and fiber tension were observed during the impregnation process to achieve the required resin content (75:25 wt. % fiber resin).

Five pipes of different designs were manufactured, marked A1 and A2, and tensile test specimens were cut from each pipe. (Figure.2.)

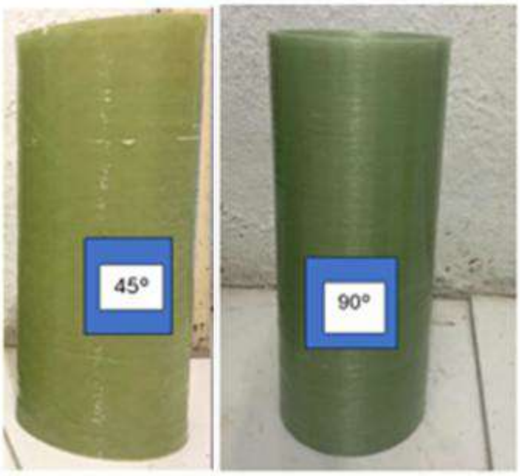


Figure 2: Produced composite pipes with filament winding technology: different winding angle

Many input factors affect the quality of composite materials, including the constituent materials of the composite structure, the mathematical model chosen for fiber winding, winding speed, winding angle, fiber tension, matrix viscosity, and more. This research analyzes the winding angle, a critical parameter that regulates the orientation and arrangement of reinforcing fibers during the fiber winding process, as the most influential factor that determines the mechanical behavior and overall performance of composite parts.

The ASTM D2290 standard (ASTM D2290, 2013) was used to test the tensile strength of composite rings from composite pipes. Following the requirements of this standard, five ring-shaped test specimens were cut from each composite pipe (labeled A1 to A3) (Figure. 3.) and further machined. The width and thickness of each specimen were measured using a micrometer with an accuracy of 0.0254 mm. A Zwick/Roell Z400 universal testing machine with a maximum force of 400 MPa and a testing speed of 0.3 inch/min was used to test the tensile strength of the ring specimens (Figure. 4.).

Figure 3: Illustration of the split-disk test specimen for tensile testing

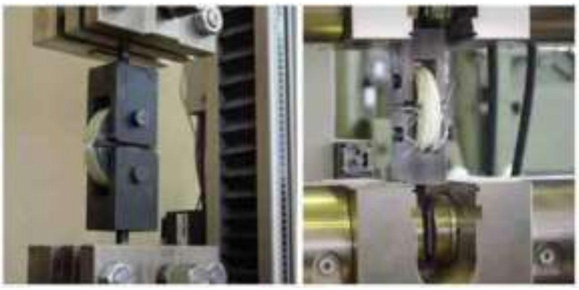


Figure 4: Universal testing machine: Zwick/Roell Z400 with max stress of 400MPa

The testing was performed at a constant speed of separation of the disk halves until the specimens ruptured.

For each tested group of ring specimens, the arithmetic mean of the measured ultimate tensile strengths was calculated. The ultimate tensile strength of the composite specimens was calculated using the equation:

$$\sigma = \frac{F_{max}}{2A_m} \qquad (1)$$

The mean value of the tensile modulus of elasticity was also calculated (equation 2). The stress-strain curves were constructed based on the force and displacement values obtained from the ring specimen testing.

$$E_m = \frac{P(0.1257 r_{mean}{}^{3})}{(\Delta y) w t^3} (2)$$

This research also involves a systematic approach to achieve precise control over fiber orientation. The goal is to optimize the mechanical properties of the composite structure while ensuring symmetry in the fiber arrangement. The algorithm considers the shape geometry, material properties, and loading conditions to generate an efficient and effective fiber path. The algorithm optimizes the fiber path to create lightweight, high-strength composite structures that meet mechanical requirements and geometry constraints.

The MATLAB software package is used for individual layer analyses, utilizing algorithms and simulations for fiber path analysis. C++ and MATLAB programming languages have developed mathematical models for this purpose. Rhinoceros 3D simulations visualize fiber path design results in a 3D CAD environment, tracing paths, patterns, and overlaps. MATLAB simulations graphically analyze dependency diagrams between participants in the path design, providing a comprehensive understanding of the fiber path's structure and functionality.

IV. SIMULATIONS AND EXPERIMENTAL RESULTS

The results of four simulation diagrams of a tube with a different winding angle of 45°, and 90° are displayed in Figure 5-6.

The simulation of a tube's path is depicted in three graphs. The first view shows the tube's path with transition sections 100 mm from the tube's ends marked in purple. The second graph shows the dependence of the winding angle along the position of the rotation axis.

The simulation results and diagrams for a layer with a 45° orientation (Figure. 5.) show milder rotations due to the 45° angle's proximity to 90°, allowing for gentler steps in the pre-matingprocess The 45° layer requires nearly two and a half times longer to rotate, a logical consequence of the curve's structure.

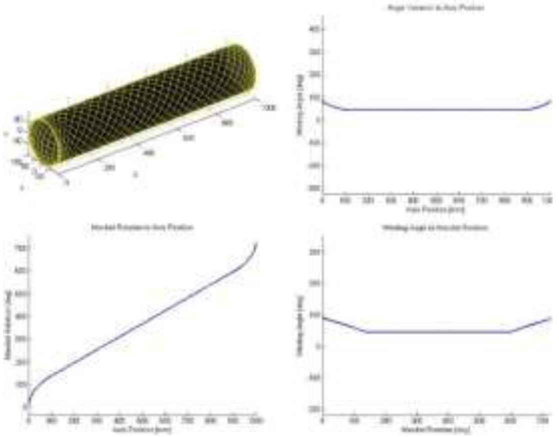


Figure 5: Diagrams of MATLAB Tube simulation on Polar 45° oriented layer and correlation graphs

In Figure.6., the graphs are more than clear, showing that the curves where the winding angle depends on the position of the axis and the rotation angle of the radial parameter are constantly 90° everywhere, while the rotation depending on the position changes rapidly quickly.

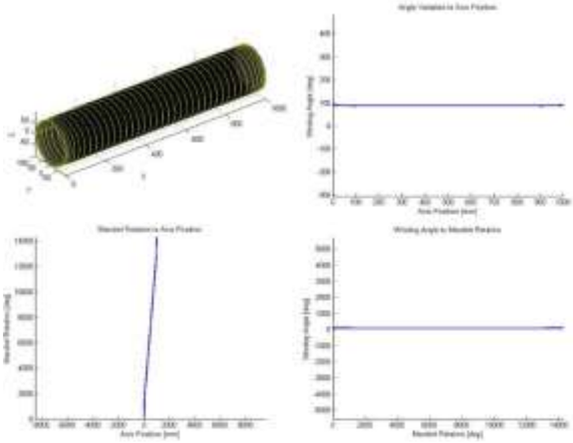


Figure 6: Diagrams of MATLAB Tube simulation on Polar 90° oriented layer and correlation graphs

At room temperature, specimens from series A1 (Glass/epoxy tube only with a winding angle of 90º), and A2(Glass/epoxy tube only with a winding angle of 45º) were tested on a Zwick/Roell Z400 universal testing machine at a speed of 5 mm/min. The results are shown in Figures. 7,8. These composites were tested on a testing machine with a higher maximum load due to their higher strength.

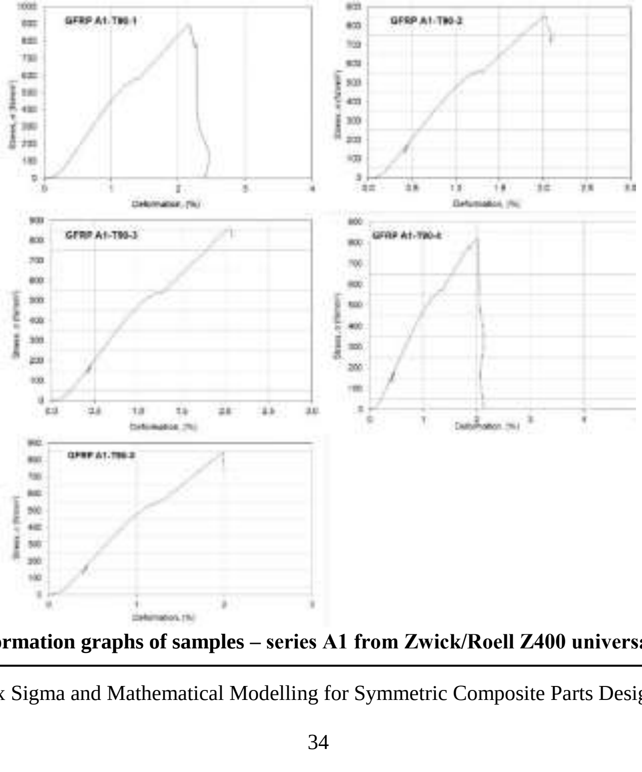


Figure 7: Stress and deformation graphs of samples – series A1 from Zwick/Roell Z400 universal tensile testing machine

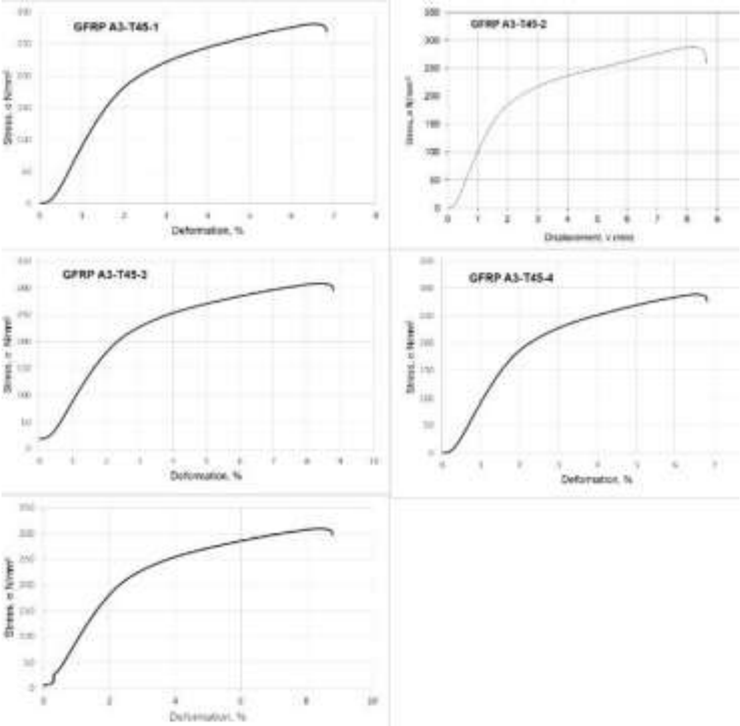
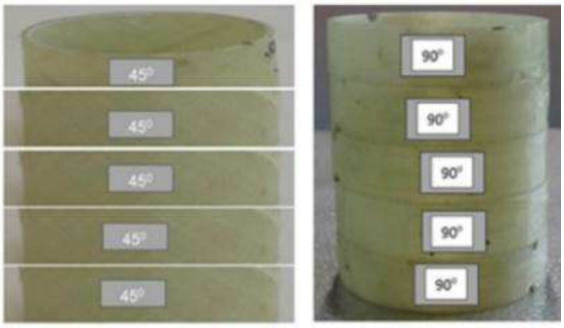


Figure 8: Stress and deformation graphs of samples – series A2 from Zwick/Roell Z400 universal tensile testing machine

Figure.9. shows the prepared samples, 5 from each series, 10 tensile samples, and the test pieces (rings) after the testing.

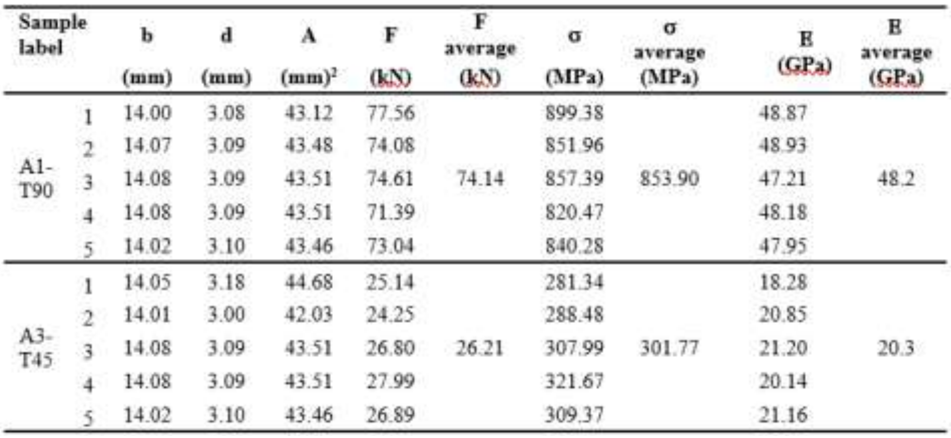


Five samples of the pipes with 45° **Five samples of the pipes with 90°**

Damage for samples of pipe with 45° **Damage for samples of pipe with 90°**

Figure. 9: Damages made during tensile tests of the split-disk specimens with different winding angles of the glass fibers

Test samples with different fiber orientation angles showed that the samples with a winding angle of ±90° have the highest tensile strength and modulus, measuring 853.94 MPa and 48.2 GPa, respectively. The results clearly show that the higher orientation angle positively influences the mechanical behavior of the samples. The higher strength of the fibers occurs when the tensile force acts in their direction. Under radial loading conditions, an increase in the winding angle brings the fiber alignment direction closer to the load direction. A winding configuration of 90°, where the load direction aligns with the fiber direction, yields the highest strength values. On the other hand, as the winding angle increases, the resistance of the composite tube to axial stress decreases. The internal pressure load is a combined load that creates both axial and radial stresses in the material, so it is expected that the maximum characteristics will be maintained in the winding configuration, which optimizes the resistance in terms of radial and axial stress [6,7].



Sample label		b (mm)	d (mm)	A $(mm)^2$	F (kN)	F average (kN)	σ (MPa)	σ average (MPa)	E (GPa)	E average (GPa)
A1-T90	1	14.00	3.08	43.12	77.56	74.14	899.38	853.90	48.87	48.2
	2	14.07	3.09	43.48	74.08		851.96		48.93	
	3	14.08	3.09	43.51	74.61		857.39		47.21	
	4	14.08	3.09	43.51	71.39		820.47		48.18	
	5	14.02	3.10	43.46	73.04		840.28		47.95	
A3-T45	1	14.05	3.18	44.68	25.14	26.21	281.34	301.77	18.28	20.3
	2	14.01	3.00	42.03	24.25		288.48		20.85	
	3	14.08	3.09	43.51	26.80		307.99		21.20	
	4	14.08	3.09	43.51	27.99		321.67		20.14	
	5	14.02	3.10	43.46	26.89		309.37		21.16	

Table 1: Hoop tensile strength results of split-disk tests

Figures.7 to 8 present the stress-strain curves obtained from testing samples with different winding angles (A1, A2). These curves show a significant increase in stress for equal deformation intervals, particularly in sample A1-T90, confirming the previously mentioned conclusions that the load-bearing capacity of composites increases with increasing winding angle in the direction of stress. The results obtained from mechanical experiments on composite tube samples provide good guidance on how to optimize their performance during manufacturing. By determining the effects of manufacturing parameters on mechanical properties, researchers can adjust the fiber-winding process to achieve appropriate material behavior, which should result in composite tubes with improved strength and durability. Mechanical testing provides the foundation for designing lightweight, high-strength composite tubes that enhance the overall performance and efficiency of structures and systems.

V. CONCLUSION

This paper applies the Six Sigma DMAIC methodology to improve the production of symmetric composite parts. Based on the Six Sigma proposed model, algorithms have been implemented with mathematical models written in two programming languages, C++ and MATLAB, resulting in appropriate simulations for designing fiber paths in symmetrical shapes. By applying the improved algorithms, precise control over the fiber path orientation was achieved. In this way, it was possible to create a composite design based on the required mechanical properties for it. Five samples of symmetrical composite structures (tubes) with different fiber orientations were produced and mechanically tested based on the developed simulations. The results of the tensile strength tests with elastic moduli showed the behavior of composites with different winding angles under stressful conditions. Composite tubes with a larger winding angle showed higher tensile strength.

The development of mathematical models for symmetrical composites sets the basis for continuous improvement in fiber winding technology and obtaining modern composite structures for application in various industries such as aerospace, automotive, renewable energy, infrastructure and construction, and marine and maritime applications. The developed models have a wide range of applications, enabling the integration of various composite structures into complex engineering constructions and challenges.

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