DOI: 10.20544/SERBE.11.01.25.P03

ENDOGENOUS BEN-PORATH MODEL AND TAXES

Dushko Josheski[1], Aneta Risteska-Jankuloska[2], Natasha Miteva[3]

Abstract

This paper is about taxes in endogenous Ben-Porath model.First it derrives exogenous Ben-Porath model with moral hazard (education effort is unobservable due to moral hazard) ,and usual Mirrlees, Pareto and Ramsey taxes.Separation theorem justifies use of labour taxes but compared to Atkinson-and Stiglitz theorem where savings and consumption should not be taxed separately,separation theorem states that human capital accumulation and consumption-saving are independent. Instead of taxing labor income heavily, education should be subsidized to encourage investment in human capital. Policy makers have justification about progressive taxation that should be paired with education subsidies to correct any underinvestment in human capital.Labour income should be primary tax base,and capital income taxation should be minimized.In endogenous determination of life expectancy or retirement model vs endogenizing by allowing individuals to choose both the level and the type of human capital accumulation the result about evolutionof human cpaital and tax revenues under different tax regimes are inverse.In the models of tax formulas with spillover effects,they(spillover effects) are present in the Pareto tax formula,while labor supply elasticity and elasticity of consumption affect Mirrleesian and Ramsey taxation respectivelly.

Keywords: Ben-Porath, endogenous, Mirrlees, Ramsey, Pareto, human capital, progressive taxation

[1] Faculty of Tourism and Business Logistics, Goce Delcev University, Stip, North Macedonia; e-mail: dusko.josevski@ugd.edu.mk ; ORCID: 0000-0002-7771-7910

[2] Faculty of Economics, University St Kliment Ohridski -Bitola,North Macedonia email: aneta.risteska@uklo.edu.mk , ORCID:0009-0007-9677-7170

[3] Faculty of Tourism and Business Logistics, Goce Delcev University, Stip, North Macedonia, natasa.miteva@ugd.edu.mk ; ORCID: 0009-0004-8082-6458

1. Introduction

Human capital[4] acquisition literature started with: Becker(1964), Ben-Porath (1967),and Heckman (1976).In Ben-Porath (1967),individuals choose optimal education and work time over their lifetime, balancing wage growth and depreciation.Heckman (1976)[5]designs a utility maximizing model with endogenous labor supply, income, human capital accumulation, consumption, and non-monetary utility of education, that contains the original model of Ben-Porath (1967) as a special case,see Fleischhauer, Kai-Joseph. (2007). According to Becker (1964), training in specific human capital is different from general training because workers do not benefit from higher productivity after changing their jobs[6]. Actually Haley(1973), wrote that the literature of human capital evolved in two ways since the beginning:Becker (1964) looks at individual investment and attempts to estimate the internal rate of return to that investment by equating properly discounted costs and returns. This foundational model; focuses on human capital investment without life-cycle considerations.Second path along which human capital literature has evolved since the beginings is exemplified by Ben-Porath (1967) which extends Becker (1964) by introducing an explicit life-cycle framework for optimal human capital investment.Here the producer of human capital has a choice between producing additions to his stock or renting his stock in the labor market or both, see Haley(1973).Our other focus is on optimal taxation literature. Optimal taxation[7]literature since Mirrlees(1971) and later developed by Saez (2001), Kocherlakota (2005), Albanesi and Sleet (2006), Golosov, Tsyvinski, and Werning (2006), Battaglini and Coate (2008) , Farhi,Werning (2013), Golosov, Troshkin, and Tsyvinski (2013) typically

[4] Human capital can be defined as knowledge, skills, attitudes, aptitudes, and other acquired traits contributing to production,see Goode (1959). Skills represent individual capacities contributing to production as an argument in the production function, see Bowles, Gintis, and Osborne (2001).

[5] Heckman (1976) extends human capital theory by introducing uncertainty and market imperfections (e.g., credit constraints). This models explains how individuals adjust labor supply and human capital accumulation over their life cycle when wages and interest rates change.This model shows that early childhood investment has dynamic complementarities, influencing later skill formation.

[6] Becker (1964) distinguishes between general human capital (skills transferable across firms) and specific human capital (skills valuable only to a specific employer). The key idea is that individuals weigh the costs of education/training (tuition, foregone earnings) against the expected future income gains.

[7] Optimal tax theory or the theory of optimal taxation is the study of designing and implementing a tax that maximises a social welfare function subject to economic constraints,see Mankiw et al.(2009).

assumes exogenous ability, thus abstracting from endogenous human capital investments, see Stantcheva (2017). Variations in human capital over the lifetime can have implications for two fundamental question : should the labour income tax be progressive[8], see Diamond and Saez (2011) and Mirrlees ,Adam (2010),and Peterman, (2016).And second question: Second, should capital be taxed?[9].Two theorems state following: Atkinson-Stiglitz Theorem- If preferences are weakly separable between consumption and labor, taxation should focus on labor income, and capital (savings) should not be taxed.Separation Theorem- Since human capital accumulation is independent of consumption-saving decisions, the same logic applies—taxation should primarily be on labor income, while capital taxation should be minimized[10].Previous research on these topics in life cycle model assumed that human capital accumulation is exogenous process, in this paper we will try to investigate optimal Mirrles,Pareto and Ramsey taxes on endogenous version of Ben-Porath human capital accumulation model. According to Stantcheva (2017) there is two way interaction between human capital and the tax system. First, investments in human capital are influenced by tax policy which was previously recognized by Schultz(1961)[11]. On the other hand,investments in human capital directly affect the available tax base and are a major determinant of the pretax income distribution.This paper will introduce three endogenous versions of Ben-Porath model with three types of tax regimes.

[8] The tax system plays a central role in all modern economies. Taxes account for between 30 and 50 per cent of national income in most developed economies,see Mirrlees et al.(2011). The way in which these huge sums of money are raised matters for economic efficiency and for fairness. Many countries look to address fiscal deficits by raising more money through their tax systems,so the importance of getting the structure of taxes right can only increase.

[9] Certain theoretical results in particular: Atkinson and Stiglitz (1976), Chamley (1986), and Judd (1985), implied no capital income taxes and one particular study Diamond and Saez (2011),did not find these results to be robust for policy makers.

[10] Jones et al. (1997) and Judd (1999) showed that "if the government can distinguish between pure consumption and human capital investment, then it can use this information to offset the distortion that labour taxation causes on human capital accumulation" see Reis (2019).

[11] Our tax laws everywhere discriminate against human capital. Although the stock of such capital has become large and even though it is obvious that human capital, like other forms of reproducible capital, depreciates, becomes obsolete and entails maintenance, our tax laws are all but blind on these matters,see Schultz (1961)

2. Ben-Porath Model with Taxation and Moral Hazard

The individual's problem in the Ben-Porath model is to maximize lifetime utility:

$$U = \int_0^{\infty} e^{-\rho t} u(c, l) dt \quad (1)$$

subject to the human capital accumulation equation:

$$\dot{h} = f(e, h) - \delta h - \tau_h h \quad (2)$$

where:e is education effort (unobservable due to moral hazard),h is human capital, τ_h is the tax on human capital,$f(e, h) = (e^{\alpha})h$ is the human capital production function.The individual's budget constraint is:

$$c + e = (1 - \tau_w)wh \quad (3)$$

Where τ_w is labor income tax.

➢ Optimal Taxation Criteria

Mirrlees Optimal Tax (Incentive-Compatible)

Mirrlees taxation is derived by maximizing social welfare while ensuring incentive compatibility.

The planner solves:

$$\max_{\tau_h} \int_0^{\infty} e^{-\rho t} u(c, l) dt \quad (4)$$

subject to:**Budget Constraint**: $c + e = (1 - \tau_w)wh$

Human Capital Dynamics:

$$\dot{h} = (e^{\alpha})h - (\delta + \tau_h)h \quad (5)$$

Incentive Compatibility (IC): The agent chooses effort eee to maximize their utility given τ_h

First-order condition (FOC) w.r.t. e:

$$u_c(1 - \tau_w)w = u_l f_e(e, h) \quad (6)$$

Now:

$$\tau_h^{Mirrlees} = \frac{\alpha e^{\alpha-1} h - \delta}{h} \quad (7)$$

Ramsey Optimal Tax (Revenue-Maximizing) : Ramsey taxation maximizes government revenue while minimizing distortions. The planner solves:

$$\max_{\tau_h} \int_0^{\infty} e^{-\rho t} G(t) dt \quad (8)$$

where government revenue is:

$$G = \tau_h h + \tau_w wh \quad (9)$$

subject to the individual's budget and human capital constraints. Differentiating revenue w.r.t. τ_h and setting to zero:

$$\frac{d}{d\tau_h} \tau_h h = h + \tau_h \frac{dh}{d\tau_h} = 0 \quad (10)$$

Solving for τ_h^{Ramsey}

$$\tau_h^{Ramsey} = -\frac{h}{\frac{dh}{d\tau_h}} \tag{11}$$

Approximating $\frac{dh}{d\tau_h}$ from the steady-state equation:

$$\tau_h^{Ramsey} = \frac{ae^{\alpha}}{\delta+\tau_h} \tag{12}$$

Pareto Optimal Tax (Balancing Efficiency and Redistribution)

Pareto taxation balances efficiency and redistribution by ensuring a minimum level of human capital while preventing excessive distortions.

The planner maximizes:

$$max_{\tau_h} \int_o^{\infty} e^{-\rho t}[u(c,l) + \lambda G(t)]\, dt \tag{13}$$

where λ is the weight on government revenue.Solving:

$$\lambda G(1-\lambda)U = 0 \tag{14}$$

Differentiating w.r.t. τ_h:

$$\lambda h(1-\lambda)\left(-u_c\frac{dh}{d\tau_h}\right) = 0 \tag{15}$$

Solving for τ_h^{Pareto}:

$$\tau_h^{Pareto} = \frac{\lambda h}{(1-\lambda)u_c\frac{dh}{d\tau_h}} \tag{16}$$

Approximating $\frac{dh}{d\tau_h}$:

$$\tau_h^{Pareto} = \frac{\lambda\alpha e^{\alpha}}{(1-\lambda)(\delta+\tau_h)} \tag{17}$$

Effectsf taxation on human capitalaccumulation are depicted in the following plot.

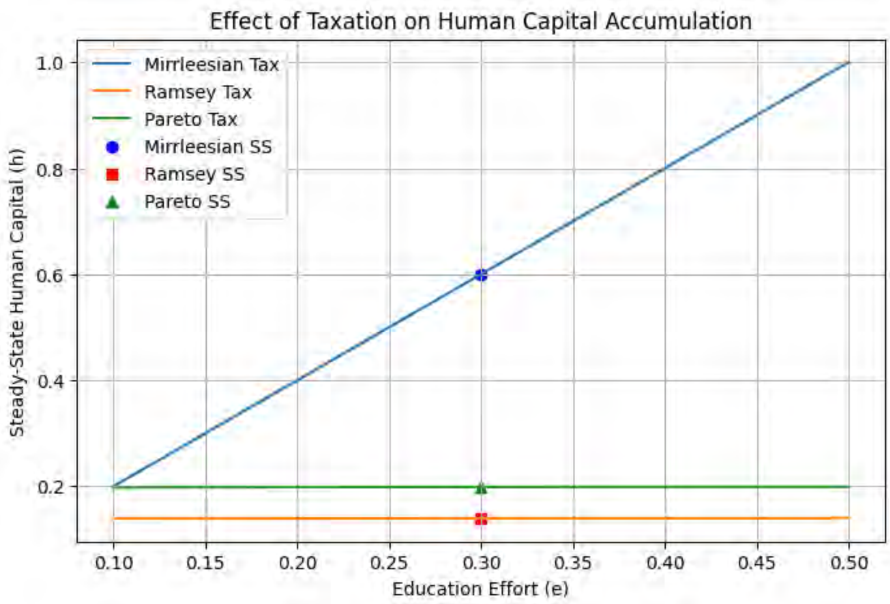

Figure 1. Effects of taxation on human capital accumulation, Source:Author's own calculations

2.1.Mirrlees (1971) non-linear tax formula, Ramsey (1927) model or Ramsey tax rule

This point actually follows Mirrleees (1971) and Diamond (1998) , in deriving non-linear optimal tax rate with no-income effects.Utility function is quasi linear:$u(c,l) = c - v(l)$,c is disposable income and the utility of suuply of labor $v(l)$ is increasing and convex in l. Earnings equal $w = nl$ where n represents innate ability. CDF of skills distribution is $F(n)$, it's PDF is $f(n)$and support range is $[0, \infty)$. Government cannot observe abilities instead it can set taxes asa function of labor income $c = w - \tau(w)$. The elasticity of the net-of-tax rate $1 - \tau$ is:

$$e = \frac{\left(\frac{n(1-\tau)}{l}\right)dl}{d\big(n(1-\tau)\big)} = \frac{v'(l)}{lv''(l)} \qquad (18)$$

By using the envelope theorem and the FOC for the individual, u_n satisfies following: $\frac{du_n}{dn} = \frac{lnv'(ln)}{n}$.Now the Hamiltonian is given as:

$$\mathcal{H} = \big[G(u_n) + \lambda \cdot \big(nl_n - u_n - v(l_n)\big)\big]f(n) + \phi(n) \cdot \frac{lnv'(ln)}{n} \quad (19)$$

In previous $\phi(n)$ is the multiplier of the state variable[12].If $-\phi(n) = \int_n^\infty [\lambda - G'(u_m)]f(m)dm$ we can write previous expression as:

$$\frac{[v'(l_n)+l_n v''(l_n)]}{n} = \left[\frac{v'(l_n)}{n}\right]\left[1+\frac{1}{e}\right] \quad (20)$$

we can rewrite FOC with respect to l_n as:

$$\frac{\tau'(w_n)}{1-\tau'(w_n)} = \left(1+\frac{1}{e}\right)\cdot\left(\frac{\int_n^\infty (1-g_m)dF(m)}{nf(n)}\right) \quad (21)$$

In previous expression $g_m = \frac{G'(u_m)}{\lambda}$ which is the social welfare on individual m . The formula was derived in Diamond (1998). In Ramsey (1927), utility function is given of type:$u = f(p_1, p_2, p_3, \ldots\ldots, w)$, $p_1, p_2, p_3, \ldots\ldots$ are prices and w is income. This result is known as Roy's identity , Roy (1947)[13], is :$\frac{\partial u}{\partial p_i} = -f_i \frac{\partial u}{\partial w}$ .Change in taxes must satisfy the following equation: $dU = \frac{\partial U}{\partial p_i} d\tau_1 + \frac{\partial U}{\partial p_2} d\tau_2 = 0$,and $\frac{d\tau_2}{d\tau_1} = -\frac{F_1}{F_2}$, change in the revenues caused by the change in taxes is :$\frac{\partial(\tau_1 f_1)}{\partial t_1} = F_1 + \frac{\tau_1 df}{dp_1} = F\left(1+\frac{\tau_1 dF_1 p_1}{p_1 dp_1 F_1}\right) = F_1\left(1-\frac{\tau_1}{p_1}\varepsilon_u^1\right)$, where ε_u^1 represents the compensated elasticity of the demand for good 1. Change of revenues as a result of change of taxes on good 2 is: $\frac{\partial(t_2 F_2)}{\partial t_2} = F_2\left(1-\frac{\tau_2}{p_2}\varepsilon_u^2\right)$. With the optimal tax structure, this identity must holds:$\frac{t_2}{p_2}\varepsilon_u^2 - \frac{t_1}{p_1}\varepsilon_u^1 = 0$, for the linear demand curve results is :$\frac{t}{p} = \frac{kQ}{bp} = \frac{k}{\varepsilon_u^d}$. This conclusion is supported by the findings of Feldstein (1978). Ramsey model was used in life cycle models, for best reference see Atkinson, A.B. and Stiglitz,J. (1976),Atkinson, A.B. and A. Sandmo (1980), Atkinson, A.B. and Stiglitz,J. (1980).For Pareto optimal taxation dueto Werning (2007):Given the utility function $u(c, y, \theta)$ and a density of skills $f(\theta)$, a differentiable;

[12] The FOC with respect to l is given as:$\lambda \cdot (n - v'(l_n)) + \frac{\phi(n)}{n} \cdot [v'(l_n) + l_n v''(l_n)] = 0$.FOC with respect to u is given as:$-\frac{d\phi(n)}{n} = [G'(u_n) - \lambda]$.

[13] The lemma relates the ordinary (Marshallian) demand function to the derivatives of the indirect utility function.

Proposition 1 Tax function $\boldsymbol{t(y)}$ inducing an allocation .$\boldsymbol{(c(\theta), y(\theta))}$ is Pareto efficient if and only if condition$\frac{\boldsymbol{\tau'(\theta) f(\theta)}}{\boldsymbol{h' y(\theta)}} + \int_{\boldsymbol{\theta}}^{\boldsymbol{\bar{\theta}}} \frac{\mathbf{1}}{\boldsymbol{u'(c(\tilde{\theta}))}} \boldsymbol{f(\tilde{\theta}) d\tilde{\theta}} \leq \mathbf{0}$ holds[14], where $\boldsymbol{\tau(\theta)} = \boldsymbol{t'(y(\theta))}$.The Pareto distribution had a density that is a power function $\boldsymbol{g(y)} = \boldsymbol{\mathcal{A} y^{-(\varphi)}}$, so that these holds: $\frac{\boldsymbol{d \log g(y)}}{\boldsymbol{d \log y}} = -\boldsymbol{\varphi In\, \bar{\tau}} \leq \frac{\boldsymbol{\sigma+\eta-1}}{\boldsymbol{\varphi+\eta-2}}$ *if* $\boldsymbol{\varphi \approx 3}$ as per Saez (2001) , then $\boldsymbol{\sigma} < \mathbf{2}$ and $\boldsymbol{\sigma}$ cannot be interpreted as risk aversion but as control variable.

3. Separating theorem: Extension to a Stochastic Setting and Numerical Simulation

Theorem 1: Separation of Consumption-Savings and Human Capital Accumulation: There exists an optimal policy where the agent's intertemporal consumption and savings decisions can be separated from their human capital accumulation decision (i.e., education effort). Specifically, the agent's optimal consumption and savings decisions are independent of their human capital accumulation decisions. The optimal consumption path $\boldsymbol{c_t^*}$ depends solely on the agent's initial wealth, interest rate, and wage rate, while the optimal education effort $\boldsymbol{e_t^*}$ is determined solely by the agent's initial human capital and the future returns to education (i.e., wages).
Now, we extend the Separation Theorem to a stochastic setting where human capital accumulation is subject to uncertainty. Then, I will outline a numerical simulation approach to verify the theoretical results.Formal setup:**Agent's Lifetime Utility:** An agent's lifetime utility depends on consumption and human capital accumulation effort:

$$U = \sum_{t=0}^{T} \beta^t u(c_t) + \gamma \cdot f(h_t, e_t) \qquad (22)$$

where:$u(c_t)$ is the utility function for consumption, β is the discount factor,$f(h_t, e_t)$ is the human capital accumulation function, which depends on the current level of human capital h_t and education effort e_t,γ is the coefficient that determines the return to human capital accumulation.
Human Capital Accumulation: The agent's human capital evolves according to:

$$h_{t+1} = h_t + f(h_t, e_t) + \sigma h_t dW_t \qquad (23)$$

[14] The starting point here is this inequality which states that marginal tax rate must be lower than 100% :$\frac{\tau(\theta)}{1-\tau(\theta)} \frac{\varepsilon_w^*}{\Phi} \left(-\frac{d\log \frac{\tau(\theta)}{1-\tau(\theta)}}{d\log w} - 1 - \frac{d\log(\varepsilon_w^*(w))}{d\log w} - \frac{d\log(h^*(w)}{d\log w} - \frac{\partial MRS}{\partial c} w \right) \leq 1$

Where $f(h_t, e_t)$ captures the deterministic part of human capital accumulation, $\sigma h_t dW_t$ represents a stochastic shock.

Consumption-Savings Decision: The agent maximizes their lifetime utility by choosing consumption c_t and savings a_{t+1} , subject to a budget constraint:

$$a_{t+1} = ra_t + w_t h_t(1 - e_t) - c_t. \tag{24}$$

where a_t is wealth, r is the interest rate, and w_t is the wage rate, which is a function of the agent's human capital h_t.

Optimal Intertemporal Consumption-Savings Decision: The agent's decision problem for consumption and savings is formulated as:

$$\max_{\{c\}} \sum_{t=0}^{T} \beta^t u(c_t) \tag{25}$$

s.t. $a_{t+1} = ra_t + w_t h_t(1 - e_t) - c_t$.

Human Capital Accumulation Decision: The agent's decision regarding human capital accumulation (education effort e_t) is made independently of their consumption-saving decisions. The optimal human capital accumulation effort e_t^* is determined by:

$$e_t^* = \arg\max_{c}(\gamma f(h_t, e) + \mathbb{E}[\beta V(a_{t+1}, h_{t+1})]) \tag{26}$$

Where $V(a_{t+1}, h_{t+1})$ is the value function , $\mathbb{E}[\beta V(a_{t+1}, h_{t+1})]$ represents the expected future value from choosing education effort e_t.

Proof 1:

- **Stochastic Model Setup**

We modify the previous deterministic model by introducing stochastic shocks to human capital accumulation. The human capital evolution equation is now:

$$dh_t = f(h_t, e_t)dt + \sigma h_t dW_t \tag{27}$$

where:W_t is a standard **Wiener process** (Brownian motion),σ is the volatility parameter capturing uncertainty in human capital accumulation.The agent maximizes expected lifetime utility:

$$U = \mathbb{E}\left[\int_0^T e^{-\rho t} u(c(t)dt\right] \tag{28}$$

subject to the **stochastic budget constraint**:

$$da_t = (ra_t + wh_t(1 - e_t) - c_t)dt \tag{29}$$

Where $\dot{a}(t) = ra + wh(1 - e) - c$.

$a(t)$ is financial wealth,,r is the exogenous interest rate.

- **Hamilton-Jacobi-Bellman (HJB) Equation**[15]

Define the **value function**:

$$V(a,h) = \max_{c,e} \mathbb{E}\left\{\int_0^T e^{-\rho t} u(c(t)dt\right\} \tag{30}$$

Using dynamic programming, the associated HJB equation is:

$$\rho V = \max_{c,e}\left\{u(c) + V_a(ra_t + wh_t(1-e_t) - c_t) + V_h f(h,e) + \frac{1}{2}\sigma^2 h^2 V_{hh}\right\} \tag{31}$$

where V_a and V_h are the derivatives of the value function with respect to a and h.

- **First-Order Conditions and Separation**

Consumption-Savings Decision

From the FOC for consumption:

$$u'(c) = V_a \tag{32}$$

which determines the consumption path **independently** of h.

Human Capital Investment Decision

The **FOC for e** is:

$$V_h \frac{\partial f}{\partial e} = wV_a \tag{33}$$

which determines $\boldsymbol{e^*(t)}$ **independently** of financial wealth a.Since $\boldsymbol{e^*(t)}$is determined solely by $f(h,e)$ and w, and does not depend on a or consumption, the **Separation Theorem still holds under uncertainty** ■.

[15] HJB equation is modeled as in Achdou et al.(2022). The deterministic optimal control problem is given as: $V(x_0) = \max_{u(t)_{t=0}^{\infty}} \int_0^{\infty} e^{-\rho t} h(x(t), u(t)dt$ s.t. $\dot{x}(t) = g\big(x(t)\big), u(t), u(t) \in U\, ; t \geq 0, x(0) = x_0$.In previous expression:$\rho \geq 0$ is the discount rate, $x \in X \subseteq \mathbb{R}^m$ is a state vector; $u \in U \subseteq \mathbb{R}^n$ is a control vector, and $h: X \times U \rightarrow R$. In vector notation previous is given as:$\rho V(x) = \max_{u \in U} h(x,u) + \nabla_x V(x) \cdot g(x,u) + \frac{1}{2} tr\big(\Delta_x V(x)\sigma^2(x)\big)$. Hessian matrix of V (dimension $m \times m$).By Ito's lemma see Kiyosi Itô (1951):$df(x) = \left(\sum_{i=1}^{n} \mu_i(x)\frac{\partial f(x)}{\partial x_i} + \frac{1}{2}\sum_{i=1}^{m}\sum_{i=1}^{m} \sigma_{ij}^2(x)\frac{\partial^2 f(x)}{\partial x_i \partial x_j}\right)dt + \sum_{i=1}^{m} \sigma_I(x)\frac{\partial f(x)}{\partial x_i} dW_i$

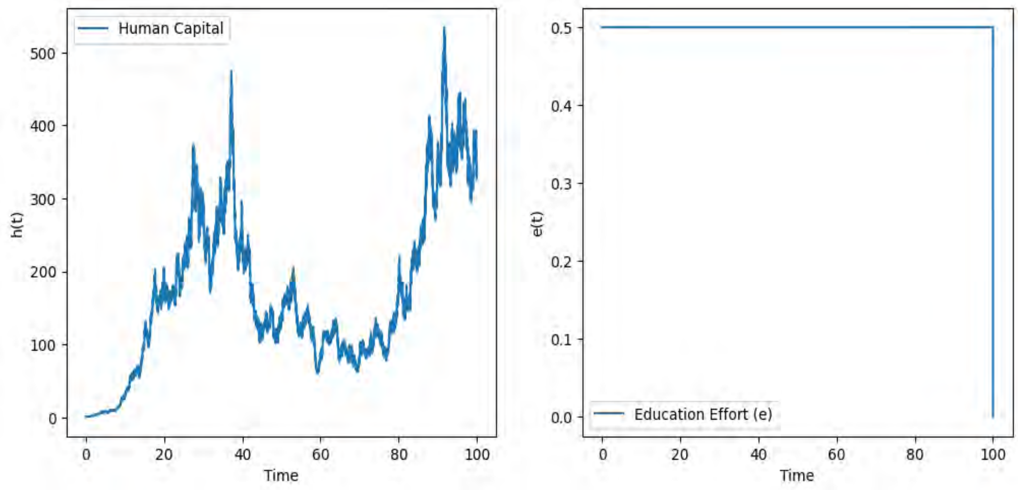

Figure 2. Human capital and education effort, Source:Author's own calculations

3.1.Atkinson-Stiglitz theorem: commodity taxation as supplementary to labor taxation

The question here is whether governments can increase social welfare by adding differentiated commodity taxation $\tau = (\tau_1, \ldots, \tau_k)$ in addition to nonlinear tax on earnings w .Atkinson and Stiglitz (1976) theorem:

Theorem 2: Commodity taxes cannot increase social welfare if utility functions are weakly separable in consumption goods versus leisure and the subutility of consumption goods is the same across individuals, i.e.,$\boldsymbol{u_i(c_1, \ldots, c_k, w)} = \boldsymbol{u_i(v(c_1, \ldots, c_k), w)}$ with the subutility function $\boldsymbol{v(c_1, \ldots, c_k)}$ homogenous across individuals. Laroque (2005) and Kaplow (2006) have provided intuitive proof of this theorem.

Next, we will compare Atkinson-Stiglitz theorem with separation theorem in a table.

Table 1. Atkinson-Stiglitz Theorem Separation Theorem in Human Capital

Aspect	Atkinson-Stiglitz Theorem	Separation Theorem in Human Capital
Focus	Optimal taxation (labor vs consumption/savings taxation)	Optimal decision-making (human capital vs consumption-saving)
Key Assumption	Weak separability of preferences between labor and consumption	No feedback loop between consumption-saving and education effort
Implication	Savings and consumption should not be taxed separately	Human capital accumulation and consumption-saving are independent
Policy Takeaway	Optimal tax should focus on labor income only	Education and human capital policy can be designed separately from savings policies

4. Endogenize Ben-Porath model and derive mathematically

Endogenizing the Ben-Porath (1967) model means incorporating decision variables within the model rather than treating them as exogenous. This typically involves allowing human capital accumulation to be driven by endogenous choices of time and resources allocated to education, training, and work.

- **Setup of the Ben-Porath Model**

An individual maximizes lifetime utility:

$$U = \int_0^{\infty} e^{-\rho t} u\big(c(t)\big) dt \qquad (34)$$

$C(t)$ is consumption ρ is discount rate, $u(c)$ is utility CRRA $u_c = \frac{c^{1-\sigma}}{1-\sigma}$ $\sigma > 0$

- **Human Capital Accumulation**

The evolution of human capital $h(t)$ follows:

$$\dot{h}(t) = f\big(s(t), h(t)\big) \qquad (35)$$

$s(t)$ is the fraction of time spent on human capital accumulation (education, training),$1 - s(t)$ is the time spent working ,$f(s, h)$ is the human capital production function, often modeled as:

$$\dot{h}(t) = As(t)h^{\phi} \qquad (36)$$

Where $A > 0, \phi > 0 < 1$

- **Budget Constraint**

Income is generated by human capital and labor supply:

$$\dot{a}(t) = w(t)\big(1 - s(t)\big)h(t) - c(t) \tag{37}$$

$\dot{a}(t)$ is asset accumulation ; $w(t)$ is wage per efficiency labor.

- **Hamiltonian Formulation**

Define the Hamiltonian:

$$\mathcal{H} = e^{-\rho t}u(c) - \lambda_a[w(1-s)h - c] + \lambda_h s h^{\phi} \tag{38}$$

Where λ_a is the co-state variable for assets ;$\lambda_h$is the co-state variable for human capital

- **First-Order Conditions**

Maximizing w.r.t. c, s, and the state equations:

Consumption FOC:

$$e^{-\rho t}u'(c) = \lambda_a \tag{39}$$

Optimal learning time $s(t)$:

$$-\lambda_a wh + \lambda_h A\phi h^{\phi-1} = 0 \tag{40}$$

Costate equations:

$$\dot{\lambda}_a = \rho\lambda_a - \lambda_a \dot{w}(1-s)$$
$$\dot{\lambda}_h = \rho\lambda_h - \lambda_h A\phi h^{\phi-1}s + \lambda_a w(1-s) \tag{41}$$

We'll implement the numerical solution and plot the equilibrium paths for the endogenized Ben-Porath model. The key equations are:Consumption Euler Equation:$\dot{c} = \frac{u'(c)}{u''(c)}(\rho - r)$.Where r is the effective return.Human Capital Accumulation: $\dot{h} = Ash^{\phi}$;Asset accumulation:$\dot{a} = w(1-s)h - c$.Optimal learning time $s(t)$:

$$s^* = \arg\max_s \left\{A\phi h^{\phi-1} - wh\frac{\lambda_a}{\lambda_h}\right\} \tag{42}$$

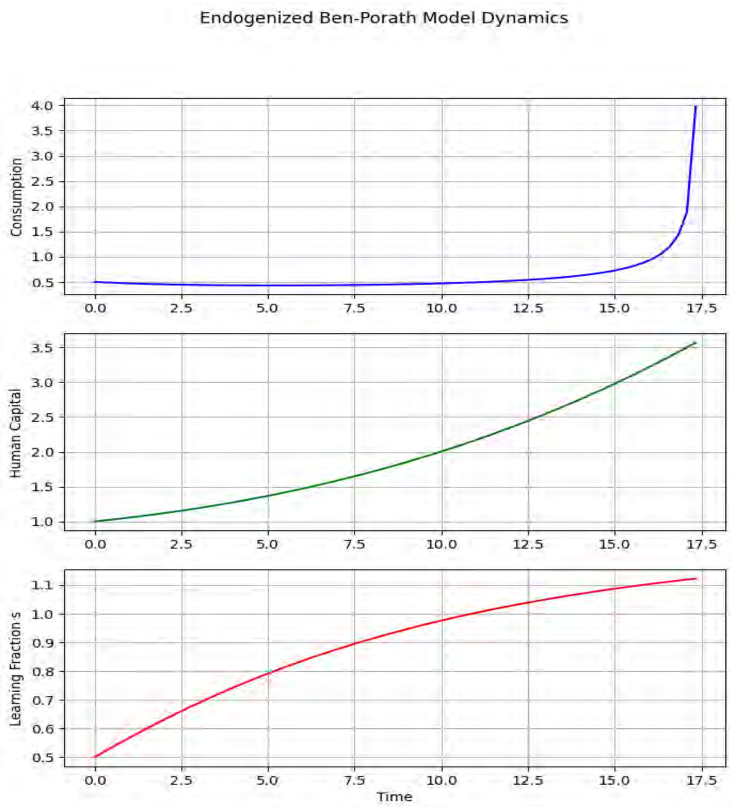

Figure 3 endogenized Ben -porath model dynamics

5. Endogenous determination of life expectancy or retirement

The endogenous Ben-Porath model is an extension of the classic Ben-Porath (1967) human capital model, incorporating endogenous determination of life expectancy or retirement. Below, we derive the key equations of the model step by step.

- **Setup of the Problem**

An individual chooses education time s and human capital investment to maximize lifetime earnings. The individual's lifetime is endogenous, meaning life expectancy T is a choice variable.

Human capital dynamics:

$$\dot{h}(t) = I(t) - \delta h(t) \tag{43}$$

where $h(t)$ is human capital,$I(t)$) is investment in human capital, and δ is the depreciation rate.Earnings function:

$$y(t) = wh(t)\big(1 - e(t)\big) \tag{44}$$

where w is the wage per unit of human capital, and $e(t)$ is the fraction of time spent on education.

Utility function:

$$U = \int_0^T e^{-\rho t} u(c(t)dt \quad (45)$$

where ρ is the discount rate and $c(t)$ is consumption.

Budget constraint:

$$\dot{a}(t) = ra(t) + y(t) - c(t) \quad (46)$$

where $a(t)$ is assets and r is the interest rate.

Endogenous life span T: Life expectancy depends on health investments, which affect mortality. We define a **survival probability function** $S(T)$ where $S(T)$ is increasing in health expenditures.

- **Hamiltonian and First-Order Conditions**

The individual maximizes lifetime utility subject to human capital accumulation, budget constraints, and the endogenous determination of T. The Hamiltonian is:

$$\mathcal{H} = e^{-\rho t} u(c) + \lambda_a[ra + wh(1-e) - c] + \lambda_h[I - \delta h\,] \quad (47)$$

First-Order Conditions (FOCs)

1.Optimal Consumption:

$$u'(c) = \lambda_a \quad (48)$$

2.Human Capital Investment:

$$\lambda_h = \lambda_a w(1-e) \quad (49)$$

3. Optimal Education Time:

$$wh = -\frac{\dot{\lambda}_h}{\lambda_h} + \delta \quad (50)$$

- **Endogenous Life Expectancy**:

  Life expectancy T is chosen optimally by setting the marginal benefit of extending life equal to the marginal cost of investment in health. That is,

$$\frac{\partial U}{\partial T} = 0 \quad (51)$$

- **Implications and Insights**

Longer life expectancy ⇒ more human capital investment: Since a longer life increases the time to recoup education costs, individuals optimally invest more in schooling.Interest rates and wages affect education time:Higher r lowers incentives to invest in education, while higher www increases schooling time.Endogenous T adds an extra margin of optimization: Unlike the original Ben-Porath model, agents can invest in health to extend their lifespan, further reinforcing the education-life expectancy complementarity.We now extend the endogenous Ben-Porath model by deriving taxation formulas under Mirrleesian, Pareto optimal, and Ramsey criteria.

- **Setup with Taxation**

Incorporating taxation, we introduce: Labor income tax τ_w affecting wage income.Capital income tax τ_r, affecting asset accumulation.Education subsidies/taxes τ_e on education investments.The budget constraint becomes:

$$\dot{a}(t) = (1-\tau_r)ra + (1-\tau_w)wh(1-e) - c - (1-\tau_e)I \quad (52)$$

$$G = \int_0^T [\tau_w wh(1-e) + \tau_r ra + \tau_e I]dt \quad (53)$$

- **Mirrleesian Taxation (Second-Best Optimal Taxation)**

Mirrleesian taxation maximizes social welfare under asymmetric information about individual ability $h(0)$. The social planner chooses taxes to optimize:

$$U = \int_0^T e^{-\rho t} u(c(t)dt \quad (54)$$

First-Order Conditions for Optimal Taxation

Labor Taxation τ_w

$$\tau_w = \frac{1-G\prime(h)}{1+\frac{h(1-e)}{c}\eta} \quad (55)$$

where η is the elasticity of labor supply.

Capital Taxation τ_r

With no savings distortions (Atkinson-Stiglitz theorem), optimal tax is zero $\tau_r = 0$ .If savings are elastic, tax follows Chamley-Judd logic: $\tau_r \to 0$

3.Education Subsidy τ_e

$$\tau_e = \frac{\partial U/\partial s}{\partial c/\partial s} \quad (56)$$

Where $\partial U/\partial s$ is the welfare gain from education. Thus, Mirrleesian taxation suggests progressive labor taxation, zero long-run capital tax, and education subsidies.

- **Pareto Optimal Taxation (First-Best Efficient Taxes)**

Pareto optimal taxation finds taxes ensuring efficiency, meaning no one can be made better off without harming others. The planner maximizes:

$$\max_{\tau_w,\tau_r,\tau_c} \int_0^T e^{-\rho t} u(c)t \quad (57)$$

subject to the economy-wide feasibility constraints.

Key Results: **Zero Capital Taxation** ($\tau_r = 0$) to avoid savings distortions.**Optimal Labor Taxation** :If redistribution is required, the **inverse elasticity rule** applies:

$$\tau_w = \frac{1}{1+\eta} \quad (58)$$

With full information, $\tau_w = 0$.Education Subsidies : If human capital is productive, the optimal subsidy equals the externality:

$$\tau_e = -\{externality\ from\ education\}. \quad (59)$$

Thus, Pareto efficiency requires zero capital taxation and education subsidies if education has positive spillovers.Thus, Pareto efficiency requires **zero capital taxation** and **education subsidies** if education has positive spillovers.

- **Ramsey Taxation (Optimal Taxes with Government Revenue Constraints)**

Ramsey taxation maximizes government revenue while minimizing distortions. The planner solves:

$$\max_{\tau_w,\tau_r,\tau_c} \int_0^T e^{-\rho t} u(c)t \qquad (60)$$

subject to the government budget constraint:

$$G = \int_0^T [\tau_w wh(1-e) + \tau_r ra + \tau_e I]dt \qquad (61)$$

Key Results: Inverse Elasticity Rule for Labor Taxation

$$\tau_w = \frac{1}{1+\eta} \qquad (62)$$

More elastic labor supply leads to lower tax rates.

Optimal Capital Tax (depends on revenue needs):

$$\tau_r = \frac{G - \tau_w wh(1-e) - \tau_e I}{rA} \qquad (63)$$

If capital is highly elastic, tax is lower.

Education Taxes/Subsidies: If education enhances wages, **subsidies** ($\tau_e < 0$) are optimal.If education has fiscal costs, a **small tax** may be needed.Thus, Ramsey taxation balances revenue needs against distortions, often leading to **moderate labor taxes, low capital taxes, and education subsidies**.

Table 2 Labor tax,capital tax education tax

Tax Type	Labor Tax τ_w	Capital Tax τ_r	Education Tax τ_e
Mirrlees	Progressive tax	Zero in long run	Subsidy if positive spillover
Pareto optimal	Zero if full info, otherwise inverse elasticity rule	Zero	Education subsidy if spillover exists
Ramsey	Inverse elasticity rule	Low or zero	Revenue-dependent

Source:Author's own calculations

Numerical example: Wage per unit of human capital: $w = 10$;Depreciation rate: $\delta = 0.02$; Discount rate: $\rho = 0.04$; Interest rate: $r = 0.03$,Elasticity of labor supply: $\eta = 0.5$, Government spending as a fraction of GDP: $\frac{G}{Y} = 0.2$.

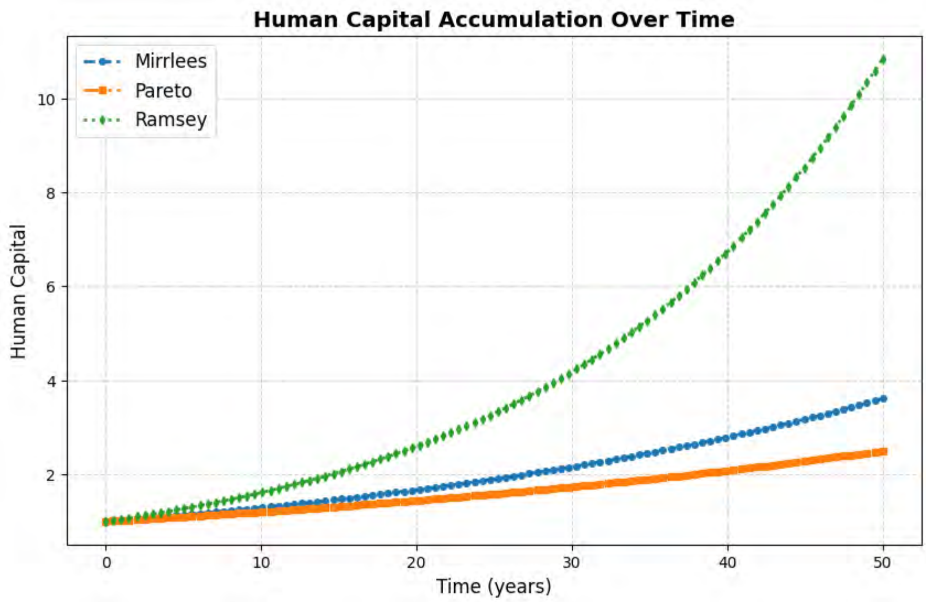

Figure 4 Human capitalaccumulation over time Source:Author's own calculations

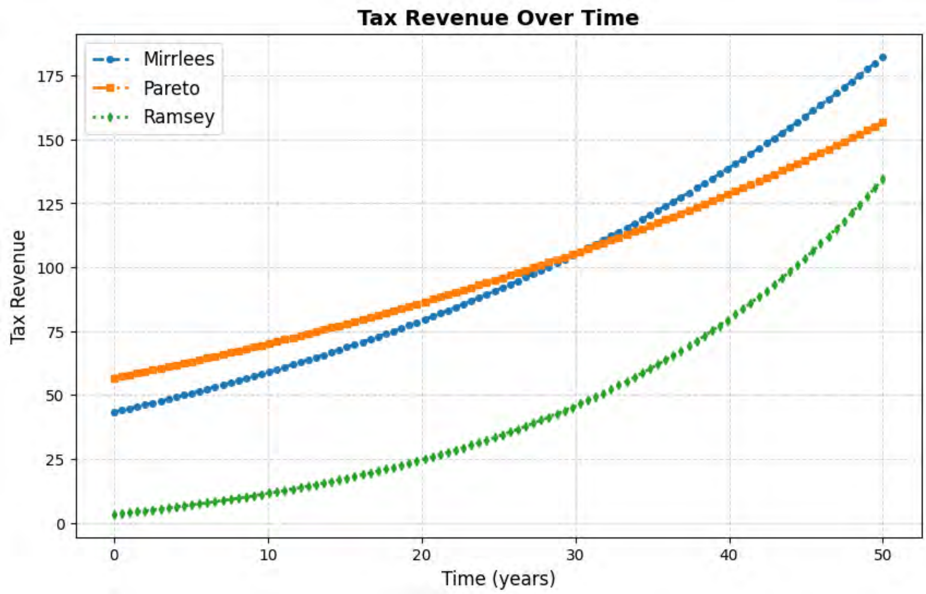

Figure 5 tax revenue over time Source:Author's own calculations

Optimal Labor Tax Rates: Mirrlees Tax: 0.53;Pareto Optimal Tax: 0.67;Ramsey Tax: 0.13;Education Subsidy: -0.10.

6. Endogenizing by alowing individuals to choose both the level and the type of human capital accumulation

The **endogenous Ben-Porath model** extends the classic Ben-Porath (1967) human capital investment model by allowing individuals to choose both the level and the type of human capital accumulation. Below is a step-by-step derivation of the model.

- **Individual's Problem in the Ben-Porath Model**

In the classic Ben-Porath model, an individual starts with an initial stock of human capital $H(t)$ and can invest time and resources into improving it over a finite lifetime T. The objective is to maximize lifetime earnings by optimally choosing human capital investment and labor supply.

- **Key Components of the Model**
- **Human Capital Accumulation Equation**

$$\dot{H}(t) = f\big(H(t), I(t)\big) \quad (64)$$

where:$H(t)$ is the stock of human capital at time t.$I(t)$ is investment in human capital (e.g., schooling, on-the-job training).$f(H, I)$ is a production function for human capital (typically assumed to be concave in I).

- **Budget Constraint**

Income at time t is given by:

$$Y(t) = wH(t)L(t) \quad (65)$$

where:w is the wage per unit of human capital.$L(t)$ is labor supply at time t, where $L(t) + S(t) = 1$ (i.e., total time is divided between work and human capital investment).

- **Lifetime Utility or Earnings Maximization**

The individual maximizes the present value of lifetime earnings:

$$V = \int_0^T wH(t)L(t)e^{-\rho t}dt \quad (66)$$

where $\rho$is the discount rate.

- **Endogenizing the Ben-Porath Model**

The endogenous version extends the model by allowing the individual to choose the human capital production function rather than it being exogenous. This introduces an additional optimization problem: deciding not just how much to invest in human capital, but also which type of human capital investment to undertake.Let $I(t)$) now be a vector representing different types of human capital investments, say $I_1, I_2, \dots, I_n$In, each affecting human capital accumulation differently. The human capital accumulation equation is now:

$$\dot{H}(I) = f(H, I_1, I_2, .., I_n) \quad (67)$$

where the choice of I_j determines:

Skill specificity: Some investments may lead to general skills, while others improve sector-specific productivity.

Returns to specialization: Different investments may have different productivity effects over time.The individual's problem now becomes:

$$\max_{I_{j(t)},L(t)} \int_0^T w(H, I_1, I_2, \ldots, I_n) HLe^{-\rho t} dt \quad (68)$$

subject to:The time constraint: $L + S = 1$, where S is the total time spent on skill acquisition.The dynamic constraint: $\dot{H}(I) = f(H, I_1, I_2, .., I_n)$.
The optimal choice of **which type of skill investment** to undertake.

- **Optimality Conditions (Hamiltonian Approach)**

Define the Hamiltonian:

$$\mathcal{H} = w(H, I_1, I_2, \ldots, I_n) HLe^{-\rho t} + \lambda(H,, I_1, I_2, \ldots, I_n) \quad (69)$$

where λ is the co-state variable representing the shadow price of human capital. The first-order conditions (FOCs) for optimization are:

Labor-Leisure Tradeoff:

$$wHe^{-\rho t} = \lambda \frac{\partial f}{\partial I_j} \quad (70)$$

where λ is the co-state variable representing the shadow price of human capital.The first-order conditions (FOCs) for optimization are:

Labor-Leisure Tradeoff:

$$\frac{\partial f}{\partial I_j} = \frac{\partial f}{\partial I_k}; \forall j, k \quad (71)$$

This ensures that the individual allocates resources efficiently across different types of human capital investment.

Co-State Evolution:

$$\dot{\lambda} = \left(\rho - \frac{\partial w}{\partial H} L\right) \lambda \quad (72)$$

The shadow price of human capital evolves over time according to the individual's discount rate and the marginal effect of human capital on wages.

- **Implications of the Endogenous Extension**

The individual optimally selects not just how much human capital to accumulate but also which kind of investment to pursue.Depending on the returns to specialization, individuals might focus more on general skills early in life and specialized skills later.The structure of the human capital function $f(H, I)$determines whether investments are complementary or substitutable.Policy implications: Government subsidies or tax incentives can influence the type of skills individuals choose to develop.

Conclusion

The endogenous Ben-Porath model extends the classical model by allowing individuals to choose the form of human capital investment, introducing an additional decision margin. This affects the optimal allocation of time, investment strategies over the life cycle, and ultimately earnings profiles. By optimizing over both the quantity and type of investment, the model provides richer insights into skill formation, specialization, and lifelong learning.The endogenous Ben-Porath model can be extended by introducing optimal taxation frameworks such as Mirrleesian, Ramsey, and Pareto taxation. Here's how we can incorporate these taxation systems into the model:

- **Mirrleesian Taxation (Optimal Income Taxation)**

In the Mirrleesian framework, taxes are designed to optimize social welfare while accounting for individuals' differing abilities or productivity. The government aims to tax individuals in such a way that the distortion on their labor supply and investment choices is minimized, given their private information on ability or human capital. Assume that income is taxed at rates that depend on the individual's human capital $H(t)$ and type of investment $I(t)$. The tax system can be described as:

$$Income\ tax: TY_t = \tau_Y\big(H(t), I(t)\big)Y(t) \tag{73}$$

where $\tau\tau_Y\big(H(t), I(t)\big)$ is the labor income tax rate, which depends on human capital and the type of investment $I(t)$.The government maximizes social welfare subject to a budget constraint while considering these taxes. The budget constraint is:

$$\int_0^T T_Y(t)dt = G \tag{74}$$

where G is the government expenditure.The individual's optimization problem with Mirrleesian taxes becomes:

$$\max_{I_{j(t)}, L(t)} \int_0^T [w(H, I_1, I_2, \dots, I_n)HLe^{-\rho t} - T_Y\big(H(t), I(t)\big)Y(t)]\, dt \tag{75}$$

subject to the same time constraint and the dynamic human capital accumulation equation. The first-order conditions now need to account for the labor income tax rate:

$$\frac{\partial w(H, I_1, I_2, \dots, I_n)HLe^{-\rho t}}{\partial L} = \frac{\partial T_Y\big(H(t), I(t)\big)Y(t)}{\partial L} \tag{76}$$

This ensures that the individual's decision balances labor supply with the marginal tax impact.

- **Ramsey Taxation (Optimal Consumption Taxation)**

The Ramsey optimal taxation problem aims to minimize the distortion on consumption choices. The government seeks to tax consumption in such a way that maximizes welfare, subject to its budget constraint.The individual's utility depends on their consumption $C(t)$ and leisure $L(t)$. The optimal tax on consumption is chosen such that the marginal cost of raising government revenue is minimized. Assuming a consumption tax τ_c, the government's revenue from consumption taxes is:

$$T_C(t) = \tau_C(t)C(t) \tag{77}$$

The individual's budget constraint then becomes:

$$C(t) + T_Y(t) = wH(t)L(t) \tag{78}$$

The optimization problem for the individual now includes both labor income tax and consumption tax:

$$\max_{I_{j(t)},L(t),C(t)} \int_0^T \left[u(C(t), L(t)e^{-\rho t} - T_Y\big(H(t), I(t)\big)Y(t) - T_C(t)C(t)\right]dt \quad (79)$$

where $U\big(C(t), L(t)\big)$ is the utility function, typically concave in both consumption and leisure. The first-order conditions will give the optimal consumption tax rate:

$$\frac{\partial u\big(C(t),L(t)\big)}{\partial C(t)} = \frac{\partial T_C(t)C(t)}{\partial C(t)} \quad (80)$$

This ensures that the marginal utility of consumption is equated to the marginal tax.

- **Pareto Efficient Taxation**

In the Pareto-efficient taxation framework, the goal is to find tax policies that maximize social welfare without making any individual worse off. The social planner maximizes the total social welfare:

$$\max_{\{\tau_Y(H(t),I(t)),\tau_C(t)\}} \int_0^T \left[\sum_i u_i\big(C_i(t), L_i(t)\big)e^{-\rho t}\right]dt \quad (81)$$

s.t. guvernment budget constraint:

$$\int_0^T T_Y(t)dt + \int_0^T T_C(t)dt = G \quad (82)$$

The Pareto-efficient taxation problem involves balancing individual utility subject to the government's revenue requirement, ensuring that taxes on labor and consumption do not distort the agents' decisions too much. The optimal tax rates under Pareto efficiency would satisfy the conditions:

$$\frac{\partial u_i\big(C_i(t),L_i(t)\big)}{\partial C_i(t)} = \lambda_i; \ \frac{\partial u_i\big(C_i(t),L_i(t)\big)}{\partial C_i(t)} = \lambda_i \times marginal\ tax\ rates \quad (83)$$

where λ_i is the shadow price of consumption and leisure for each individual.

- **Incorporating Taxes into the Endogenous Ben-Porath Model**

The incorporation of these taxes into the endogenous Ben-Porath model affects both the individual's investment decisions and labor supply. The optimal allocation of time between human capital investment $S(t)$ and labor $L(t)$ is influenced by the taxation structure. The first-order conditions and Hamiltonian would be adjusted as follows.Hamiltonian with Taxes:

$$\mathcal{H} = w(H, I_1, \dots, I_n)HLe^{-\rho t} - T_Y\big(H(t), I(t)\big)Y(t) + \lambda\big(f(H, I_1, \dots, I_n)\big) - T_C(t)C(t) \quad (84)$$

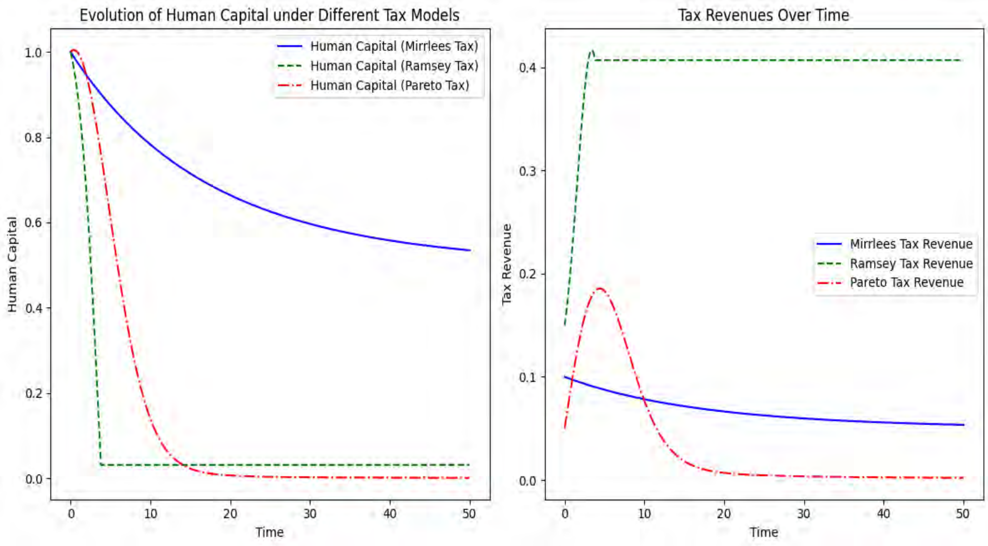

Figure 6 evolution of human capital under different tax regimes and tax revenues over time, Source:Author's own calculations

7. Derivation of Mirrlees, Ramsey, and Pareto Optimal Tax Rate Formulas in the Ben-Porath Model with Spillovers

We derive the optimal tax formulas for:

- ✓ Mirrleesian Optimal Tax Rate τ_m^* – Optimal taxation on labor income.
- ✓ Ramsey Optimal Tax Rate τ_r^*-optimal taxation on consumption
- ✓ Pareto Optimal Tax Rate τ_p^* -optimal taxation on human capital investment

➢ **Mirrleesian Optimal Tax Rate ($\boldsymbol{\tau_m^*}$)**

Mirrlees taxation is based on the principle of optimal income taxation, where a planner maximizes social welfare while considering individuals' responses to taxation.Government's Problem:The government maximizes the social welfare function:

$$W = \int_0^T e^{-\rho t} u(c, l) dt \quad (85)$$

subject to: The household's budget constraint

$$c(t) = (1 - \tau_r)(1 - \tau_m) w(1 - s - l) \quad (86)$$

– The human capital accumulation equation:

$$\dot{h} = (1 - \tau_p) A \quad (87)$$

– The government's budget constraint:

$$G = \tau_m wh(1 - s - l) + \tau_r c + \tau_p A h^{\alpha} \bar{h}^{\gamma_{ext}} s^{\beta} \quad (88)$$

Optimality Condition for **($\boldsymbol{\tau_m^*}$) (Mirrleesian Labor Tax Rate)**

By differentiating the indirect utility function with respect to τ_m the optimal Mirrleesian labor tax rate is:

$$\tau_m^* = \frac{1-G'(h)}{1+\epsilon_n} \tag{89}$$

where: $G'(h)$is the marginal cost of public funds ;ϵ_n is the elasticity of labor supply .If labor supply is inelastic $\epsilon_n \to 0$, the tax rate is higher. If labor supply is very elastic, the tax rate is lower.

- **Ramsey Optimal Tax Rate τ_r^***

Ramsey taxation minimizes distortions in consumption choices while ensuring revenue sufficiency.Ramsey Problem:The government solves:

$$\max_{\tau_r} \int_0^T e^{-\rho t} u(c,l)dt \tag{90}$$

subject to the **Euler equation**:

$$\dot{c} = c\left(r - \rho - \frac{\dot{\tau}_r}{1-\tau_r}\right) \tag{91}$$

Where

$$r = (1-\tau_r)Ah^{\alpha}\bar{h}^{\gamma_{ext}}s^{\beta} \tag{92}$$

is the return on human capital investment.

Using the inverse elasticity rule for optimal taxation, the Ramsey optimal tax on consumption is:

$$\tau_r^* = \frac{1}{\eta_c}\frac{G'(h)}{1+G'(h)} \tag{93}$$

Where η_c is the elasticity of consumption with respect to tax rate.If consumption is highly inelastic $\eta_c \to 0$, a higher Ramsey tax is optimal. If consumption is elastic, taxation should be lower.

- **Pareto Optimal Tax Rate τ_p^***

Pareto taxation ensures efficiency while accounting for spillovers in human capital accumulation.Planner's Problem:The social planner maximizes:

$$\max_{\tau_p} \int_0^T e^{-\rho t} u(c,l)dt \tag{94}$$

Subject to:

- **Human capital accumulation with spillovers**:

$$\dot{h} = (1-\tau_p)Ah^{\alpha}\bar{h}^{\gamma_{ext}}s^{\beta} \tag{95}$$

- **Steady-state growth of human capital**:

$$g_h = (1-\tau_p)Ah^{\alpha-1}\bar{h}^{\gamma_{ext}}s^{\beta} \tag{96}$$

Optimality Condition for**τ_p^*(Pareto-Efficient Tax on Human Capital Investment).**From the modified Samuelson Rule with spillovers, we obtain:

$$\tau_p^* = 1 - \frac{\alpha+\gamma_{ext}}{\alpha+\gamma_{ext}+\beta} \tag{97}$$

where:α is private return to human capital;γ_{ext} is the spillover effect ;β is the return to learning effort.If externalities are strong $\gamma_{ext} \gg 0$,then τ_p^* should be lower to encourage human capital accumulation.Summary of Optimal Tax Formulas.

Table 3 summary of tax formulas in Ben-Porath mode with spillovers

Tax Type	Formula	Key Elasticities	Effect
Mirrleesian Labor Tax (τ_m^*)	$\tau_m^* = \frac{1 - G'(h)}{1 + \epsilon_n}$	Elasticity of labor supply ϵ_n	Higher labor supply elasticity reduces (τ_m^*)
Ramsey Consumption Tax (τ_r^*)	$\tau_r^* = \frac{1}{\eta_c} \frac{G'(h)}{1 + G'(h)}$	Elasticity of consumption η_c	Higher consumption elasticity reduces τ_r^*
Pareto Human Capital Tax (τ_p^*)	$\tau_p^* = 1 - \frac{\alpha + \gamma_{ext}}{\alpha + \gamma_{ext} + \beta}$	Spillover effect γ_{ext}	Larger spillovers reduce τ_p^*

Source:Author's own calculations

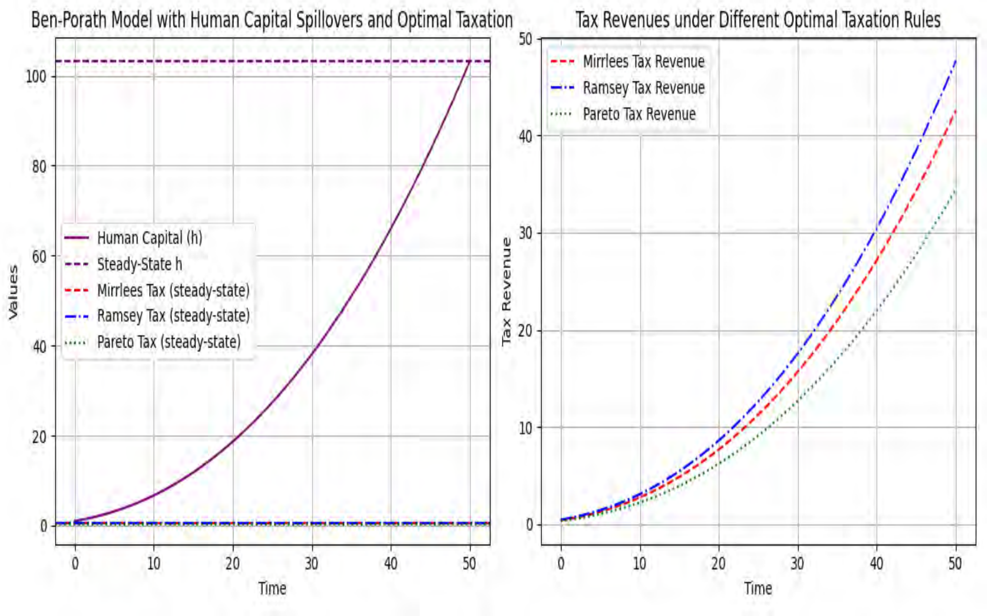

Figure 7 Ben-Porath model with human capital spillovers and tax revenues under optimaltaxation rules, Source:Author's own calculations

Table 4 Tax revenues under different tax regimes

time	Mirrlees Tax Revenue	Ramsey Tax Revenue	Pareto Tax Revenue
0	0.411765	0.461538	0.333333
1	0.549020	0.615385	0.444444
2	0.707835	0.793398	0.573009
3	0.888868	0.996314	0.719560
4	1.092.772	1.224.865	0.884625
5	1.320.187	1.479.771	1.068.723
6	1.571.747	1.761.739	1.272.367
7	1.848.070	2.071.463	1.496.057
8	2.149.761	2.409.623	1.740.283
9	2.477.415	2.776.882	2.005.526

Source:Author's own calculations

8. Conclusion

In our Separation Theorem, we found that human capital investment decisions (education effort) are independent from consumption-saving decisions.This means:The choice of consumption and savings does not affect human capital accumulation.Human capital accumulation can be optimized separately from the consumption-saving problem.Policy interventions targeting education and human capital formation can be made separately from consumption-saving policies. Atkinson-Stiglitz argues that savings and labor supply should be separate for tax policy design.The Separation Theorem argues that human capital accumulation is independent of consumption-saving for individual decision-making. Atkinson-Stiglitz argues that savings and labor supply should be separate for tax policy design.The Separation Theorem argues that human capital accumulation is independent of consumption-saving for individual decision-making.In Ben-Porath model with moral hazard (exogenous version of the model) steady-state levels of human capital are different under different tax regimes,SS level is highest under Mirrleesian tax regime, higher than Pareto optimal tax regime, and SS level of human capital is lowest under Ramsey tax regime.While the chosen effort level at which these steady-state human capital levels are evaluated is the same for all tax regimes. In the endogenized human capital curve is convex,and learning fraction s is concave.While consumption in this model is constant until higher levels of human capital accumulation and learning fraction where instantly grows exponentially.In Ben-Porath model that incorporates endogenous determination of life expectancy or retirement,human capital accumulation is highest in Ramsey model, higher than Mirrlees taxes and Pareto taxes.While Pareto taxation generates highest revenue up until 30 years which later is surpassed by Mirrlees taxation. Ramsey taxation in this model generates lowest revenues.In endogenous Ben-Porath model where individuals are allowed to choose both the level and the type of human capital accumulation level of human capital is highest under Mirrlees tax regime, folowed by Pareto but later is being surpassed by Ramsey after 15-20 years.Ramsey tax regime generates highest tax revenues.Pareto taxes generate higher tax revenues than Mirrlees but after 10-15 years are surpassedby the later tax regime.In Ben-Porath Model with Spillovers human capital without taxes is not in stedy-state compared with taxes where three types of regimes. Underinvestment can occur due to:Borrowing constraints if individuals cannot finance education upfront.Externalities if the social return to education is higher than the private return.Imperfect information: Individuals may underestimate the long-term returns to education. Ramsey tax regime generates highest taxe revenues, flowed by Mirrlees and Pareto and this gap increases over time horizon.

References

Achdou, Y. Han, J. Lasry, J-M. Lions, P-L. Moll, B.(2022). Income and Wealth Distribution in Macroeconomics: A Continuous-Time Approach, *The Review of Economic Studies*,Volume 89,Issue 1,Pages 45-86

Albanesi, S., Sleet, C. (2006). Dynamic Optimal Taxation with Private Information. The Review of Economic Studies, 73(1), 1–30. http://www.jstor.org/stable/3700615

Atkinson, A.B. and A. Sandmo (1980). Welfare Implications of the Taxation of Savings, Economic Journal, Vol. 90, 1980, 529-49. (web)

Atkinson, A.B. and Stiglitz,J (1976). The design of tax structure: Direct versus indirect taxation,Journal of Public Economics, Vol. 6, pp.55-75. (web)

Atkinson, A.B. and Stiglitz,J. (1980).Lectures on Public Economics, Chap 14-4 New York: McGraw Hill, 1980. (web)

Battaglini, Marco, Stephen Coate. (2008). Pareto Efficient Income Taxation with Stochastic Abilities. J. Public Econ. 92 (3): 844–68.

Becker, Gary S. (1964). Human Capital: A Theoretical and Empirical Analysis, with Special Reference to Education. Chicago: Univ. Chicago Press.

Ben-Porath, Yoram. (1967). The Production of Human Capital and the Life Cycle of Earnings. J.P.E. 75 (4): 352–65.

Bowles, S., Gintis, H., & Osborne, M. (2001). The Determinants of Earnings: A Behavioral Approach. Journal of Economic Literature, 39(4), 1137–1176. http://www.jstor.org/stable/2698522

Chamley, C. (1986). Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives. Econometrica, 54(3), 607–622. https://doi.org/10.2307/1911310

Diamond, P. (1998). Optimal income taxation: An example with a U-shaped pattern of optimal marginal tax rates. American Economic Review, 88, 83–95.

Diamond, P., & Saez, E. (2011). The Case for a Progressive Tax: From Basic Research to Policy Recommendations. Journal of Economic Perspectives, 25, 165-190.

Farhi E, Werning I. (2013). Insurance and taxation over the life cycle. Rev. Econ. Stud. 80:596–635

Feldstein, M.(1978), The Welfare Cost of Capital Income Taxation, Journal of Political Economy, Vol. 86, No. 2, Part 2: Research in Taxation (Apr., 1978), pp. S29-S51

Fleischhauer, Kai-Joseph. (2007). A Review of Human Capital Theory: Microeconomics. SSRN Electronic Journal. 10.2139/ssrn.957993.

Golosov, M., Troshkin, M., Tsyvinski, A. (2016). Redistribution and Social Insurance. The American Economic Review, 106(2), 359–386. http://www.jstor.org/stable/43821455

Golosov, Mikhail, Aleh Tsyvinski, and Ivan Werning. (2006). New dynamic public finance: A users guide. NBER Macroeconomics Annual 21:317-363.

Goode, R. B. (1959).Adding to the Stock of Physical and Human Capital, The American Economic Review, 49(2), 147—155

Haley, W. J. (1973). Human Capital: The Choice between Investment and Income. The American Economic Review, 63(5), 929–944. http://www.jstor.org/stable/1813915

Heckman, James J. (1976). A Life-Cycle Model of Earnings, Learning, and Consumption. J.P.E. 84, no. 4, pt. 2 (August): S9–S44.

Itô,K.(1951). On stochastic differential equations. Memoirs, American Mathematical Society 4,pp. 1–51. Online

Jones, L., Manuelli, R., and Rossi, P. (1997) On the optimal taxation of capital income, Journal of Economic Theory, 73, 93–117

Judd, K. (1999). Optimal taxation and spending in general competitive growth models, Journal of Public Economics, 71, 1–26

Judd,K.L.(1985). Redistributive taxation in a simple perfect foresight model. *Journal of Public Economics*. 28 (1): 59–83.

Kaplow, L. On the undesirability of commodity taxation even when income taxation is not optimal. Journal of Public Economics, 90(6–7), 2006,pp.1235–50.

Kocherlakota, N. R. (2005). Zero Expected Wealth Taxes: A Mirrlees Approach to Dynamic Optimal Taxation. Econometrica, 73(5), 1587–1621. http://www.jstor.org/stable/3598884

Laroque, G. R. : Indirect taxation is superfluous under separability and taste homogeneity:A simple proof. Economics Letters, 87(1), 2005,pp.141–144.

Mankiw, N. Gregory; Weinzierl, Matthew; Yagan, Danny (2009). Optimal Taxation in Theory and Practice. Journal of Economic Perspectives. 23 (4): 147–174

Mirrlees, J. A. (1971). An exploration in the theory of optimal income taxation. Review of Economic Studies,38, 175–208.

Mirrlees, J. A., & Adam, S. (2010). Dimensions of tax design: the Mirrlees review. Oxford University Press.

Mirrlees, James & Adam, Stuart & Besley, Timothy & Blundell, Richard & Bond, Stephen & Chote, Robert & Gammie, Malcolm & Johnson, Paul & Myles, Gareth & Poterba, James. (2011). The Mirrlees Review: Conclusions and Recommendations for Reform*. Fiscal Studies. 32. 331 - 359. 10.1111/j.1475-5890.2011.00140.x.

Peterman, W. B. (2016). The effect of endogenous human capital accumulation on optimal taxation. Review of Economic Dynamics, 21, 46–71. doi:10.1016/j.red.2016.04.001

Ramsey, F. (1927). A Contribution to the Theory of Taxation. Economic Journal. 37: 47 61. doi:10.2307/2222721.

Reis, B. C. (2019). Optimal taxation with unobservable investment in human capital. Oxford Economic Papers. doi:10.1093/oep/gpz038

Roy, R. (1947). La Distribution du Revenu Entre Les Divers Biens. Econometrica. **15** (3): 205–225.

Saez,E(2001).Using elasticities to derive optimal income tax rates, The Review of Economic Studies, 68(1),2001,pp.205- 229

Schultz, T. W. (1961). Investment in Human Capital. The American Economic Review, 51(1), 1–17. http://www.jstor.org/stable/1818907

Stantcheva, S.(2017). Optimal Taxation and Human Capital Policies over the Life Cycle, Journal of Political Economy 125, 1931–90.

Werning,I.(2007).Pareto efficient income taxation. NBER Public Economics meeting.