Automatic Hexahedral Mesh Generation for FEM Using Shape Recognition Technique and Tree Method

Satoshi Nagakura, So Noguchi, Hideo Yamashita, and Vlatko Cingoski

***Abstract*—In this paper, a method for automatic hexahedral (Hex) mesh generation is presented. In this method, the shape recognition technique and the tree method are used. Utilizing the shape recognition technique, a mesh is generated in a complicated analysis domain, and a high-quality Hex mesh is obtained. By using the tree method, control of the mesh density is easily enabled. In this proposed system, a modest amount of input data is necessary, like object parameters and the mesh density data.**

***Index Terms*—Finite-element method, hexahedral element, mesh generation, shape recognition technique, tree method.**

I. Introduction

In order to perform a finite element analysis, the analysis domain has to be divided into finite elements. In three-dimensional (3-D) space, the mesh generation in the analysis domain is usually very laborious and time-consuming process, compared with that of the two-dimensional (2-D) space. This is one of the main reasons why 3-D analysis is usually avoided if not necessary. However, there are increasingly more problems where a true 3-D FEA has to be performed. Therefore, it is strongly desirable to develop a suitable 3-D automatic mesh generator. In practice, there are several automatic mesh generators, which usually generate tetrahedral (Tet) mesh. The rationale behind this is that the Tet mesh can be easier generated even for complex analysis domains because of the existence of a suitable mathematical approach. However, if a Tet mesh is compared with a Hex meshes for the same number of nodes, the number of Tet elements is about five times as many as that of Hex mesh. Thus, FEA using Tet mesh requires longer computation time compared with FEA using a Hex mesh with the same number of nodes. On the other hand, if the number of Tet elements is decreased to decrease the computation time, one cannot get the desired accuracy of the results. Consequently, a large interest in development of an automatic Hex mesh generator for FE electromagnetic field analysis has been observed mainly due to the higher accuracy and better convergence. Unfortunately, the generation of Hex mesh is more difficult than that of Tet because of 1) lack of suitable numerical method such as Delaunay Algorithm and 2) inconveniences for meshing complex domains, especially for curvilinear models. Therefore, the development of the automatic Hex mesh generator is strongly desired.

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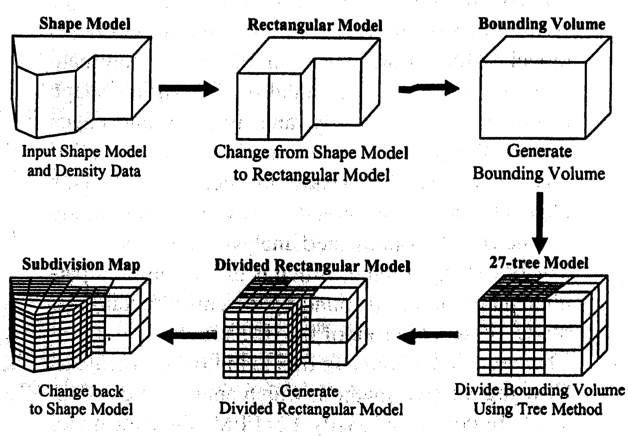

Fig. 1. Simplified algorithm for the automatic mesh generation system.

It is well established among the meshing experts that there are few necessary conditions for considering one automatic mesh generator as a good one, such as

1) correct meshing with small amount of input data;
2) easy mesh density control;
3) generation of finite element with better shapes;
4) easy applicability to complicated analysis domains.

Taking all of the above into consideration, in this paper, we propose a new method for automatic Hex mesh generation. In our method, we implemented two well-known computer algorithms: first, the shape recognition technique to simplify the complex 3-D analysis domains [1] and, second, the tree method [2] to actually generate a Hex mesh. The proposed automatic mesh generator also enables interactive mesh density control and generation of meshes with desired and regulated mesh densities. In what follows, first, a detailed explanation of the proposed method is described, followed by two applications for meshing of complex 3-D analysis domains. Conclusions and some remarks for future work in this area are also addressed at the end.

II. Automatic Hex Mesh Generation Using Shape Recognition Technique and Tree Method

A. Flow of Processes

The computation flow of the proposed system is shown in Fig. 1. It can be simply divided into several steps.

Step 1) In this method, the input data are only the shape of the model and the radii of bubbles, which are used to control the mesh density inside the entire analysis domain later on. The shape of the model can be made by CAD software using DXF file format, and the

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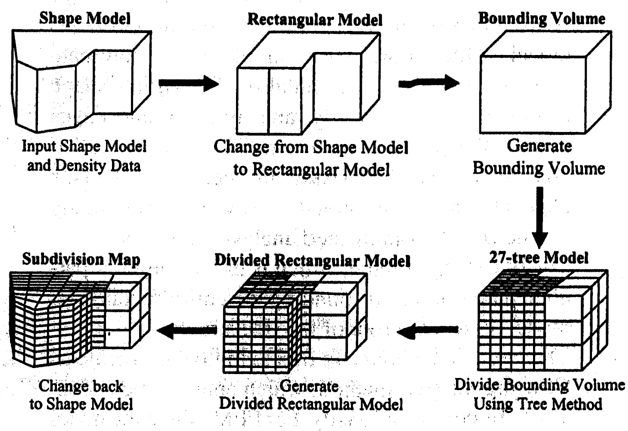

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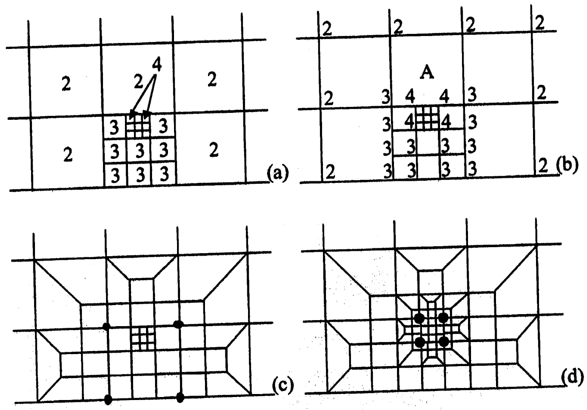


Fig. 3. Templates are applied to a 2-D model by using nine-tree method.

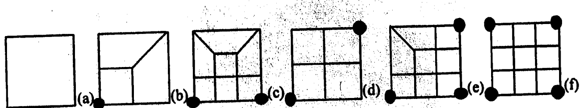


Fig. 4. 2-D templates.

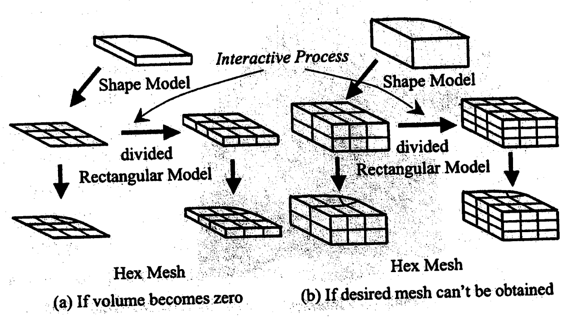


Fig. 5. Interactive mesh control.

- *Control the Mesh Density:* Users can generate meshes with desired mesh density.
- *Dropping the Total Number of Elements:* Users can avoid unwanted increase in the number of finite elements such as inside thin areas.

With the proposed generation method, one can modify the shape of a Hex mesh and the number of generated elements by transforming the shape of the divided rectangular model. For example, when the shape model contains a very thin area as shown in Fig. 5(a), there are some cases when the divided rectangular model, which is generated from shape model, no longer has any volume. At that time, user can modify the "divided rectangular model" interactivity. Thus, user can act interactively to "push up" the upper face of the model. On the other hand, when user wants to develop a mesh that is divided 3 × 3 × 3, as shown in Fig. 5(b), and the "divided rectangular model" does not correspond to the desired division map, the user can agaiin modify the mesh interactively in order to get the desired mesh.

The above interactive process can be performed easily by editing faces or edges of the "divided rectangular model" using the mouse or keyboard directly.

IV. Applications

A. Mesh Quality

The mesh quality is very important factor in order to obtain accurate results of the finite element analysis. Therefore, it is extremely important to evaluate element quality of the generated finite elements as well. In this paper, we evaluate the quality of generated Hex meshes in terms of topological and geometrical irregularity.

When the inside node of the Hex mesh connects six neighboring nodes, the topology of the mesh is the best. For estimation of the topological irregularity, we use the following expression:

$$\varepsilon_t = \frac{1}{n}\sum_{i=1}^{n}|\delta_i - 6| \tag{1}$$

where n represents the total number of inside nodes, and δ_i represents the number of nodes that are connected with a single inside node i. If the mesh topology is nearly good, ε_t is nearly zero. However, when the mesh topology is bad, ε_t is bigger than zero.

When the Hex element is the regular square, the geometry of the mesh is the best. For estimation of the geometrical irregularity, we use the following expression

$$\varepsilon_g = \frac{1}{m}\sum_{i=1}^{m} g_i \tag{2}$$

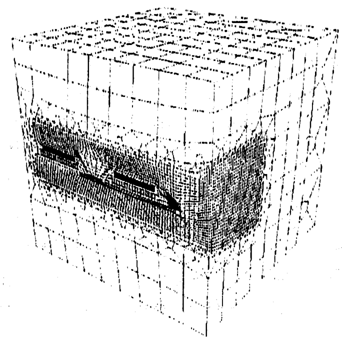
where m represents the total number of inside elements, and g_i represents the geometrical irregularity of element i, which is calculated using the following expression:

$$g_i = 1 - \frac{1}{6}\sum_{j=1}^{6}\frac{l_{ij,\max}}{l_{ij,\min}} \tag{3}$$

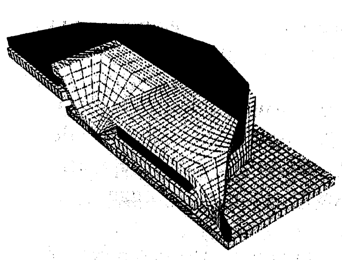
where $l_{ij,\min}$ and $l_{ij,\max}$ represent the minimum and the maximum length of the edge, which consists of facet j of element i, respectively. If the element has a good geometrical shape that is near square, ε_g is near zero, and on the other hand, if ε_g is bigger than zero, the shape of the elements are far from regular square.

B. Applications

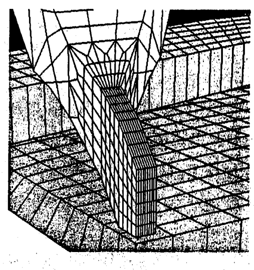
The first application model is the model of a thin-film writing head, which records and plays back thin film shown in Fig. 6. Fig. 6(a) shows the whole generated subdivision map, Fig. 6(b) shows the bodies of the thin-film writing head, and Fig. 6(c) shows the point of the thin-film writing head. In general, it is extremely difficult to develop suitable mesh for the thin-film writing head. However, by utilizing our proposed method, a good Hex mesh was developed. The reasons are twofold: 1) By using the shape recognition technique, a very good shape model was generated, and 2) using the tree method provides a suitable method for quality meshing of such complex domains. We can see that the point area of the thin-film writing head is perfectly divided into a suitably dense and high-quality mesh.



(a) Whole Area



(b) Body



(c) Point

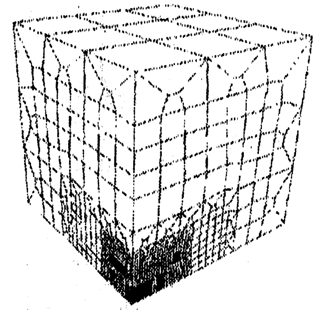
Fig. 6. thin-film writing head.

The second model is the test model of the JIEE and is constructed of a ferrite, a coil, and an aluminum plate, as shown in Fig. 7. Fig. 7(a) shows the whole generated subdivision map, and Fig. 7(b) shows the bodies of the analysis domain. In this model, the eddy current streams though the surface of the aluminum plate because of the induced magnetic flux. In such a case, we must develop dense mesh along the aluminum plate surface. Because the automatic division routine was not able to do that [see Fig. 7(c)], we applied the interactive mesh control module to this model and obtained the desired shape and density of the finite element mesh [see Fig. 7(b)].

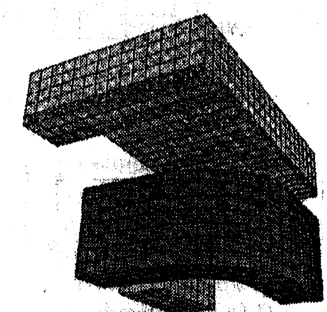
In Table I, the number of nodes and finite elements, coefficients $\varepsilon_t, \varepsilon_g$, and the computation time for the generation of finite element meshes using a Pentium III 800 MHz workstation is presented. As can be seen, using the proposed mesh generator, we could confirm that elements with good quality are generated for short computation time.

V. Conclusion

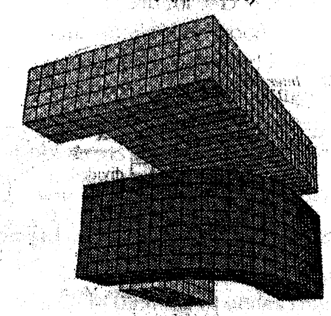
In this paper, a new method for automatic Hex mesh generation using the shape recognition technique and the tree method is presented. The main features of the proposed method are the small input data, easy control of mesh density, and the generation of finite element with good shapes. The method also enables interactive mesh control. In future work, we would like to develop an adaptive routine for Hex mesh generation based on the herein proposed method.



(a) Whole Area



(b) Body (Interactive)



(c) Body (No Interactive)

Fig. 7. Test model of JIEE.

TABLE I
Generated Finite-Element Mesh Data

Model	*Thin Film Writing Head*	*Test Model of JIEE*
Nodes	72,264	10,930
Elements	70,328	10,047
ε_t	0.368	0.492
ε_g	0.374	0.252
Time(s)	422.34	21.23

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