Generalized Functions Online Workshop

May 12th, 2025

Book of Abstracts

Anabela S. Silva, 9:45 – 10:05

Development of existence and uniqueness theorems in Fractional Calculus based on the classical theory

In this talk, we will look at a fractional initial value problem from the point of view of the existence and uniqueness of solutions, based on the classical theory of integer derivatives. We will start with a historical contextualisation of a well-known Nagumo theorem and show its evolution into fractional calculus. We will then construct a Nagumo type theorem for a fractional initial value problem using Hilfer's derivative.

Olena Atlasiuk, 10:05 – 10:25

On continuity in a parameter of solutions to generic boundary-value problems in Sobolev spaces

We consider the most general class of linear inhomogeneous boundary-value problems for systems of ordinary differential equations of arbitrary order whose solutions and right-hand sides belong to appropriate Sobolev spaces. For the parameter-dependent problems from this class, we found necessary and sufficient conditions for their solutions to be continuous in the Sobolev space with respect to the parameter. We also prove a two-sided estimate for the degree of convergence of these solutions to the solution of the nonperturbed problem. Necessary and sufficient conditions are found that guarantee strong and uniform convergence of operators. Moreover, a theorem on the unique representation of an arbitrary linear continuous operator in a Sobolev space is proved and the norm of this operator is estimated from above.

Danijela Milenković, 10:25 – 10:45

Moment-based approach for determining probabilistic properties of generalized stochastic processes via chaos expansions

The polynomial chaos expansion method has gained prominence for solving stochastic differential equations in recent years. Representing solutions as stochastic processes via orthogonal series expansions raises key questions: how to compute fundamental probabilistic properties – such as expectation, variance, covariance, and finite-dimensional distributions (via cumulative distribution functions or probability density functions) – and how to model input parameters (e.g., coefficients, driving forces, initial conditions) with prescribed probability distributions in the framework of chaos expansion. Here, we observe both issues. Building on the Wiener-Itô chaos expansion, based on Hermite polynomials, we derive explicit formulas for calculating the cumulative distribution function and probability density function from the chaos expansion coefficients. Since moments and moment determinacy play a critical role in these expressions, we also provide a formula for evaluating nth order moments from the chaos expansion coefficients. Conversely, we propose an algorithm to determine the Wiener- Itô chaos expansion for a random variable with a known distribution, utilizing a generalized Legendre polynomial expansion. Both approaches are illustrated with simple yet instructive examples, demonstrating the applicability of our findings.

Ani Tumanyan, 10:45 – 11:05

Fredholm Solvability for Special Classes of Hypoelliptic Operators

We study the normal solvability and Fredholm properties of special classes of hypoelliptic operators in multianisotropic Sobolev spaces. We establish normal solvability and a priori estimates for hypoelliptic operators with special variable coefficients, associated with multi-quasi-

elliptic symbols, acting in weighted Sobolev spaces. Fredholm criteria are derived for specific classes of regular hypoelliptic operators in various scales of multianisotropic spaces. These results are applied to the analysis of solution smoothness, index invariance on the scales, and the spectral properties of such operators.

Panel discussion: "Navigating Funding Pathways: Opportunities and Insights for Early-Career Researchers"

11:15 – 12:15

This panel discussion will explore funding opportunities available to early-career researchers, with a special focus on female mathematicians. We will highlight European and international funding programs, including those aimed at promoting gender equity in science. After a short introduction, the discussion will shift toward practical strategies for writing successful research proposals and avoiding common pitfalls. Panel participants — including speakers and attendees — are warmly invited to share their experiences, ask questions, and contribute to a lively and supportive exchange of ideas.

The panel will be led by Wil Schilders.

Wil Schilders worked in industry for 30 years, before entering academia. He has led several European organisations related to mathematics for industry and is currently the president of the worldwide organization ICIAM. Both in industry and in academia he has initiated and led more than 20 projects, on the national (Dutch) and European level. At Eindhoven University of Technology, he set up a Project Development Office specifically aimed at mathematicians and computer scientists.

Suzana Aleksić, 13:45 – 14:05

Some properties of finitely generated shift-invariant spaces

In this study, we investigate the properties of finitely generated shift-invariant (FGSI) subspaces of Sobolev spaces, with a particular focus on their behavior under operations such as convolution and product. These subspaces are defined as the closure of the span of integer translations of a finite set of generators from a Sobolev space. We give a structural characterization of the dual spaces of FGSI spaces. We also demonstrate that certain types of equations with right-hand sides in FGSI spaces admit solutions within some FGSI space. Additionally, we present a characterization of the convolutors of FGSI spaces and determine the wave front sets associated with convolutions and products within these spaces.

This is joint work with Stevan Pilipović and Aleksandar Aksentijević.

Astrit Ferizi, 14:05 – 14:25

Short-time coupled fractional Fourier transform and asymptotic behavior of generalized functions

We investigate the continuity of both the short-time coupled fractional Fourier transform (STCFRFT) and its synthesis operator on the appropriate space of Schwartz test functions. Using the obtained continuity results, we follow the duality approach to develop a distributional framework for the STCFRFT on the space of tempered distributions $S'(\mathbb{R}^2)$. Furthermore, we prove the consistency of this definition with the STCFRFT defined via direct evaluation of a distribution on its kernel. The desingularization formula and a characterization of boundedness in $S'(\mathbb{R}^2)$ are also provided. Additionally, in the last section, we present several Abelian- and Tauberian-type results that fully characterize the quasiasymptotic behavior of tempered distributions at the origin in terms of the asymptotic behavior of their STCFRFT.

The talk is based on a joint work with Katerina Hadzi-Velkova Saneva.

Jasmina Veta Buralieva, 14:25 – 14:45

Abelian- and Tauberian-type results for the generalized Stockwell and wavelet transforms

Tauberian-type results characterizing the quasiasymptotic behavior of polynomial multiplication of Lizorkin distributions in terms of their Stockwell transforms are obtained. Some Abelian-type results relating the quasiasymptotic behavior and quasiasymptotic boundedness of Lizorkin distributions to the asymptotic behavior of their Stockwell transforms are given. Several Abelian-type results for the generalized wavelet transform are also presented.

Bakhyt Alipova, 14:45 – 15:05

Dynamics of thermoelastic half-plane using generalized functions

This study develops a mathematical model to analyze the dynamic behavior of a thermoelastic half-plane subjected to periodic mechanical and thermal loads at its boundary. The problem is formulated as a boundary value problem, incorporating initial and boundary conditions that describe the displacement of the material and its temperature distribution. Surface forces and heat fluxes are introduced as generalized functions to accurately represent localized effects at the boundary.

A key aspect of the approach is the construction of Green's tensor using Fourier transformation, ensuring the correct representation of wave propagation and radiation effects. The study examines the distribution of both longitudinal and transverse waves within the medium, providing insights into their interactions. The final solution is obtained through functional convolution, offering a comprehensive understanding of the system's response to periodic external influences. These findings contribute to the broader study of wave mechanics in thermoelastic materials and have potential applications in engineering, material science, and geophysics.

Sarita Singh, 15:25 – 15:45

Role of PDE in Computational Biology

PDEs are a powerful and versatile tool for modeling and simulating biological systems in computational biology. They allow researchers to study the dynamics, interactions, and behavior of biological systems at various scales, from the cellular level to the organismal level. While there are challenges associated with using PDEs, the benefits of their accuracy and flexibility make them an essential tool for understanding complex biological processes.

Ashish Pathak, 15:45 – 16:05

Characterizing wavefront sets of the solutions to the time-dependent Schrödinger equations using the Stockwell transform

In this paper, we characterize the usual and Sobolev wavefront sets and propose a novel representation of the Schrödinger equation with sub-quadratic perturbation using the Stockwell transform. Additionally, we discuss its application in determining the wavefront set of solutions to the Schrödinger equation with sub-quadratic perturbation.

Hazal Yüksekkaya, 16:05 – 16:25

Mathematical behaviors of solutions for the delayed wave equation

Time delay effects arise in many applications and practical problems such as physical, chemical, biological, thermal and economic phenomena. In this paper, we study the delayed wave equation. We prove the mathematical behaviors results for the wave equation.