International journal for science, technics and innovations for the industry

MACHINES TECHNOLOGIES MATERIALS

YEAR XIX Issue 4 / 2025 ISSN PRINT 1313-0226
ISSN WEB 1314-507X

Published by
Scientific technical
Union of Mechanical Engineering

MACHINES. TECHNOLOGIES. MATERIALS

INTERNATIONAL SCIENTIFIC JOURNAL

ISSN PRINT 1313-0226, ISSN WEB 1314-507X, YEAR XIX, ISSUE 4 / 2025

All articles are published after peer review by two independent reviewers.

Scientific and technical union of mechanical engineering
108 R. S. Rakovski str., Sofia, Bulgaria
www.stumejournals.com, office@stumejournals.com

C O N T E N T S

MACHINES

TECHNOLOGIES

MATERIALS

Changes in the mass moment of inertia of the planetary gear mechanism reduced to the axis of the sun gear depending on the ratio between the radii of the sun and planet gears

Sasko Milev[1]*, Blagoja Nestorovski[2], Darko Tasevski[2], Zoran Dimitrovski[1]
Faculty of Mechanical Engineering, Goce Delcev University, Stip, North Macedonia, Krste Misirkov,10A, 2000, Stip[1],
email: sasko.milev@ugd.edu.mk[1*]
Faculty of Mechanical Engineering, SS. Cyril and Methodius University in Skopje, North Macedonia[2]

Abstract: *Direct Planetary gear mechanism normally consists of a single central gear at the center, an internal (ring) gear around the outside, some number of planet gears that go in between and a planet carrier. They are used to reduce or multiply the number of revolutions. For correct selection of the torque of the driving machine, it is necessary to know the moment of inertia of the planetary gear mechanism, which directly depends on the mass moment of inertia of the planetary gear mechanism and the angular acceleration. In this paper is analyzed the impact of the ratio of the radii of the central and planet gears on the mass moment of inertia of the planetary gear mechanism reduced to the axis of the central gear at a constant diameter of the internal gear. The changes in the reduced mass moment of inertia depending on the number of planet gears was also analyzed.*
Keywords*: PLANETARY MECHANISM, REDUCED MASS MOMENT OF INERTIA, PLANET CARRIER, SUN GEAR, PLANET GEARS*

1. Introduction

Planetary gears can provide high transmission accuracy and support high torque transmission. They belong to the group of epicyclic mechanisms and are most often used for torque transmission and speed transformation. They can be used as reducers, when the angular velocity of the output shaft is lower than the angular velocity of the input shaft through which energy is supplied to the system, or they can be used as multipliers, when the angular velocity of the input shaft is lower than the output shaft angular velocity.

Planetary gears normally consist of; a single sun gear at the center (a), an internal (ring) gear around the outside (b), some number of planets (c) that go in between and a planet carrier H. Planets are the same size, at a common center distance from the center of the planetary gear, and held by a planetary carrier. During the operation of the planetary train, the input power drives the sun gear to rotate, and further drives the surrounding planetary gears which are housed in a rotating planetary carrier. Then the output shaft connected with the planetary carrier connects rotates to output the torque and rotating speed.

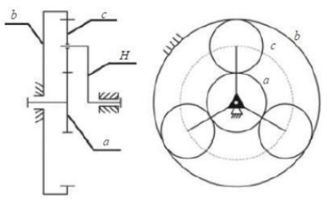

Fig.1.Planetary gear: (a) sun gear, (b) internal (ring) gear, (c) planets, H (planet carrier)

Number of planet gears can be different and they are typically spaced at regular intervals around the sun. More planet gears allow better force balance and lower forces by more load sharing. However, number of the additional planet gears do not change the kinematic but dynamic characteristics of the mechanism.

During the operation of planetary mechanisms, there are three phases in their movement: the start-up phase, the stationary mode of operation and the stopping phase. During the first phase, the speed of the driving member increases from zero to maximum and the work of the driving forces is greater than the work of the resistance forces. In the stationary mode, the work of the driving forces for a full cycle is equal in absolute value to the work of the resistance forces. During the stopping phase, the speed of the driving member decreases to zero, and the work of the resistance forces is greater than the work of the driving forces. (The resistance forces are understood as all the forces that oppose the movement of the mechanism).

It is necessary the drive unit to enable uninterrupted operation of the mechanism in all three phases, i.e. to provide a sufficiently large input torque that will be transmitted to the drive shaft of the planetary gear and to provide appropriate angular velocity of the input shaft at which the material moment of inertia is reduced.

The general form of the Lagrange equation of motion

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{q}_i}\right) - \frac{\partial E_k}{\partial \dot{q}_i} + \frac{\partial E_p}{\partial \dot{q}_i} = Q_i$$

applied to a planetary gear train is ;

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\varphi}}\right) - \frac{\partial E_k}{\partial \varphi} + \frac{\partial E_p}{\partial \varphi} = M_i \tag{1}$$

where E_k is kinetic energy of the mechanism, φ is angle of rotation (generalized coordinate), $\dot{\varphi}$ is angular velocity (generalized velocity), M_i is generalized moment. The member $\frac{\partial E_p}{\partial \varphi}$ takes over the action in the mechanism of the forces from the weight and the elasticity of the members. These forces can be neglected since their influence is very small and the elasticity of the members of the planetary mechanism can be avoided by increasing their stiffness [1]. For efficient use of equation (1), we reduce the moments Mi of all members of the planetary mechanism to the input shaft and replace the total action of the thus reduced moments with a single member M* for which we further use the term reduced moment of inertia. The work of the reduced moments with respect to the possible displacements is equal to the total work of all forces acting on the mechanism with respect to the possible displacements, so equation (1) takes the form [2-3]

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\varphi}}\right) - \frac{\partial E_k}{\partial \varphi} = M^* \tag{2}$$

We replace the mass material moments of inertia of all the components of the planetary mechanism with a single material moment of inertia I* reduced to the input shaft. I* is called the reduced mass material moment of inertia.

$$J^* = \sum_{i=1}^{n}\left[J_{Si}\left(\frac{\omega_i}{\omega}\right)^2 + m_i\left(\frac{v_{Si}}{\omega}\right)^2\right] \tag{3}$$

The following equations apply to planetary mechanisms:

$$E_K = \frac{J^* * \dot{\varphi}^2}{2} \tag{4}$$

$$\frac{\partial E_k}{\partial \dot{\varphi}} = J^* * \dot{\varphi} \tag{5}$$

$$\frac{\partial E_k}{\partial \dot{\varphi}} = \frac{1}{2}\frac{dJ^*}{\partial \varphi} * \dot{\varphi}^2 \quad (6)$$

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{q}_1}\right)^{\cdot\cdot} = J^* * \ddot{\varphi} + \frac{dJ^*}{\partial \varphi} * \dot{\varphi}^2 \quad (7)$$

By substituting (4), (5), (6) and (7) into (2) we obtain the following nonlinear differential equation (8) :

$$J^* * \ddot{\varphi} + \frac{1}{2}\frac{dJ^*}{\partial \varphi} * \dot{\varphi}^2 = M^* \quad (8)$$

By solving (8), we determine the value of the actual angular velocity of the input (drive) shaft on which the sun gear is mounted and on which the reduction is performed.

$$\omega_i = \sqrt{\frac{2}{J^*{}_i}\int_{\varphi_0}^{\varphi_i} M^* * d\varphi + \frac{J_0^*}{J_i^*} * \omega_o^2} \quad (9)$$

M*= M*(φ, ω, t) =M_p-M_o, where M_p is moment of motion, M_o is moment of resistance forces. J_0^* is the reduced mass moment at the initial moment.

Considering that in gear mechanisms the transmission ratio does not change during operation, the reduced mass moment of inertia remains unchanged, i.e. $J_0^* = J_i^*$ equitation (9) will be [4] :

$$\omega_i = \sqrt{\frac{2}{J^*{}_i}\int_{\varphi_0}^{\varphi_i} M^* * d\varphi + \omega_o^2} \quad (10)$$

From equitation (10) it is obvious that the angular velocity depends on the reduced mass moment of inertia I*.

2. Mass moment of inertia reduced to the axis of the sun gear

In this paper we will further examine changes in the mass moment of inertia of the planetary gear mechanism reduced to the axis of the sun gear depending on the ratio between the radii of the sun gear and planet gears. For this purpose, we first determine the expression for calculating the mass moment of inertia reduced on the sun gear using the law of kinetic energy [5-6].

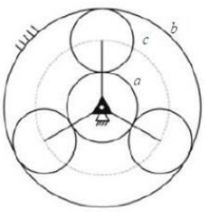

***Fig.2.** Scheme of planetary mechanism*

$$E_K^* = E_{Ka} + NE_{Kc} + E_{KH} \quad (11)$$

E_K^* – kinetic energy of the system

E_{Ka} -sun gear kinetic energy

E_{Kc} - kinetic energy of one planetary gear

E_{KH}- planet carrier kinetic energy

N- number of planetary gears

$$E_K^* = J^* * \frac{\omega_a^2}{2} \quad (12)$$

ω_a [1/s] - angular velocity of the sun gear

$$E_{Ka} = \frac{J_a * \omega_a^2}{2} = \rho_a \pi h R_a^4 \frac{\omega_a^4}{4} \quad (13)$$

$J_a = \frac{m_a * R_a^2}{2}$; $m_a = \rho_a \pi h R_a^2$, where

J_a - mass moment of inertia of sun gear [kg*m^2],

m_a[kg]–sun gear mass, R_a[m]– sun gear radius, h-gear thickness

ρ_a[kg/m^3] - specific density of the material from which the sun gear is made

$$E_{Kc} = \frac{J_c * \omega_c^2}{2} + \frac{m_c * v_{sr}^2}{2} \quad (14)$$

$J_c = \frac{m_c * R_c^2}{2}$; $m_c = \rho_c \pi h R_c^2$, where

J_c -mass moment of inertia of one planet gear [kg*m^2]

R_c-planetary gear radius, v_{sr}- planetary gear center speed

$v_{sr} = (R_a + R_c) * \omega_H$

ω_H [1/s] - angular velocity of the planet carrier

ρ_c[kg/m^3] - specific density of the material from which the planetary gears are made

At the mutual contact points of the sun gear with the planetary gears, the peripheral speed is

$R_a * \omega_a = R_c * \omega_c \rightarrow \omega_c = (R_a \backslash R_c) * \omega_a$

By substituting the previously mentioned equations into equation (13) we obtain

$$E_{Kc} = \rho_c * \pi * h * \frac{R_a^2}{2} * \frac{R_c^2}{2} * \omega_a^2 + \rho_c * \pi * h * \frac{R_c^2}{2} * (R_a + R_c)^2 * \omega_H^2 \quad (15)$$

$$E_{KH} = \frac{J_H * \omega_H{}^2}{2} \quad (16)$$

where

$J_H = m_H * \frac{(R_a + R_c)^2}{2}$, $m_H = \rho_H * \pi * (R_a + R_c)^2 * h$

m_H [kg] - mass of of the carrier H,

J_H - mass moment of inertia of the carrier H [kg*m^2]

ρ_H [kg/m^3] - specific density of the material from which the carrier H is made,

Using above equatations for J_H and m_H equitation (15) will be transformed in

$$E_{KH} = \rho_H * \pi * h * \frac{(R_a + R_c)^4}{2} * \frac{\omega_H^2}{2} \quad (17)$$

According to the Willis Method for epicyclic mechanisms, for the ratio of angular velocities in the planetary mechanism we obtain[7-9]

$$i_{1H} = \frac{\omega_a}{\omega_H} = 1 - \frac{d_b}{d_a} = 1 - \frac{(R_a + 2R_c)}{R_a}$$

$$\frac{\omega_a}{\omega_H} = \frac{-R_a}{2 * R_c} \quad (18)$$

where R_b=R_a+2R_c

By substituting equations (12), (13), (15), (17) and (18) into equation (11), the following expression is obtained:

$$J^* = (\rho_a * \pi * h) * \frac{R_a^4}{2} + N(\rho_c * \pi * h) * \frac{2R_a^2R_c^2 + R_a^2(R_a+R_c)^2}{4} +$$
$$+\rho_H * \pi * h_H * (R_a + R_c)^4 * \frac{R_a^2}{4 * R_c^2} \quad (19)$$

Further, using equation (19), the values for the reduced mass moment of inertia were calculated. It was assumed that all parts of the planetary mechanism are made of same material with a density ρ [kg/m³]. The radius of the internal (ring) gear (b) remains constant during the calculations. Thickness of the carrier h_m, which has shape of a circular plate, is $h_H=h/k$, where h is thickness of gears.

$$J^* = (\rho * \pi * h) * \left(\frac{R_a^4}{2} + \frac{N * (2R_a^2R_c^2 + R_a^2(R_a + R_c)^2)}{4} + \frac{(R_a + R_c)^4 * R_a^2}{4 * k * R_c^2}\right) \quad (20)$$

From equation (20) it is clear that for a given density of the material from which the mechanism is made and a given thickness of the gears, the value of the reduced mass moment depends on the ratio between the radii of the central and planetary gears, on the thickness of the carrier h which is taken to have the shape of a circular plate and on the number of planet gears.

3. Results and discussion

The calculation performed below is for $h_H=h/4$, i.e. the thickness of the planet carrier is 4 times less than the thickness of the gears and reduced mass moment of inertia $J^*_{(N=k)} = J^*/(\rho\pi h)$ for N planet gears. In table 1 are given values for reduced mass moment of inertia depending on the ratio between the radii of the sun gear and planet gears.

Table 1: *Reduced mass moment of inertia of the planetary gear mechanism reduced to the axis of the sun gear depending on the ratio between the radii of the sun and planet gears for 3,4,5,7 and 9 planet gears*

Reduced mass moment of inertia for 3,4,5,7 and 9 planet gears depending on the ratio between the radii of the sun gear and planet gears								
i_{aH}	**Ra**	**Rc**	**Ra/Rc**	**$J^*_{(N=3)}$**	**$J^*_{(N=4)}$**	**$J^*_{(N=5)}$**	**$J^*_{(N=7)}$**	**$J^*_{(N=9)}$**
0,176	0,85	0,075	11,3	6,608	6,764	6,921	7,234	7,547
0,250	0,8	0,1	8,00	3,228	3,360	3,493	3,759	4,024
0,333	0,75	0,125	6,00	1,813	1,925	2,037	2,262	2,486
0,429	0,7	0,15	4,67	1,113	1,207	1,301	1,489	1,677
0,538	0,65	0,175	3,71	0,724	0,802	0,880	1,037	1,194
0,667	0,6	0,2	3,00	0,490	0,554	0,619	0,749	0,878
0,818	0,55	0,225	2,44	0,340	0,393	0,446	0,552	0,658
1,000	**0,5**	**0,25**	**2,00**	**0,239**	**0,282**	**0,325**	**0,411**	**0,497**
1,222	0,45	0,275	1,64	0,170	0,204	0,238	0,307	0,375
1,500	0,4	0,3	1,3	0,120	0,147	0,173	0,227	0,281
1,857	0,35	0,325	1,08	0,084	0,104	0,125	0,166	0,206
2,333	0,3	0,35	0,86	0,057	0,072	0,087	0,117	0,147
3,000	0,25	0,375	0,67	0,038	0,048	0,059	0,080	0,101
4,000	0,2	0,4	0,50	0,023	0,030	0,037	0,050	0,064
5,667	0,15	0,425	0,36	0,013	0,017	0,021	0,028	0,036
9,000	0,1	0,45	0,22	0,006	0,007	0,009	0,013	0,016
19,00	0,05	0,475	0,10	0,001	0,002	0,002	0,003	0,004

In the upper half of Table 1, when the gear ratio is greater than 1, the planetary gear mechanism functions as a multiplier. In this case, the angular velocity of the sun gear is less than the angular velocity of the carrier. For a gear ratio of 1, the radius of the sun gear is twice the radius of the planet gears and in this case the angular velocities of the sun gear and the planet gears are equal. When the gear ratio is greater than 1, the planetary gear mechanism functions as a reducer and in this case the angular velocity of the sun gear is greater than the angular velocity of the carrier.

The dependence of the reduced mass moment of inertia on the ratio between the radii of the sun gear and planetary gears is shown at figure 3.

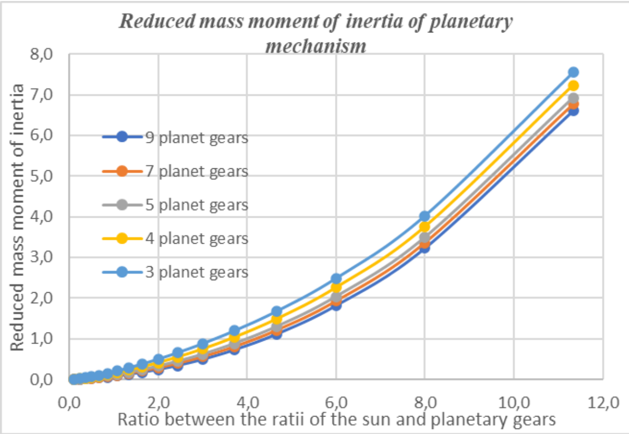

Figure 3: *Reduced mass moment of inertia of planetary mechanism*

When the number of planet gears is 9, the reduced moment of inertia as a function of the ratio of the radii of the sun gear and planet gears is given by the approximate equation

$$y=0.0517x^2-0.0073x+0.0243 \quad (21)$$

Based on the data in the Table 1 and Figure 3 it can be concluded that with an increase in the ratio between the radii of the sun gear and the planet gears, the value of the mass moment of inertia of the planetary mechanism reduced to the sun gear axis also increases. This dependence is described by second order curve - parabola. By increasing the number of planet gears, the coefficient in front of the quadratic element (21) also increases.

In table 2 are represented calculated the percentage increases of the reduced mass moment of inertia when the planetary mechanism has 4, 5, 7 or 9 planet gears compared to the reduced moment when there are only 3 planet gears.

Table 2. *Percentage increases of the reduced mass moment of inertia `when the planetary mechanism has 4, 5, 7 or 9 planet gears*

Increase in the reduced mass moment of inertia with 4, 5, 7 and 9 planetary gears compared to the reduced mass moment of inertia with 3 planetary gears [in percentage]					
IaH ratio	Ra/Rc	J*N=4/ J*N=3	J*N=5/ J*N=3	J*N=7/ J*N=3	J*N=9/ J*N=3
-0,176	11,33	2,37	4,74	9,48	14,22
-0,250	8,000	4,11	8,23	16,46	24,69
-0,333	6,000	6,18	12,36	24,72	37,08
-0,429	4,667	8,45	16,90	33,80	50,70
-0,538	3,714	10,83	21,65	43,31	64,96
-0,667	3,000	13,24	26,47	52,94	79,41
-0,818	2,444	15,62	31,25	62,50	93,75
-1,000	**2,000**	**17,96**	**35,92**	**71,84**	**107,76**
-1,222	1,636	20,21	40,42	80,85	121,27
-1,500	1,333	22,36	44,71	89,42	134,14
-1,857	1,077	24,37	48,73	97,46	146,19
-2,333	0,857	26,21	52,42	104,84	157,26
-3,000	0,667	27,86	55,71	111,43	167,14
-4,000	0,500	29,28	58,56	117,12	175,67
-5,667	0,353	30,45	60,91	121,81	182,72
-9,000	0,222	31,37	62,74	125,47	188,21
-19,00	0,105	32,02	64,05	128,09	192,14

The results of the table 2 are graphically shown on the figure 4.

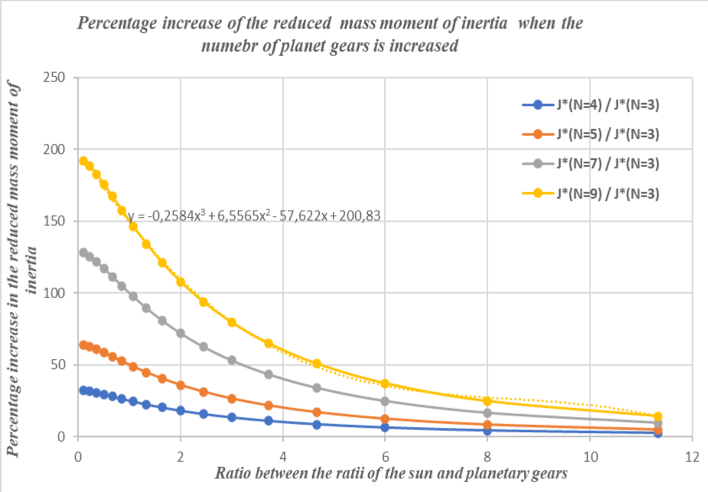

Figure 4: *Percentage increases of the reduced mass moment of inertia `when the planetary mechanism has 4, 5, 7 or 9 planet gears compared with 3 planet gears mechanism*

From the data in Table 2 represented at Figure 4 it can be concluded that at the same ratio of the radii of the sun gear and planetary gears, with an increase in the number of planetary gears, the value of the reduced mass moment of inertia also increases. The percentage increase in the reduced mass moment of inertia when the number of planet gears increases is greatest at the largest planetary mechanism ratios, that is, at the smallest ratios of the radii of the sun gear and planet gear. With an increase in the ratio between the radii of the sun gear and planetary gear, this percentage increase decreases. It follows that the percentage increase is smaller in cases when the planetary mechanism works as a multiplier than when it works as a reducer.

4. Conclusion

By changing the ratio of the radii of the sun gear and planet gears in planetary mechanisms, the mass moment of inertia reduced to the sun gear axis is also changed, and with it the reduced torque M^* is also changed, which directly affects the characteristics of the planetary mechanism. On the other hand, an increase in the reduced mass moment of inertia can be achieved by increasing the number of planetary gears. The number of planetary gears does not affect the kinematic characteristics, but it does affect the moment of inertia.

5. References

[1] J.J.Uicker Jr., G.R.Penock., J.E.Shigley, Oxford University Press (2017) *Theory of Machines and Mechanisms, 5-th edition*

[2] R.Budinas , J.Nissbet., Mc Graw Hill Education (2020) *Shigley's Mechanical Engineering Design*

[3] T.Petrovic, T.Ivanov and M.Milosevic, Forsch Ingenieures (2009), *A new structure of combined gear trains with high transmission ratios*

[4] Y.Chen, Springer China Machine Press (2021) *Automotive Transmissions, Design, Theory and Applications*

[5] K.Arnaudov., D.Karaivanov, CRC Press, Taylor & Francis Group (2018), *Planetary Gear Trains*

[6] D.Stamboliev, Ss.Cyril and Methodius University in Skopje, Macedonia ((1976) *Prenosnici na vozilata*

[7] W.Jiang., Wiley (2019). *Analyses and Design of Mechanical Elements.*

[8] J.Meriam, L.Kraige., J.Bolton, Wiley (2018) *Engineering Mechanics,* Volume **2**, *Dynamics*

[9] S.Simeonov, S.Milev, Goce Delcev University Stip (2019), *Praktikum po Masinski elementi*