KANTIAN ECONOMICS, IMPURE ALTRUISM, BERGE EQUILIBRIUM, ARROW-LIND PRINCIPLE, WITH LINEAR-COST EQUILIBRIUM

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Abstract

In this paper we will review s-ome results in Kantian economics, (im)pure altruism (warm-glow model), Berge equilibrium altruism and social welfare Arrow-Lind principle and a linear-cost-share equilibrium which is a special case of a Lindahl equilibrium. A strategy profile is a Kantian equilibrium if no player would like all players to alter their contributions by the same multiplicative factor. We study Kantian equilibrium here in Laffont,1975 setting of macroeconomic constraints. Simulations show Kantian equilibrium is in the upper part of efficiency curve, while the ethics utility is maximized, meaning that Kantian equilibrium is efficient but with ethics taken into consideration. Also, as in the case of Nash equilibrium individual utility is more maximized than societal. In a setting with money neutrality as macroeconomic constraint, impure altruism equilibrium or warm-glow equilibrium is at the same point as Berge equilibrium and that is the highest point of efficiency utility line. Kantian and Nash equilibrium are on the same point in this setting while modified Lindahl equilibrium (linear-cost share equilibrium) is as efficient as Pareto equilibrium, other equilibriums are all settled on the efficiency line except for the Kantian, Nash that also are on the warm glow utility line. Pareto equilibrium together with Kantian-Nash equilibrium are on warm glow, and ethics utility line. In the second simulation Kantian is same as Pareto and modified Lindahl equilibrium, impure altruism remains the most efficient equilibrium, Berge equilibrium now is the second most efficient. If marginal cost (Arrow-Lind principle) line crosses sum of marginal benefits (Samuelson condition), then the best equilibrium based on welfare is Nash Equilibrium

Keywords: Kantian equilibrium, warm-glow model, Arrow-Lind principle, Linear-Cost equilibrium, ethics in economics

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1. Introduction

This paper is about cooperation in economics, and (im)pure altruism. Cooperation may be the only means of satisfying one's own self-interested preferences, see Roemer (2019). Altruism and cooperation are frequently confounded in literature. Economic theory has focused on the competitive tendencies of economic agents instead of cooperative. Indeed, the two great theoretical contributions of microeconomics are both models of competition: the theory of competitive or Walrasian equilibrium, and game theory, with its associated stability concept, Nash equilibrium. But there is growing attention to the claim that humans are a cooperative species, see Bowles,S. and Gintis,H.(2011), and Henrich, N., and J. Henrich.(2007). The basic idea behind Kantian equilibrium is that in a cooperative situation everyone asks: 'What would be best for me if everyone were to do it?' When everyone answers in the same way, then that is what everyone does, see Sher,I.(2020). In Nash equilibrium, one chooses one's own strategy to maximize one's own utility holding others' strategies fixed at the equilibrium. In contrast, in Kantian equilibrium, one chooses the common strategy to be adopted by everyone to maximize one's own utility.Impure altruism or Warm-glow giving is an economic theory describing the emotional reward of giving to others. Such warm-glow giving has been acknowledged in the literature, see Arrow (1975), Sen (1977), Sudgen (1982),Sudgen (1984),Hartmann et al.(2017).

2. Kantian equilibria and efficiency (due to Roemer (2010), Roemer (2019))

Under classical behavior, at least in large economies, individuals ignore the externalities induced by their choices. This behavior can be called autarkic and can be contrasted with behavior that can be called interdependent. The equilibrium associated with this behavior is caled Nash equilibrium.[4] Formally there is a set of n agents with payoff function $V_i\colon \mathbb{R}^n_+ \to \mathbb{R}$,We define effort also as: $\mathcal{L}^{-1} = (\mathcal{L}^1, \dots \mathcal{L}^{1-1}, \mathcal{L}^{i+1}, \mathcal{L}^n)$, and payoff function V_i is strictly monotone and decreasing in $\mathcal{L}^{-1}$ $\forall i$. This set-up describes the fisher's problem, where $V_i(\mathcal{L})$ is the utility of fisher i if the vector of labors expended by all fishers is $\mathcal{L}$.The production function is strictly concave in labor. We can also represent this problem by a collection of functions $\{V_i \mid i = 1, \dots, n\}$ defined on the domain $\mathbb{R}^n_+$;

Definition *1*: A vector of strategies $\boldsymbol{\mathcal{L}} = (\boldsymbol{\mathcal{L}}^1, \dots, \boldsymbol{\mathcal{L}}^n)$ is a multiplicative Kantian equilibrium of the game $\boldsymbol{G} = \boldsymbol{S}(\boldsymbol{V}^1, \dots, \boldsymbol{V}^n)$ for $\forall \boldsymbol{i} = \boldsymbol{1}, \dots, \boldsymbol{n}$

[4] For example a classic tragedy of commons example**:** A set of fishers must expend labor on a lake to catch fish, but there is a congestion problem, so the fish caught per unit of time decreases with the number of total hours expended in fishing by the community. Each fisher has a utility function over fish caught and labor expended.In the Nash equilibrium of the game where each fisher's strategy is a labor choice, there is over-fishing: the equilibrium is Pareto inefficient, and everyone would gain in welfare from a small decrease in labor expenditures, Some kind of cooperation is necessary to solve the problem.See Roemer (2010)

$$\arg_{\alpha\in\mathbb{R}_+} \max V^i(\alpha\mathcal{L}) = 1 \quad (1)$$

Proposition 1 : Kant's categorical imperative: one should take those actions and only those actions that one would advocate all others take as well. Thus, one should expand one's labor by a factor $\boldsymbol{\alpha}$ if and only if one would have all others expand theirs by the same factor. Kant's categorical imperative is a cooperative norm. The contrast is with the non-cooperative concept of Nash equilibrium, where the counterfactual envisaged by the individual is that one changes one's labor while the labor of all others remains fixed.

Proposition 2 : $\boldsymbol{\mathcal{L}} = (\boldsymbol{\mathcal{L}^1},..,\boldsymbol{\mathcal{L}^n}) \in \boldsymbol{S^n}$ is a multiplicative Kantian equilibrium of the game $\boldsymbol{G} = \boldsymbol{S}(\boldsymbol{V^1},\dots,\boldsymbol{V^n})$ if :

$$(\forall i = 1,..,n)(\forall\alpha \in \mathbb{R}_+)(V^i(\mathcal{L}) \geqq V^i(\alpha\mathcal{L}) \quad (2)$$

In previous S is a common strategy space.

Definition 2:$\boldsymbol{\mathcal{L}}$ is $\boldsymbol{G}$ -efficient if $\nexists\boldsymbol{\mathcal{L}'} \in \boldsymbol{S^n}$ that Pareto dominates $\boldsymbol{\mathcal{L}} \in \boldsymbol{G}$.

Definition 3: Some game $\boldsymbol{G} = \boldsymbol{S}(\boldsymbol{V^1},\dots,\boldsymbol{V^n})$ is monotone increasing (decreasing) if $(\forall\boldsymbol{i} = \mathbf{1},..,\boldsymbol{n})(\mathbf{V^i}(\cdot)$ is strictly increasing (decreasing) in $\boldsymbol{\mathcal{L}^{-i}}$.

Theorem 1: Suppose that $\boldsymbol{G} = \boldsymbol{S}(\boldsymbol{V^1},\dots,\boldsymbol{V^n})$ is monotone increasing (decreasing).And let $\boldsymbol{\mathcal{L}^*}$ be Kantian equilibrium of $\boldsymbol{G}$ with $\boldsymbol{\mathcal{L}^i} > \mathbf{0}, \forall\boldsymbol{i} = \mathbf{1},..,\boldsymbol{n}$,then $\boldsymbol{\mathcal{L}^*}$ is $\boldsymbol{G}$ efficient.

Proof: Now, let V^i be monotone increasing. Suppose now that $\mathcal{L}^*$ is Kantian but is not G -efficient and is Pareto dominated by allocation $\hat{\mathcal{L}}$ then:

$$r = \max_i \frac{\hat{\mathcal{L}}_i}{\mathcal{L}_i^*} \quad (3)$$

$\exists j, r > \frac{\hat{\mathcal{L}}_j}{\mathcal{L}_j^*}$; for if not then $\mathcal{L}^*$ is not Kantian equilibrium, because all agents would weakly prefer to change to $r\,\mathcal{L}^*$ and some would prefer the change ,and let i^* be an agent for whom $r\mathcal{L}_i^* = \hat{\mathcal{L}}_i$ .So now $r \neq 1$ or else agent i^* would be worse of at $\hat{\mathcal{L}}_i$ than $\mathcal{L}_i^*$ by V^i as monotone increasing. Now by vector $\mathcal{L} = r\mathcal{L}^*$ and we have:

$$V^{i*}(r\mathcal{L}^{i*}) > V^{i*}(\hat{\mathcal{L}}) \geq V^{i*}(\mathcal{L}^{i*}) \quad (4)$$

The first inequality follows from the fact that in $r\,\mathcal{L}^*, i^*$ expends the same labor as the agent does in $\hat{\mathcal{L}}$, while some other agents expend strictly more labor, and none expends less labor than in $\hat{\mathcal{L}}$; the second inequality follows by Pareto domination. Previous This contradicts the assumption that $\alpha^{i*}(\mathcal{L}) = 1$, which proves the claim. About the effort similar argument applies for $V^i(\cdot;e)$ and is monotone decreasing, $r = \min \frac{\hat{\mathcal{L}}_i}{\mathcal{L}_i^*}$ ∎ . So If the game G is monotone increasing (decreasing), then there are positive (negative) externalities to individual effort. In Nash equilibrium, these are not taken into account by individuals, and so, unsurprisingly, equilibriums are often inefficient. By this theorem Roemer (2010) names Kantian equilibrium to be cooperative solution.

Theorem 2 : Now let $\boldsymbol{V^i}|\boldsymbol{i}=\boldsymbol{1},..,\boldsymbol{n}$ be concave real valued payoff functions defined in $\mathbb{R}^{\boldsymbol{n}}_{+}$ and $\forall\boldsymbol{\mathcal{L}}\in\mathbb{R}^{\boldsymbol{n}}_{++}$ it is defined $\boldsymbol{\alpha_i}(\boldsymbol{\mathcal{L}})=\{\boldsymbol{a}|\boldsymbol{argmax_\alpha V^i}(\boldsymbol{\alpha\mathcal{L}})\}$ and $\exists\boldsymbol{b}\in\mathbb{R}^{\boldsymbol{n}}_{++},\exists\boldsymbol{B}\in\mathbb{R}^{\boldsymbol{n}}_{++}$ so that $\boldsymbol{b}\leqq\boldsymbol{\mathcal{L}}\leqq\boldsymbol{B}\Rightarrow(\forall\boldsymbol{i}=\boldsymbol{1},..,\boldsymbol{n})(\boldsymbol{b}\leqq\boldsymbol{\alpha_i}(\boldsymbol{\mathcal{L}})\boldsymbol{\mathcal{L}^i}\leqq\boldsymbol{B})$. Then there exists a strictly positive Kantian equilibrium for the game $\boldsymbol{V^i}|\boldsymbol{i}=\boldsymbol{1},..,\boldsymbol{n}$.

Proof: $\alpha_i(\mathcal{L})$ is convex set $\forall i,\mathcal{L}$, and by the mapping $\mathbb{R}^n_{++}:\Theta(\mathcal{L}^1,..,\mathcal{L}^n)=(\alpha_1(\mathcal{L})\mathcal{L}^1,..,\alpha_n(\mathcal{L})\mathcal{L}^n)$ and by $\exists b\in\mathbb{R}^n_{++},\exists B\in\mathbb{R}^n_{++}$ Θ maps the rectangle $R=\{b\leqq\mathcal{L}\leqq B\}$,Θ is upper-hemicontinuous[5], and by Kakutani fixed point theorem fixed point of Θ exists in R and at fixed point $\mathcal{L}^*$ we have $(\forall i=1,..,n)(1\in\alpha_i(\mathcal{L}^*)$ so $\mathcal{L}^*$ is Kantian equilibrium ■.

3. Macroeconomics constraints, efficiency, ethics (due to Laffont, 1975)

This model assumes that as in Akerlof (2002),by near-rational rules of thumb: losses to individuals (individual firms) are small (second-order),but the effect on economy is large. Here rules of thumb involve money illusion. For example if money supply increases by fraction $\varepsilon=0.05$,losses are approximately $\varepsilon^2=0.0025$,agents adjust prices slowly, but a change n money supply alters real balances $\frac{M}{P}$ by a first rder amount causing firs-order changes in output and employment. In Laffont (1975) paper it is introduced a concept of a new ethics, we postulate that a typical agent assumes (according to Kant's moral) that the other agents will act as he does and he maximizes his utility function under this new constraint. Instead, money illusion this paper uses money neutrality. In this model all agents have the same differentiable, increasing, concave utility function. Measurable space of agents is: $A=[0,1]$ endowed with Lebesque measure μ .There are two commodities in this economy with initial endowments $(1,1)$.The two goods can be transformed $X\rightarrow Y$ by $y\leqq\alpha x$. The $p(Y)=1/\alpha$. The optimization problem is :

$$\begin{gathered} maxU(x,y) \\ s.t.(x+\left(\frac{1}{\alpha}\right)y=1+\left(\frac{1}{\alpha}\right)1 \end{gathered} \tag{5}$$

Where $\frac{U_2}{U_1}=\frac{1}{\alpha}$.The only macroeconomic constraints in this model are additive resource constraints : $\int_A xd\mu=1$; $\int_A yd\mu=1$. In this framework it is fruitless in terms of economic efficiency for consumers to go beyond their selfish behavior, since equilibrium price is $p=\frac{1}{\alpha}$. If agents act Kantian, they will only realize

[5] $\boldsymbol{f}:\boldsymbol{X}\rightarrow\boldsymbol{Y}$ $\boldsymbol{f}$ is upper hemicontinuous if for for every open set $\boldsymbol{U}\in\boldsymbol{Y}$ the set $\{\boldsymbol{x}\in\boldsymbol{X}:\boldsymbol{f}(\boldsymbol{x})\in\boldsymbol{U}\}$ is upper semicontinuous, meaning that its closure is contained in the preimage of $\boldsymbol{U}$ i.e.: $\overline{\boldsymbol{x}\in\boldsymbol{X}:\boldsymbol{f}(\boldsymbol{x})\in\boldsymbol{U}}\subseteq\boldsymbol{f^{-1}}(\boldsymbol{U})$. This condition essentially means that the preimage of any open set in the codomain$\overline{\boldsymbol{Y}}$ has a "nice" behavior with respect to the topology of the domain $\overline{\boldsymbol{X}}$

equilibrium without tâtonnement process. Now this model is modified by externality of consumption. This is done by adding another argument to the utility function: $\int_A yd\mu 1$.So, now the optimization problem is given as: $maxU\left(x, y, \int_A yd\mu\right)$,and $s.t.(x+\left(\frac{1}{\alpha}\right)y = 1+\left(\frac{1}{\alpha}\right)1$so now FOC is : $\frac{U_2}{U_1} = \frac{1}{\alpha}$. But here marginal conditions of the Pareto optimum with equal distribution are:$\frac{U_2}{U_1} = \frac{1}{\alpha} - \frac{U_3}{U_1}$.Since when agent he choose quantity y from Y it is assumed that: $\int_A yd\mu = y$ .Maximization problem here is:max $U(1+\frac{1}{\alpha}-\left(\frac{1}{\alpha}\right)y, y, y)$.Marginal conditions again are: $\frac{U_2}{U_1} = \frac{1}{\alpha} - \frac{U_3}{U_1}$. This Kantian behavior realizes the optimum that would be attained with selfish behavior only if appropriate taxes $\tau = -\frac{U_3}{U_1}$ were imposed on the consumption of good Y.This model then is transformed in OLG setting with identical agents that live in two periods: young, and old, while the total population is constant. In the first period consumer bears no uncertainty but there are S states of nature in second period. Now let $w_s > 0$ be the endowment of good 0 in period 1,and $s = 1, \ldots, S$.Now, $x \in \mathbb{R}^L$ is a consumption vector for a consumer in period 1. And probability of being in any state is Π_s .In period 1 consumer buys insurance at price q_s against states $s = 1, \ldots S$, now z_s is the amount of claims against the state s bought (if positive) or sold short (if negative) by a representative consumer.Technology is $l = 1, \ldots, L$ with price p_l.Now te maximization problem is:

$$\begin{gathered} \max \textstyle\sum_{s=1}^{S} \Pi_s(x_0)U(x_0, x_s) \\ s.t. px_0 \leqq w_0 - \textstyle\sum_{s=1}^{S} q_s z_s; px_s \leqq w_s + z_s, s = 1, \ldots S \\ x_s \geqq 0, s = 0,1,..,S \end{gathered} \tag{6}$$

Model assumes insurance company with zero profit constraint, made possible by having the number of consumers go to infinity so that frequencies in all states of nature converge to probabilities. The price of insurance at inefficient stationary equilibrium will be :$q_s = \Pi_s(x_0)$.This is variable insurance premium, and is strictly equivalent in this model to a form of Kantian behavior. With the neutrality of money now in first period consumers receive wage w and transfers t in first period of their life. They can consume c_1 or hold money m with a real return r_m , or to hold an asset a with a return r which is stochastic. In the second period they consume c_2, their receipts from money $r_m m$ and from their asset holdings ra.Maximization problem now is:

$$\begin{gathered} \max U(c_1) + EU(c_2) \\ s.t. c_1 = w + t - a - m \\ c_2 = r_m m + ra \end{gathered} \tag{7}$$

M is nominal per capita money, r is real money per capita. Now, by definition: $r_m - 1 = -\frac{\Delta P}{P} = \frac{\Delta m}{m} - \frac{\Delta M}{M}$,and when $\frac{\Delta m}{m} = 0$ implies that $\frac{\Delta M}{P} = m(1-r_m)$.There is a macroeconomic constraint that relates the per capita transfer to young consumers

t to the amount of real money created by the government:$t = \frac{\Delta M}{p} = m(1 - r_m)$. Neutrality of money is defined as the impossibility of the government affecting the long-run equilibrium. Now maximization problem will be: $\max U(c_1) + EU(r_m m + ra)$ s.t. $c_1 + a + r_m m = w$. Then the real yield of money turns out to be 1, whatever the government does. Money is neutral.

4. A theory of warm glow giving: Impure altruism by Andreoni (1991)

Warm-glow giving is an economic theory describing the emotional reward of giving to others. This warm-glow model was developed by Andreoni (1989),and Andreoni(1990).Warm glow-represents the selfish pleasure derived from "doing good", regardless of the actual impact of one's generosity. Impurely altruistic maximization for all except for individual $G_{-i} = \sum_{j \neq i} g_j$ in Andreoni(1990) is given as:

$$\max_{x, g_i, G} U_i(x, g_i, G) \\ s.t. x_i + g_i = w_i; G_{-i} + g_i = G \tag{8}$$

Where G is public good,U_i is utility strictly quasi-concave,g_i is gift of public good, $i = I, \dots n$ individuals,w_i is individual endowment with wealth. Total amount of public good is :$G = \sum_{i=1}^{n} g_i$ .When $U_i = U_i(x_i, G)$ he individual cares nothing for the private gift and is purely altruistic. But when $U_i = U_i(x_i, g_i)$ the individual is motivated to give only by warm-glow and is purely egoistic. When G and g_i are arguments in the utility function individuals are impurely altruistic. If we substitute $g_i = G - G_{-i}$ the maximization problem will be:$\max_G U_i(w_i + G_{-i} - G, G, G - G_{-i}$. Differentiating with respect to G and solving yields a donations function that takes as arguments the exogenous parts of the maximand:

$$G = f_i(w_i + G_{-i}, G_{-i}) \\ g_i = f_i(w_i + G_{-i}, G_{-i}) - G_{-i} \tag{9}$$

The derivative with respect to public good is f_{ia} marginal propensity to donate for altruistic reason and if charity and public good are normal $0 < f_{ia} < I$. Second argument comes from the private goods dimension of the utility function and is called f_{ie} marginalpropensityto donate for egoistic reasons. Assuming that both warm glow and the private good are normal, then some of the new dollar of wealth w_i will go towards increasing consumption of each. Nash equilibrium is stable when: $0 \leqq f_{ia} + f_{ie} \leqq I$.Now, w_i and G_{-i}will be perfect substitutes in the individual's donations function so $\frac{\partial f_i}{\partial G_{-i}} = I$, $f_{ia} + f_{ie} = 1$.And or the case of impure altruism :$\frac{\partial f_i}{\partial w_i} < \frac{\partial f_i}{\partial G_{-i}} < I$. So the altruism coefficient is defined as:

$$\alpha = \frac{\frac{\partial f_i}{\partial w_i}}{\frac{\partial f_i}{\partial G_{-i}}} = \frac{f_{ia}}{f_{ia} + f_{ie}} \tag{10}$$

Where $0 < \alpha_i \leqq I$ .For pure altruists $f_{ie} = 0$ $\alpha_i = 1$ and for pure egoists $f_{ia} + f_{ie} = I$ sp $\alpha = f_{ia}$. For impure altruists $f_{ie} > 0$ and $f_{ia} < \alpha < I$.

Proposition 3. The change in total giving resulting from a transfer between any two people, say person $\boldsymbol{I}$ and 2, such that $\boldsymbol{dw_1 = -dw_2 = dw}$:

$$\frac{dG}{dw} = c(\alpha_1 - \alpha_2) \tag{11}$$

Where $0 < c < I$. Hence, the income transfer will increase(decrease, or not change)the total provision of the public good if and only if the income gainer is more altruistic than (less altruistic than, or equally as altruistic as) the income loser. Now let $\mathcal{T} = \sum_{i=1}^{n} \tau_i - s_i g_i$ where $\mathcal{T}$ are net government receipts and $Y = G + T$, τ_i are the lump-sum taxes, The impure altruism preferences are:$U_i = U_i(x_i, Y, g_i)$ .Now let $y_i = g_i(I - s_i) + \tau_i$represent i's total contribution to the public good. Assume that the government subsidizes private giving at a rate s_i, and pays for this subsidy by levying lump sum taxes,τ_i .Since $Y = \sum_{i=1}^{n} y_i$ then $Y_{-i} = Y - y_i$. and write the budget constraint as:$x_i + Y = w_i + Y_{-i}$.now the maximization problem and FOC'S are:

$$\max_{Y} U_i(w_i + Y_{-i} - Y, Y, \frac{Y - Y_{-i} - \tau_I}{I - s_i}$$
$$-\frac{\partial U_i}{\partial x} + \frac{\partial U_i}{\partial Y} + \frac{\partial U_i}{\partial g}\frac{I}{I - s_i} = 0 \tag{12}$$

Solving previous yields : $y_i = f_i\left(w_i + Y_{-i}, \frac{Y_{-i} + \tau_I}{I - s_i}, s_i\right) - Y_{-i}$ .Altrusim coefficient now is: $\alpha = \frac{\frac{\partial f_i}{\partial w_i}}{\frac{\partial f_i}{\partial G_{-i}}} = \frac{f_{ia}}{\frac{f_{ia} + f_{ie}}{I - s_i}}$.

Proposition 4 : Now given the preferences $\boldsymbol{U_i = U_i(x_i, Y, g_i)}$ and an interior equilibrium, any increase (decrease) in the lump sum tax $\boldsymbol{\tau_i}$ will increase (decrease) the total provision of the public good if and only if $\boldsymbol{\alpha_i \leqq I}$ and $\boldsymbol{\alpha_j \leqq I}$ for some $\boldsymbol{j}$.That is: $\boldsymbol{dY = c\sum_{i=1}^{n}(I - \alpha_i)d\tau_i}$, where $\boldsymbol{0} \leq \boldsymbol{c} \leq 1$.Now any increase (decrease) of subsidy rate $\boldsymbol{s_i}$ will increase (decrease) the total provision of public good if and only if $\boldsymbol{\alpha_i \leq I, \forall i, \alpha_j < I}$ for some $\boldsymbol{j}$, that is: $\boldsymbol{dY = c\sum_{i=1}^{n}\left(\alpha_i \frac{f_{is}}{f_{ia}} + (I - \alpha_i)\frac{Y_{-i} + \tau_I}{I - s_i}\right) ds_i}$. It can be seen that the distribution of income as well as government tax policies are crucial in determining the total supply of the public good. By totally differentiating donations giving functions we get:

$$dy_i = \left(f_{ia} + \frac{I}{I - s_i} f_{ie} - I\right) dY_{-i} + \frac{I}{I - s_i} f_{ie} d\tau_i + \left[\frac{Y_{-i} + \tau_i}{(I - s_i)^2} f_{ie} + f_{is}\right] ds_i$$
$$dY = c\sum_{i=1}^{n}(I - \alpha_i)d\tau_i + c\sum_{i=1}^{n}\left(\alpha_i \frac{f_{is}}{f_{ia}} + (I - \alpha_i)\frac{Y_{-i} + \tau_I}{I - s_i}\right) ds_i \tag{13}$$

Where in the last term $\frac{dY}{d\tau} + \frac{dY}{ds} > \frac{dY}{d\tau}$. And for any given level of taxes collected, the taxes will have a bigger impact on total giving if they are spent on subsidizing gifts rather than on direct grants. Unlike in Feldstein (1980) result does not depend on theprice elasticity of giving. In this formulation, all that is required is that the $\alpha_i < 1$ .One application of this theorem is under Rotten kid theorem by Becker (1974).

Theorem 3: Rotten kid : If a head exists, other members also are motivated to maximize family income and consumption, even if their welfare depends on their own consumption alone. Or each beneficiary, no matter how selfish, maximizes the family income of his benefactor and thereby internalizes all effects of his actions on other beneficiaries, see Becker (1981).

Proof: To derive the Rotten Kid Theorem with impure altruism, we introduce the concept of impure altruism, where parents derive utility not only from their children's consumption but also from their own consumption. Maximization problem here is:

$$\max_{W_p, W_c} U_{total}(W_p, W_c) = U_p(W_p) + U_c(W_c) \quad s.t.\, W_p + W_c = Y \tag{14}$$

Where W_p, W_c are wealth allocated by parents for themselves. And Wealth allocated by parents for their children, respectively. We define Lagrangian as:

$$\mathcal{L}(W_o, W_c, \lambda) = U_p(W_o) + U_c(W_c) + \lambda(Y - W_p - W_c) \tag{15}$$

Taking partial derivatives of $\mathcal{L}$ and setting them to zero we get:

$$\frac{\partial \mathcal{L}}{\partial W_p} = \frac{\partial U_p}{\partial W_p} - \lambda = 0 \qquad \frac{\partial \mathcal{L}}{\partial W_c} = \frac{\partial U_c}{\partial W_c} - \lambda = 0 \qquad \frac{\partial \mathcal{L}}{\partial \lambda} = Y - W_p - W_c = 0 \tag{16}$$

$Y - W_p - W_c = 0$ is constraint representing that the total wealth allocated by parents does not exceed their lifetime income. Solving these equations simultaneously gives us the optimal allocation of wealth for parents and children, maximizing their total utility while considering impure altruism■.

5. Berge equilibrium (due to Berge (1957))

Definition 4: We are considering normal form of a game $\boldsymbol{G = \langle N, S_i, u_i \rangle}$, where $\boldsymbol{N = \{1, 2, .., n\}}$ the set of $\boldsymbol{n}$ players,$\boldsymbol{S_i}$ is non-empty strategy set of player $\boldsymbol{i}$ and $\boldsymbol{i \in N}$, and $\boldsymbol{u_i}$ is that player's utility function. Strategy profile here is $\boldsymbol{s = \{s_1, s_2, \dots, s_n\} \in S}$ and denote and incomplete strategy profile:$\boldsymbol{s_{-i} = (s_1, s_2, \dots, s_{i-1}, s_{i+1}, \dots, s_n)}$ .A strategy profile $\boldsymbol{s^* \in S}$ is called **Berge equilibrium** if for any player $\boldsymbol{i \in N}$and any $\boldsymbol{s_{-i} \in S_{-i}}$ the strategy profile satisfies $\boldsymbol{u_i(s_i^*, s_{-i}) \le u_i(s^*)}$

Definition 5: Consider this game $\boldsymbol{G = (x_i, u_i)_{i \in I}}$ where $\boldsymbol{X = \Pi_{i \in I} X_i}$ and $\boldsymbol{I \in (1, .. n)}$ is the set of strategy profiles in the game$\boldsymbol{X_i \subset E_i}$,subset of vector space,and let $\boldsymbol{R = \{R_i\}_{i \in M}}$ is a set of subsets of $\boldsymbol{I}$.A feasible strategy $\boldsymbol{\overline{x} \in X}$ is an equilibrium point for the set $\boldsymbol{R}$ relative to the set $\boldsymbol{S}$ or a coalitional Berge equilibrium if $\boldsymbol{u_{rm}(\overline{x}) \geq u_{rm}(\overline{x} - S_m, xS_m), \forall m \in M, \forall r_m \in R_m, x_{S_m} \in X_{S_m}}$

6. Arrow-Lind theorem (due Arrow-Lind (1970))

Expected payoff of the project is $\Pi = \overline{\overline{\Pi}} + X$ where $\overline{\overline{\Pi}} = E(\Pi)$ и $E(X) = 0$.For the individual households $0 \leq s \leq 1$ where s is their share of projects return.Now, $k(s)$ -is risk premium and $\mathrm{E}\left[U\left(M + s\overline{\overline{\Pi}} + sX\right)\right] = U\left(M + s\overline{\overline{\Pi}} - k(s)\right)$.Where $k(s)$-are the risk bearing costs for projects ,M-is medial income.

Theorem 4 :Arrow-Lind : Suppose $\boldsymbol{cov(M, \Pi) = 0}$ and $\boldsymbol{s = \frac{1}{H}}$ .Total risk bearing costs are , $\boldsymbol{Hk(s) = Hk\left(\frac{1}{H}\right) = Hk(H)}$, and they move to zero as $\boldsymbol{H} \to \infty$

Proof: by differentiation of expected utility with respect to s we get

$$\frac{\partial}{\partial s}\mathrm{E}\left[U\left(M + s\overline{\overline{\Pi}} + sX\right)\right] = \mathrm{E}\left[U'\left(M + s\overline{\overline{\Pi}} + sX\right)\left[\overline{\overline{\Pi}} - X\right]\right] \tag{17}$$

If $s = 0$ and $cov(M, \Pi) = 0$, in turn implies that $cov(M, X) = 0$ it follows: :

$$\mathrm{E}\left[U'(M)\left[\overline{\overline{\Pi}} + X\right]\right] = \overline{\overline{\Pi}}E[U'(M)]$$

$$\lim_{s\to 0}\frac{\mathrm{E}\left[U\left(M + s\overline{\overline{\Pi}} + sX\right) - U(M)\right]}{s} \to \overline{\overline{\Pi}}E[U'(M)] \tag{18}$$

Or equivalent by the assumption $s = \frac{1}{H}\lim_{H\to\infty} HE\left[U\left(M + \frac{\overline{\overline{\Pi}} + \bar{X}}{H}\right) - U(M)\right] = \overline{\overline{\Pi}}E[U'(M)]$ it follows: $\lim_{H\to\infty}\left[\frac{\overline{\overline{\Pi}}}{H} - k(H)\right] = \overline{\overline{\Pi}}$; $\lim_{H\to\infty} Hk(H) = 0$ ■

7. Linear-cost share equilibrium

Definition 6: A linear cost-share equilibrium is a vector of shares $\boldsymbol{b \in (b^1, .., b^n) \in [0,1]^n}$such that $\boldsymbol{\sum b^i = 1}$ and acontribution vector $\boldsymbol{(E^1, .., E^n)}$ and a public good l evel $\boldsymbol{y}$, which is feasible, such that: $(\forall \boldsymbol{i})(\boldsymbol{E^i = bC(y)})$ and $\boldsymbol{y}$ maximizes $\boldsymbol{u^i\left(y, b^iC(y)\right)}$. A linear-cost-share equilibrium is a special case of a Lindahl equilibrium, $\boldsymbol{E^i}$ is i's contribution to the public good, and the cost function is $\boldsymbol{C(y) = E}$,the production function $\boldsymbol{G}$ is inverse cost function.The payoff function for each individual is:$\boldsymbol{u^i(G(E^s), E^i}$ multiplicative Kantian equilibrium is:

$$\frac{d}{dr}|_{r=1}u^i\left(G(rE^s)rE^i\right) = 0, u_1^i(G'(E^s)E^s + u_2^iE^i = 0 \Rightarrow -\frac{u_1^i}{u_2^i} = \frac{E^i}{E^S}\frac{1}{G'(E^S)}$$
(19)

Since $\frac{1}{G'(E^S)} = C'(y)$ by adding previous we can write $\sum \frac{1}{MRS^i} = C'(y)$ which Samuelson condition for provision of public goods. Where $E^S = \sum E^i$.If $b^i = \frac{E^i}{E^S}$, these equations are identical to $\frac{u_1^i}{u_2^i} = \frac{E^i}{E^S} \frac{1}{G'(E^S)}$, and so Kantian optimization decentralizes the Lindahl equilibrium, see Roemer (2019).

8. Simulations of the Kantian equilibrium result

In this section we will simulate Kantian equilibrium result. First, we will plot Kantian Economics: Individual vs. Societal Utility, next we will plot a global graph Macroeconomics Constraints, Efficiency, and Ethics, And Macroeconomics Constraints, Efficiency, Ethics, Warm-glow Giving, Nash Equilibrium, Money Neutrality, Arrow-Lind Principle, and Linear Cost-Share Equilibrium will be the last plot. All the simulations and plots re coded in Python. These simulations are done with artificial data they do not use real observations. Their main purpose and sole one is to depict graphically this economy described in previous sections.

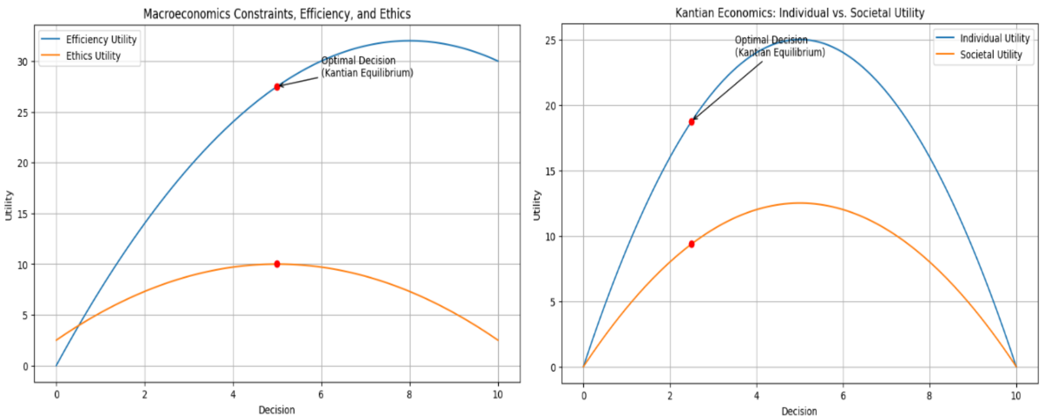

Figure 1 Macroeconomics Constraints, Efficiency, and Ethics; Kantian Economics: Individual vs. Societal Utility

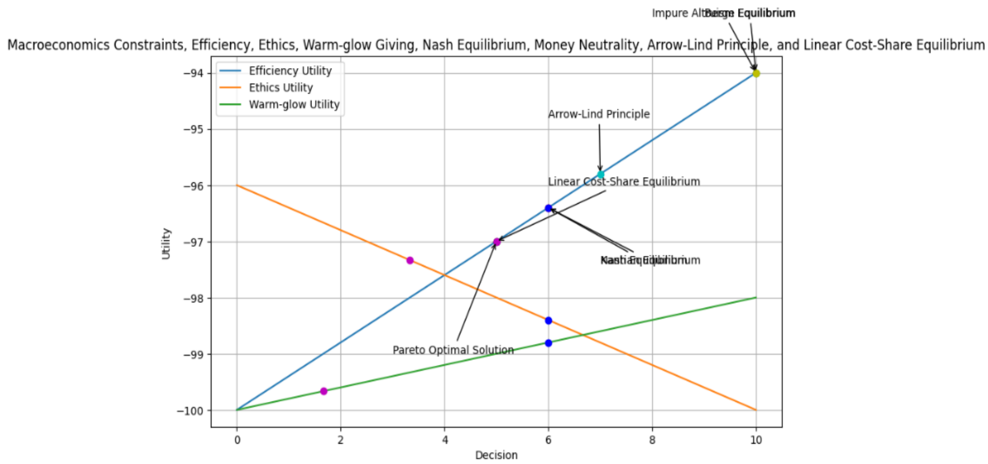

Figure 2 Macroeconomics Constraints, Efficiency, Ethics, Warm-glow Giving, Nash Equilibrium, Money Neutrality, and Arrow-Lind Principle

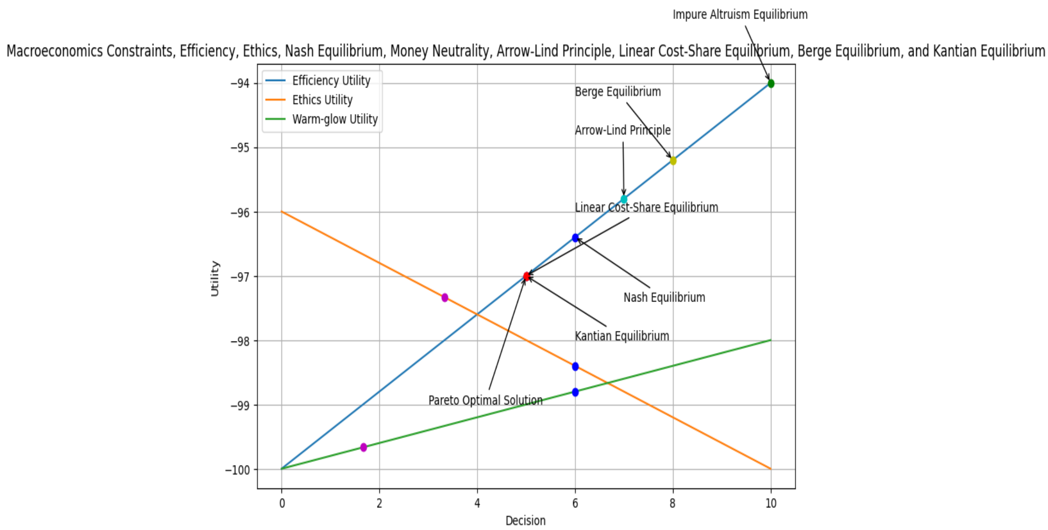

Figure 3 second simulation of the whole model

In previous examples: Kantian Equilibrium is calculated based on the balance between efficiency and ethics. Nash Equilibrium and Impure Altruism Equilibrium are specific points on utility curves. Pareto Optimal solution and Arrow-Lind Principle are specific points chosen to illustrate these concepts. Linear Cost-Share Equilibrium is derived by sharing the decision proportionally according to the importance of each factor. Berge Equilibrium is marked at a specific decision point as an example. Next, we will show two examples of the usage of these equilibria in the provision of public goods.

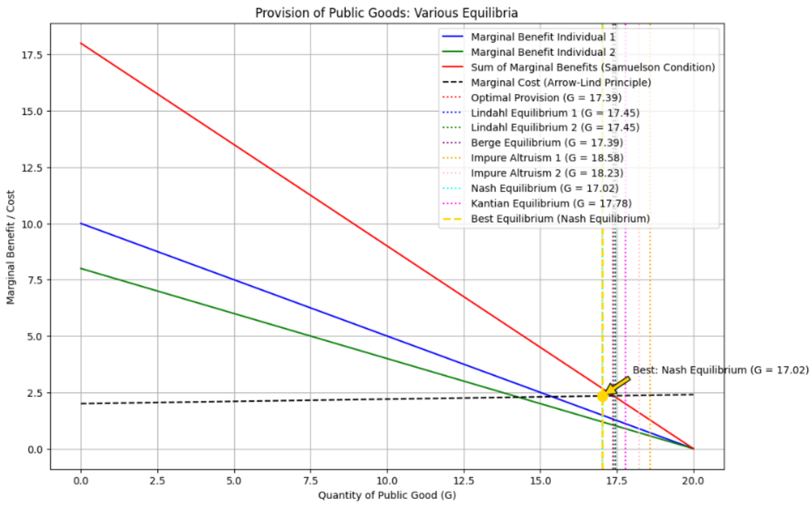

Figure 4 Provision of public goods: Various equilibria.

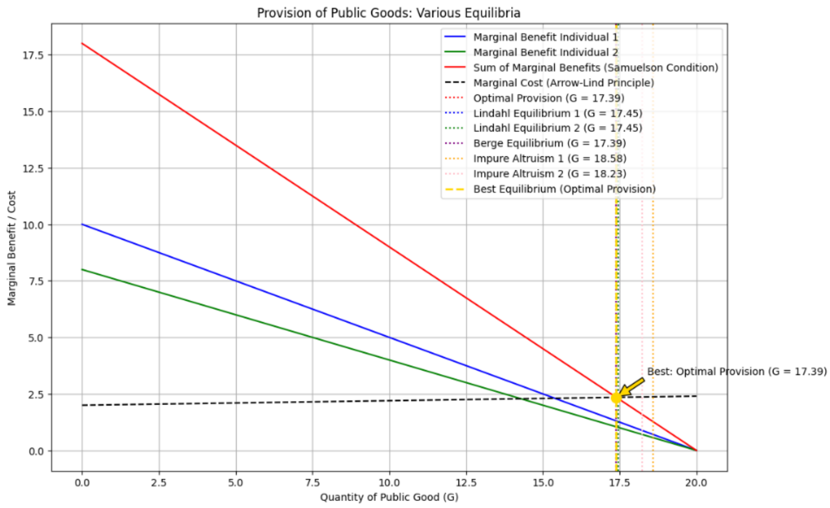

Figure 5 second simulation of the whole model.

Optimal provision of public goods is:

$$\sum_{i=1}^{n} MB_i(G) = MC(G) \qquad (20)$$

For two individuals:$(a_1 - b_1 G) + (a_2 - b_2 G) = c + \frac{d}{\sqrt{n}} G$.Solving for G: $a_1 + a_2 - \left(b_1 + b_2 + \frac{d}{\sqrt{n}}\right) G = c \Rightarrow G_{optimal} = \frac{a_1 + a_2 - c}{b_1 + b_2 + \frac{d}{\sqrt{n}}}$.Where c is a fixed component of the marginal cost, independent of the quantity G of the public good. This term $\frac{d}{\sqrt{n}} G$ represents the variable part of the marginal cost, which increases with the quantity G of the public good. Where n is the number of individuals and $\sqrt{n}$ is used to distribute the cost among individuals. So, d could be interpreted as the sensitivity of the marginal cost to the provision level of the public good.Also, a_i represents marginal benefit when $G = 0$.And b_i is a coefficient that reflects how the marginal benefit (marginal disutility) decreases as more of the public good is provided marginal benefit function can usually be expressed as:$MB_i(G) = a_i - b_i G$. In **Lindahl equilibrium**, each individual pays a price proportional to their marginal benefit, i.e. $P_i = MB_i(G)$. For two individuals:$P_1 = MB_1(G); P_2 = MB_2(G)$. Where the total cost for each individual must sum to the marginal cost:$P_1 + P_2 = c + \frac{d}{\sqrt{n}} G$.And solving for G:

$$G_{Lindahl_1} = \frac{a_1 - \lambda_1 c}{b_1 + \lambda_1 \frac{d}{\sqrt{n}}}; G_{Lindahl_2} = \frac{a_2 - \lambda_2 c}{b_2 + \lambda_2 \frac{d}{\sqrt{n}}} \quad (21)$$

In **Berge equilibrium**, each individual's marginal benefit equals the marginal cost of providing the good,i.e. $MB_1(G) = MC(G); MB_2(G) = MC(G)$.Solving for G: $a_1 - b_1 G = c + \frac{d}{\sqrt{n}} G; G_{Berge_1} = \frac{a_1 - c}{b_1 + \frac{d}{\sqrt{n}}}$.Same applies for the second individual. In **Impure altruism**, individuals gain utility from the act of giving itself. So utility function is given as: $U_i = MB_i(G) - \alpha \cdot MC(G)$. The parameter α captures the relative importance or weight of this altruistic component. If α is high, the individual derives significant utility from others benefiting from the public good. If α is low, the individual's utility is more focused on their direct benefit from the public good. FOC is given as: $\frac{\partial U_i}{\partial G} = 0$.For individual i:$a_i - b_i G - \alpha \cdot \left(c + \frac{d}{\sqrt{n}} G\right) = 0$. Solving for $G_{impurealtruism_1}; G_{impurelatruism_2}$:

$$G_{impurealtruism_1} = \frac{a_1 - \alpha c}{b_1 + \alpha \frac{d}{\sqrt{n}}}; G_{impurealtruism_2} = \frac{a_2 - \alpha c}{b_2 + \alpha \frac{d}{\sqrt{n}}} \quad (22)$$

In **Nash equilibrium**, each individual's contribution is optimal given the contributions of others. Condition here is: $MB_1(G) = \frac{MC(G)}{2}$.For individual i:$a_i - b_i G = \frac{c + \frac{d}{\sqrt{n}} G}{2}$. Solving for G_{Nash}:

$$G_{Nash} = \frac{a_1 + a_2 - c}{b_1 + b_2 + \frac{d}{\sqrt{n}}} \quad (23)$$

In **Kantian equilibrium**, individuals contribute equally to the public good. Solving for two individuals $G_{Kantian}$:

$$G_{Kantian} = \frac{a_1 + a_2 - c}{b_1 + b_2} \quad (24)$$

The **Arrow-Lind principle** states that the marginal cost of providing a public good is divided among all individuals. Marginal cost are: $MC(G) = c + \frac{d}{\sqrt{n}} G$.FOC is given:$MB_i(G) = MC(G)$. For individual i:$a_i - b_i G = c + \frac{d}{\sqrt{n}} G$. Solving for two individuals $G_{ArrowLind_1; G_{ArrowLind_2}}$:

$$G_{ArrowLind_1} = \frac{a_1 - c}{b_1 + \frac{d}{\sqrt{n}}}; G_{ArrowLind_2} = \frac{a_2 - c}{b_2 + \frac{d}{\sqrt{n}}} \quad (25)$$

These derivations provide the mathematical conditions for each equilibrium concept in the context of public goods provision.

9. Conclusion

Kantian equilibrium in our model is almost same as Nash equilibrium so we can call him Kantian-Nash equilibrium. There is a literature or there exists a concept of generalized Kantian-Nash equilibrium see Grafton et al.(2017).This equilibria are about forming stable coalitions in static game of climate change mitigation. The results might have been different if we assumed that the population of Kantians in the society increases more than Nashians. But as the general population grows Arrow-Lind is far more efficient than Kantian and Nash equilibrium. And warm-glow model or impure altruism together with Berge equilibrium where agents cooperate instead of defect are much more efficient than Kantian and Nash equilibria. Linear cost equilibrium (modified Lindahl equilibrium)seems to be only Pareto efficient solution from those offered. In the second simulation of the model, impure altruism is again the most efficient equilibrium suggesting that private provision of public goods is the most efficient way of supply of public goods. The applicability of these equilibria was tested in the case of provision of public goods. Lindahl equilibrium, Kantian equilibrium and especially Impure altruism led to overprovision of public goods. This in turn leads to: Inefficient resource allocation, crowding out of private investment, increased tax Burden, government budget deficits, distorted Incentives: Overprovision of certain public goods might create distorted incentives for individuals and businesses. For example, if public goods like healthcare or education are overprovided, it may reduce the incentive for individuals to invest in their own health or education. Moral Hazard: Overprovision of public goods can create a moral hazard problem, where individuals or businesses take on more risk because they believe the government will always provide for their needs, leading to irresponsible behavior. Inequitable distribution of benefits: Public goods are typically non-excludable, meaning that everyone can benefit from them. However, overprovision may disproportionately benefit certain groups over others, leading to inequities. For example, if more resources are allocated to public parks in wealthier areas, those communities benefit more than others. Berge equilibrium is same as optimal provision of public goods equilibrium. Nash equilibrium is located where marginal cost (Arrow-Lind principle) line crosses sum of marginal benefits (Samuelson condition).

References

Akerlof, G. A. (2002). Behavioral Macroeconomics and Macroeconomic Behavior. The American Economic Review,92(3),411–433. http://www.jstor.org/stable/3083349

Andreoni, J. (1990). Impure Altruism and Donations to Public Goods: A Theory of Warm-Glow Giving. The Economic Journal, 100(401), 464–477. https://doi.org/10.2307/2234133

Andreoni, James (1989). Giving with Impure Altruism: Applications to Charity and Ricardian Equivalence. Journal of Political Economy. 97 (6): 1447–1458

Andreoni, James (1990). Impure Altruism and Donations to Public Goods: A Theory of Warm-Glow Giving. The Economic Journal. 100 (401): 464–477

Arrow, K. J., Lind, R. C. (1970). Uncertainty and the Evaluation of Public Investment Decisions. The American Economic Review, 60(3), 364–378. http://www.jstor.org/stable/1817987

Arrow, KennethJ. (1975). Gifts and exchanges.In Altruism, Morality and EconomicTheory(ed. E. S. Phelps). Russell Sage Foundation.

Becker, G. S. (1974). A Theory of Social Interactions. Journal of Political Economy, 82(6), 1063–1093. http://www.jstor.org/stable/1830662

Becker, G. S.(1981).A Treatise on the Family. Cambridge, Mass.: Harvard Univ. Press,

Berge, C. (1957). Theorie générale des jeux á n−personnes. Paris, Gauthier Villars.

Bowles,S. and Gintis,H. (2011), A cooperative species: experimental economics, anthropology, and evolutionary biology

Feldstein M.(1980). A contribution to the theory of tax expenditures: the case of charitable contributions. In The Economics of Taxation (ed. Henry Aaron and Michael J. Boskin). Washington, D.C.: Brookings Institution

Grafton, R. & Kompas, Tom & Long, Ngo. (2017). A Brave New World? Kantian-Nashian Interaction and the Dynamics of Global Climate Change Mitigation. European Economic Review.

Hartmann, Patrick & Eisend, Martin & Apaolaza, Vanessa & D'Souza, Clare. (2017). Warm glow vs. altruistic values: How important is intrinsic emotional reward in proenvironmental behavior?. Journal of Environmental Psychology. 52. 43-55. 10.1016/j.jenvp.2017.05.006.

Henrich, N., and J. Henrich.(2007). Why Humans Cooperate: A Cultural and Evolutionary Explanation. New York: Oxford University Press.

Laffont, J.-J. (1975). Macroeconomic Constraints, Economic Efficiency and Ethics: An Introduction to Kantian Economics. Economica, 42(168), 430–437. https://doi.org/10.2307/2553800

Roemer, J. E. (2010). Kantian Equilibrium. Scandinavian Journal of Economics, 112(1), 1–24.

Roemer, J. E. (2019). How We Cooperate: A Theory of Kantian Optimization. Yale University Press

Sen, Amartya K. (I977). Rational fools: a critique of the behavioral foundations of economic theory. Journal of Philosophy and Public Afairs, vol. 6, pp. 317-44.

Sher,I. (2020). Normative Aspects of Kantian Equilibrium. Erasmus Journal for Philosophy and Economics,Volume 13, Issue 2, pp. 43–84

Sugden, Robert (1982). On the economics of philanthropy. Economic Journal, vol. 92, pp. 341-50.

Sugden, Robert (1984). Reciprocity: the supply of public goods through voluntary contributions. Economic Journal ,vol. 94, pp. 772-87