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RELIABIITY: RELIABILITY OF THE IMPROVED NETWORK

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Abstract. One of the problems of interest in the field of multi-state network reliability is to found a minimal number of components that should be improved, in order to obtain a network with higher level of work. The main purpose in this paper is how to find the minimal number of components that should be exchanged, so that we will have system that works in a higher level and have the greatest possible reliability.

Keywords: Reliability, multi-state systems, network reliability, minimal path vectors.

1. Introduction

The two-terminal reliability (2TR) is a well-known network binary reliability problem. Usually, when reliability of the systems is analyzed, it is assumed that the system and its components can be in either in a working or in a failed state. These models are known as binary state models. It is found that such models are unable to describe some systems, as telecommunication and transportation systems, water distribution, gas and oil production and hydropower generation systems. These systems may provide a service or function at degraded component performance levels and better results may be obtained using a multi-state reliability approach.

Important tools in reliability theory are importance measures. They are used to evaluate and rank the impact of individual components within a system. There are few authors that present and evaluate composite importance measures for multi-state system with multi-state components (MSMC). Ramirez-Marquez and Coit [1] proposed two main importance measures for multi-state systems: quantification of the impact of a component as a whole on system reliability and quantification how a particular component state or set of states affect system reliability. Ramirez-Marquez, Roco, Gebre, Coit and Tortorella [2] gives another importance measures for multi-state systems with multi-state components: unsatisfied demand index (UDI), multi-state failure frequency index (MFFI) and multi-state redundancy importance (MRI). The first one provides insight regarding a component or component state contribution to unsatisfied demand and the second one elaborates on an approach that quantifies the contribution of a particular component or component state to system failure. MRI identifies where to allocate component redundancy as to improve system reliability.

In this paper we do not introduce new measure, but we analyze which of the components of the multi-state system is better to be improved in order to obtain system with greater reliability. The networks we considered are with an added constraint that the capacity of the arcs is an integer-valued variable taking values from the set $\{0, 1, 2,\ldots, M_i\}$. In this case, the capacity of the entire network is an integer value from the set $\{0, 1, 2, \ldots, M\}$. In [4] and [5] there are described algorithms for obtaining minimal path vectors for each level of such network. If we want to improve the network in order to get a network with greater capacity, we can change some of the components with better ones. In this paper we are focusing on two problems:

- How to use the minimal path vectors for the old network to find minimal path vectors for the improved one.
- We analyze how to decide which component to be changed, in order to obtain best performance with minimal cost. We suppose that cost of the changing of each link is the same.

2. Minimal path vectors of multi-state network

Let us regard a stochastic capacitated two-terminal network from a specified source node s to a specified sink node t. By $\mathcal{N}$ we will denote the set of nodes and by $\mathcal{A} = \{a_i \mid 1 \le i \le n\}$ the set of arcs (links). The set of available capacities of the arc a_i is denoted by S_i, $S_i=\{0, 1, \ldots, M_i\}$, (0 means no capacity, M_i means full capacity). The set S_i is known as **capacity space set** of the arc a_i and the capacity set space of the whole network is note by $S=\{0,1,\ldots, M\}$. Also we note $S=\{S_i | 1\le i\le n\}$. Let x_i be the state of the arc a_i, then the vector $\vec{x} = (x_1, x_2, \ldots, x_n)$ denotes the state of all the arcs of the network and it is called **state vector**. The **vector of maximal states** of the system, $(M_1, M_2,\ldots,M_n)$, will be denote by **M** and $E=S_1\times S_2\times\ldots\times S_n$ will be the set of all state vectors.The function φ: $E\rightarrow S$ where $\varphi(\vec{x})$ is available capacity from source to sink under system state vector $\vec{x}$ is called **multi-state structure function**.

Multi-state two terminal reliability of level d ($M2TR_d$) is the probability that a flow equal to or greater to d can be successfully delivered form source node to sink node.

$$M2TR_d = P(\varphi(\vec{x}) \ge d). \qquad (1)$$

A vector $\vec{y}$ is said to be less than $\vec{x}$, $\vec{y} < \vec{x}$, (or dominated by $\vec{x}$) if $\forall i$, $y_i \le x_i$ and for some k, $y_k < x_k$. A vector $\vec{x}\in E$ is said to be a **minimal path vector to level** d (MPV_d) if $\varphi(\vec{x})\ge d$ and for all other $\vec{y} < \vec{x}$, $\varphi(\vec{y})<d$ and a vector $\vec{x}\in E$

is said to be a **minimal cut vector to level** d if $\varphi(\vec{x})<d$ and for all other $\vec{y}>\vec{x}, \varphi(\vec{y})\geq d$.

In order to specify the structure of the network we will define binary minimal path vector. Let we have a multi-state network with set of nodes N and set of links $A=\{a_i \mid 1\leq i\leq n\}$. We will regard the network with the same nodes and links in which all links are binary, i.e. their state sets are $\{0, 1\}$. Let $\vec{v}$ be minimal path vector for this network. Then we will say that $\vec{v}$ is a **binary minimal path vector** for the multi-state network. By BPV we will denote the set of all binary minimal path vectors.

Let us denote a network by G=($\mathcal{N}$, $\mathcal{A}$, BPV, S, $\mathcal{VP}$), where $\mathcal{N}$ is the set of nodes, $\mathcal{A}$ the set of arcs, BPV the set of binary minimal path vectors, S the set of capacity state sets of the components, and $\mathcal{VP}$ the set of probabilities of components levels, where $\vec{p}_i$ is the vector of probabilities of the i-th link, i.e. $p_{id}=P(x_i=d)$. It is clear that each network is totally described by these elements.

Proposition 1. Let $\vec{x}$ be a MPV_d for the network G=(N,A,BPV,S,VP). Then, in order to be delivered d units from the source to the sink, when the system is in state $\vec{x}$, each link is used in only one direction.

Proof: Let $\vec{x}$ be a MPV_d such that the link a_i from the node u_1 to the node u_2 is used once in direction from u_1 to u_2, and once in the opposite direction. Then, one unit goes through the path w_1 from the source s to u_1, then through the link u_1u_2 and from u_2 the sink t through the path w_2, and another one goes through the path w'_1 from the source s to u_2, then through the link u_2u_1 and from u_2 the sink t through the path w'_2. Now, we can choose the way $w_1w'_2$ for one unit and the way w'_1w_2 for another one. Therefore, we obtain a smaller path, so $\vec{x}$ is not MPV_d. □

Using the Proposition 1, for each MPV_d, $\vec{x}$, the links of the networks graph can be oriented in respect to $\vec{x}$. Next we give more rigorous property about the orientation on links on a network.

Proposition 2. Let $\vec{x}$ be a MPV_d for the network G=(N,A,BPV,S,VP). When we want to deliver d units from the source to the sink when the system is in state $\vec{x}$, each link is used only in one direction. If the links are oriented as they are used, then the obtained subgraph is acyclic.

Proof: Let $\vec{x}$ be a MPV_d. From the Proposition 1 we can orient the links. Suppose that there is a cycle v in the obtained oriented subgraph. Since each unit goes throw acyclic path, we have that the cycle is used from different units. We can suppose that $v=w_1\ldots w_k$ such that the path w_i is used from the i-th unit. So, the path of the i-th unit can be written as $u_iw_iu_i'$, $i=\overline{1,k}$. Now, may be chosen another path for each of those k units, u_1u_k' for the first one, and $u_iu'_{i-1}$ for the i-th, $i=\overline{2,k}$. Therefore we obtained smaller path, so $\vec{x}$ is not MPV_d. □

Using the Proposition 2, we can define **ordering of the nodes in respect to $\vec{x}$**.

Definition. Let $\vec{x}$ be a MPV$_d$ for the network G=(N,A,BPV,S,VP) and the links are oriented in respect to $\vec{x}$. For two links a and b we will say that $a <_{\vec{x}} b$ if there is a path from a to b in the oriented graph. We will call this ordering as **ordering of the nodes in respect to $\vec{x}$**.

Note that this ordering is not always a linear ordering.

Corollary 1. Let $\vec{x}$ be a MPV$_d$ for the network G=(N,A,BPV,S,VP) and the links are oriented in respect to $\vec{x}$. Then there are not cycles in the corresponding subgraph.

Definition. We will say that $<_{\vec{x}} \Leftrightarrow <_{\vec{y}}$ i.e. the ordering in respect to $\vec{x}$ and the ordering in respect to $\vec{y}$ are equivalent, iff for all nodes a and b, $a <_{\vec{x}} b \Rightarrow \neg(b <_{\vec{x}} a)$ and $a <_{\vec{y}} b \Rightarrow \neg(b <_{\vec{y}} a)$.

Definition. Let $\vec{x}$ be a MPV$_d$ and $\vec{y}$ be a MPV$_{d'}$ such that $<_{\vec{x}} \Leftrightarrow <_{\vec{y}}$. We define $<'_{\vec{x}+\vec{y}}$ by $a <'_{\vec{x}+\vec{y}} b$ if $a <_{\vec{x}} b$ or $a <_{\vec{y}} b$. Then the ordering $<_{\vec{x}+\vec{y}}$ is a transitive closer of $<'_{\vec{x}+\vec{y}}$.

It is easy to show that $<_{\vec{x}+\vec{y}}$ is a ordering. From the Kirchhoff's Current law follows that if the network works in the state $\vec{x}$, where $\vec{x}$ is a MPV$_d$, then exactly d units get out from the source node and come into the sink node. For all other nodes we have that the number of units that get into the node is the same to the number of units that get out from it.

Proposition 3. Let $\vec{x}_k \in$ BPV, $k = \overline{1,d}$ for the network G=(N, A, BPV, S, VP) such that $<_{\vec{x}_i} \Leftrightarrow <_{\vec{x}_j}$ for $i \neq j$. Then $\vec{x} = \sum_{k=1}^{d} \vec{x}_k$ is a minimal path vector for level d.

Proof: It is clear that the proposition is true for $d = 1$. Suppose that it is true for all integers $d \leq r$. So, for $d = r$, $\vec{y}_r = \sum_{k=1}^{r} \vec{x}_k$ is a minimial path vector for level r, and, exactly r units get out from the source node and come into the sink node and for other nodes, the number of units that get into the node is equal to the number of units that get out from it.

Let $\vec{x}_{r+1}$ is a minimal path vector for level 1 and $<_{\vec{y}_r} \Leftrightarrow <_{\vec{x}_{r+1}}$. The vector $\vec{y}_{r+1} = \vec{y}_r + \vec{x}_{r+1} = \sum_{k=1}^{r+1} \vec{x}_k$ is a path vector for level $r + 1$, since the first r units will use the performance of the vector $\vec{y}_r$ and the $r + 1$-th unit will pass over from the source to the sink using the links corresponding to the vector $\vec{x}_{r+1}$. Moreover, when the network works in state $\vec{y}_{r+1}$, exactly $r + 1$ units get out from the source node and come into the sink node and for other nodes, the number of units that get into the node is equal to the number of units that get out from it.

Let us suppose that it $\vec{y}_{r+1}$ is not a minimal path vector for level $r + 1$ i.e. there is a smaller minimal path vrector $\vec{z}$. Since $\vec{z}$ is a minimal path vector for level $r + 1$, when the network works in state $\vec{z}$, exactly $r + 1$ units get out from the source node and come into the sink node and for other nodes, the number of units that get into the node is equal to the number of units that get out from it.

Let us regard the vector $\vec{y}_{r+1} - \vec{z} > \vec{0}$. If the network is in this state, then there no units that gets out from the source node and comes into the sink node. Also, for other nodes, the number of units that get into the node is equal to the number of units that gets out from it and there are nodes in which at last one unit gets in and out. If the links are oriented in respect to $\vec{y}_{r+1} - \vec{z}$, this will be possible only if exist at last one cycle, which is in contradiction with our assumption that $<_{\vec{y}_r} \Leftrightarrow <_{\vec{x}_{r+1}}$. □

Proposition 4. If $\vec{x}$ is a MPV$_d$ for the network G=(N, A, BPV, S, VP), then there are vectors $\vec{x}_k \in$ BPV, k=1,…, d, such that $\vec{x} = \sum_{k=1}^{d} \vec{x}_k$.

Proof: It is clear that the proposition is true for d=1. Suppose that it is true for all integers $d \le r$. Let $\vec{x}$ be a minimal path vector for level r+1. From the Proposition 2, the vertexes of the graph can be oriented in respect to $\vec{x}$. Suppose that one of those units uses the sequence of links corresponding to the vector $\vec{y} \in$ BPV. Now, this vector has the same orientation as the vector $\vec{x}$ and $\vec{y} \le \vec{x}$. Let us observe the vector $\vec{x} - \vec{y}$. It is a path vector for level r, since we subtract capacity only from the links that are used by one of the units, so the rest r units will pass over from the source to the sink if the system is in this state. Suppose that $\vec{x} - \vec{y}$ is not a minimal path vector for level r, i.e. there is a smaller path vector $\vec{x}'$. Then, $\vec{x}' + \vec{y}$ is a path vector for level r+1 and $\vec{x}' + \vec{y} < \vec{x} - \vec{y} + \vec{y} = \vec{x}$, which is in contradiction with our assumption that $\vec{x}$ is a minimal path vector for level r+1. So $\vec{x} - \vec{y}$ is a minimal path vector for level r.

Now, from the inductive assumption we have that there are vectors $\vec{x}_k \in \text{BPV}$, $k=1\ldots r$, such that $\vec{x} - \vec{y} = \sum_{k=1}^{r} \vec{x}_k$. Taking $\vec{y} = \vec{x}_{r+1}$, we obtain

$$\vec{x} = \sum_{k=1}^{r} \vec{x}_k + \vec{y} = \sum_{k=1}^{r+1} \vec{x}_k . \qquad \square$$

Now, we will show how the system reliability can be computed, when the minimal path vectors are known. For binary systems, using minimal path sets, the reliability can be computed through the inclusion/exclusion formula [3]. This formula has to be extended to account for the new vector structure of the minimal sets. For the multi-state case, M2TR_d can be obtained with the following modification of the inclusion/exclusion formula:

$$\text{M2TR}_d = \sum_{h=1}^{T} P(\vec{x} \geq \vec{y}_h) - \sum_{h<k}^{T} P(\vec{x} \geq \vec{y}_h \wedge \vec{x} \geq \vec{y}_k) + \ldots (-1)^T P(\vec{x} \geq \vec{y}_1 \wedge \ldots \wedge \vec{x} \geq \vec{y}_T), \quad (2)$$

where T is the number of MPV_d and $\mathbf{y}_h \in \text{MPV}_d$. Using the following notation,

$$\max(\mathbf{z}_1,\ldots,\mathbf{z}_s) = (\max(z_1^{(1)},\ldots,z_s^{(1)}),\ldots,\max(z_1^{(l)},\ldots,z_s^{(l)})), \quad (3)$$

where $z_u^{(v)}$ is the v-th coordinate of $\mathbf{z}_u$, the equation (2) can be write as:

$$\text{M2TR}_d = \sum_{h=1}^{T} P(\vec{x} \geq \vec{y}_h) - \sum_{h<k}^{T} P(\vec{x} \geq \max(\vec{y}_h, \vec{y}_k)) + \ldots (-1)^T P(\vec{x} \geq \max(\vec{y}_1,\ldots,\vec{y}_T)) \quad (4)$$

We use the formula (4) for calculation of the reliability of level d.

3. Improvement of the network and calculation of the new reliability

Suppose that we have a two-terminal network G=(N, A, BPV, S, VP) with S={0, 1,... M}. We want to improve that network for one level. In order to achieve this we are lead from two criteria: changing a minimal number of components and obtaining a network with greater reliability.

We improve the network by changing some of the links with better one, that working in higher level. From that reason we look at the relation between MPV_d on two networks that differs just in maximal levels of the links.

Proposition 5. Let G = (N, A, BPV, S, VP) be a network with vector of maximal states **M** and let G' = (N, A, BPV, S', VP') be a network with vector of maximal state **M' ≥ M**. Then, if $\vec{x}$ is a MPV_d for the network G, it is also a MPV_d for the network G'.

Next two propositions follows directly from Proposition 2 and Proposition 3.

Proposition 6. Let G=(N, A, BPV, S, VP) be a network with vector of maximal states **M**, and let G'=(N, A, BPV, S', VP') be a network with vector of maximal states **M' > M**. Let $\vec{x}$ be a MPV_d for a network G, such that $x_j = M_j$ and $\vec{y} \in$ BPV, such that $y_j = 1$. Also let $<_{\vec{x}} \Leftrightarrow <_{\vec{y}}$ and $\vec{x}+\vec{y} \le$ **M'**. Then $\vec{x}+\vec{y}$ is a MPV_{d+1} for the network G'.

Proposition 7. Let $MPV_{d,}$ $d=\overline{1,M}$, of the network G=(N, A, BPV, S, VP) for all are known, and let G'=(N, A, BPV, S', VP') be a network with vector of maximal states **M' > M**. Then, the set of MPV_{d+1} of the network G' is $\{\vec{x} \mid \vec{x}$ is MPV_{d+1} for G$\} \cup \{\vec{x}+\vec{y} \mid \vec{x}$ is MPV_d for G, $\vec{y} \in BPV, <_{\vec{x}} \Leftrightarrow <_{\vec{y}}\}$.

Using Proposition 6 *k* times we have following Corollary:

Corollary 1. Let G = (N, A, BPV, S, VP) be a network with vector of maximal states **M**, and let G'=(N, A, BPV, S', VP') be a network with vector of maximal states **M' > M**. Suppose that $\vec{x}$ is a MPV_d for G, such that $x_j = M_j$ and $\vec{y} \in MPV_k$, such that $y_j = k$, $<_{\vec{x}} \Leftrightarrow <_{\vec{y}}$ and $\vec{x}+\vec{y} \le$ **M'**. Then, $\vec{x}+\vec{y}$ is a MPV_{d+k} for the network G' if and only if $\vec{x}$ is a MPV_d for the network G.

Definition 1: Let $\vec{x}$ be a MPV_M and let $\vec{y}$ be a BPV of a network G=(N, A, BPV, S, VP), with maximal capacity **M**. We define function ξ by

$$\xi(\vec{x},\vec{y}) = (z_1, z_2, \ldots, z_n), \qquad z_i = \begin{cases} 1, & x_i + y_i > M_i \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

Next Proposition helps to select best combination of links that should be changed:

Proposition 8: Let G = (N, A, BPV, S, VP) be a network with maximal capacity **M**. The minimal numbers of components that need to be changed with ones that work with greater level, in order to get a network that works in level *M*+1 is $\min\{\sum_{i=1}^{n} \xi(\vec{x},\vec{y})_i \mid \vec{x} \in MPV_M, \ \vec{y} \in BPV$ and $<_{\vec{x}} \Leftrightarrow <_{\vec{y}}\}$.

Suppose that for each link of the network G=(N, A, BPV, S, VP), we have a new one that works with one level greater. In fact, the *i*-th link can be replaced with a link with $S_i^{'} = \{0,1,\ldots, M_i+1\}$ and have probability vector $\vec{p}_i^{\,'}$. For each $\vec{x} \in MPV_M$ and $\vec{y} \in BPV$ such that $<_{\vec{x}} \Leftrightarrow <_{\vec{y}}$ and $\sum_{i=1}^{n} \xi(\vec{x},\vec{y})_i = \min\{\sum_{i=1}^{n} \xi(\vec{x},\vec{y})_i \mid \vec{x} \in MPV_M, \ \vec{y} \in BPV$ and $<_{\vec{x}} \Leftrightarrow <_{\vec{y}}\}$ we define a new network $G_{\varphi(x,y)}$=(N, A, BPV, S', VP') with $S_i^{'}=S_i$ and $\vec{p}_i=\vec{p}_i$ when $\varphi(\vec{x},\vec{y})_i=0$ and $S_i^{'}$={0,1,..., M_i+1} and, $\vec{p}_i=\vec{p}_i^{'}$ when $\varphi(\vec{x},\vec{y})_i=1$. When there is more then one such network, we will choose that one with greater reliability.

At the end, we give how to choose which links to be improved in order to obtain a most reliable network that works with one level greater.

Step 1 We will find all $\vec{x}\in$ MPV$_M$ and $\vec{y}\in$ BPV such that $<_{\vec{x}}\Leftrightarrow<_{\vec{y}}$ and

$$\sum_{i=1}^{n}\xi(\vec{x},\vec{y})_i=\min\{\sum_{i=1}^{n}\xi(\vec{x},\vec{y})_i \mid \vec{x}\in \text{MPV}_M,\ \vec{y}\in \text{BPV and } <_{\vec{x}}\Leftrightarrow <_{\vec{y}}\}.$$

Step 2 Using Proposition 5, we will find the MPV$_d$, $d=\overline{1,M+1}$ for all candidates.

Step 3 Using (4) we will calculate the reliability of level d, $d=\overline{1,M+1}$ for all candidates, and we will select that one that has greatest reliability.

4. Case study

In this section we will explain the proposed procedure on an example. Regard the network G given in Figure 1 with S_1={0,1,2}, S_2={0,1,2,3}, S_3={0,1}, S_4={0,1,2} and S_5={0,1,2,3}. The probability vectors are $\vec{p}_1$=(0.1,0.1,0.8), $\vec{p}_2$=(0.1,0.1,0.2,0.6), $\vec{p}_3$=(0.1,0.9), $\vec{p}_4$=(0.1,0.1,0.8) and $\vec{p}_5$=(0.1,0.1,0.1,0.7). They can be replaced by links with $\vec{p}_1^{'}$=(0.1,0.1,0.1,0.7), $\vec{p}_2^{'}$=(0.1,0.1,0.2,0.6), $\vec{p}_3^{'}$=(0.1,0.9), $\vec{p}_4^{'}$=(0.1,0.1,0.8) and $\vec{p}_5^{'}$=(0.1,0.1,0.1,0.7). It is clear that the vector of maximal state **M**=(2,3,1,2,3) and BPV={(1,1,0,0,0), (0,0,0,1,1), (1,0,1,0,1), (0,1,1,1,0)}.

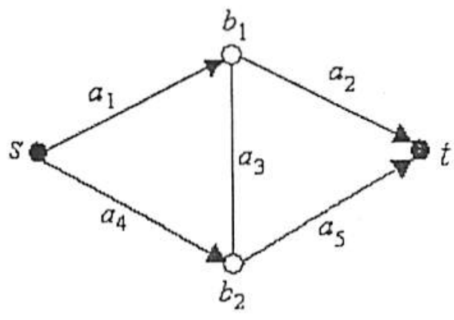

Figure 1

The minimal path vectors of all levels, obtained using algorithm given in [4], are given in Table 1.

d	MPV_d	*d*	MPV_d	*d*	MPV_d	*d*	MPV_d
1	1 1 0 0 0	2	2 1 1 0 1	3	2 2 0 1 1	4	2 3 1 2 1
	0 0 0 1 1		1 1 0 1 1		2 1 1 1 2		2 2 0 2 2
	1 0 1 0 1		1 0 1 1 2		1 2 1 2 1		2 1 1 2 3
	0 1 1 1 0		2 2 0 0 0		1 1 0 2 2		
			1 2 1 1 0		1 0 1 2 3		
			0 1 1 2 1		2 3 1 1 0		
			0 0 0 2 2				

Table 1. MPV_d *of network* G.

Now, in order to obtain a network that works with level 5, to each MPV_4, we will add a binary vector. Links that should be changed are given in Table 2.

MPV_4	$MPV_5=MPV_4+BPV$	Components for changing
2 3 1 2 1	3 4 1 2 1	1,2
	3 3 0 2 2	1
	2 3 1 3 2	4
	2 4 2 3 1	2,3,4
2 2 0 2 2	3 3 0 2 2	1
	3 2 1 2 3	1
	2 2 0 3 3	4
	2 3 1 3 2	4
2 1 1 2 3	3 2 1 2 3	1
	3 1 2 2 4	1,3,5
	2 1 1 3 4	4,5
	2 2 0 3 3	4

Table 2:. MPV_5 *of the new networks and components that should be changed*

The minimal number of component that should be improved is 1, and we have two candidates for the new network: to change the first component and to change the fourth component.

Let G_1 be the network obtained from G, such that the first is replaced with a component that have capacity set S1={0, 1, 2, 3} and probability vector $\vec{p}_1$ =(0.1, 0.1, 0.1, 0.7). The reliability of this network is given in the first part of Table 3. Let G_2 be the network obtained from G, such that the fourth component is replaced with a component that have capacity set S_4={0,1,2, 3} and probability vector $\vec{p}_4$=(0.1,0.1,0.1,0.7). The reliability of this network is given in the second part of Table 3. It is obvious that we need to improve the fourth component, because in this case we have greater reliability.

Network G_1					
d	MPV_d	$M2TR_d$	d	MPV_d	$M2TR_d$
1	1 1 0 0 0 0 0 0 1 1 1 0 1 0 1 0 1 1 1 0	0,97848	3	2 2 0 1 1 2 1 1 1 2 1 2 1 2 1 1 1 0 2 2 1 0 1 2 3 2 3 1 1 0 3 2 1 0 1 3 3 0 0 0	0,78008
2	2 1 1 0 1 1 1 0 1 1 1 0 1 1 2 2 2 0 0 0 1 2 1 1 0 0 1 1 2 1 0 0 0 2 2	0,92522	4	2 3 1 2 1 2 2 0 2 2 2 1 1 2 3 3 2 1 1 2 3 3 0 1 1	0,53572
			5	3 3 0 2 2 3 2 1 2 3	0,33936
Network G_2					
d	MPV_d	$M2TR_d$	d	MPV_d	$M2TR_d$
1	1 1 0 0 0 0 0 0 1 1 1 0 1 0 1 0 1 1 1 0	0,97848	3	2 2 0 1 1 2 1 1 1 2 1 2 1 2 1 1 1 0 2 2 1 0 1 2 3 2 3 1 1 0 0 1 1 3 2 0 0 0 3 3	0,78204
2	2 1 1 0 1 1 1 0 1 1 1 0 1 1 2 2 2 0 0 0 1 2 1 1 0 0 1 1 2 1 0 0 0 2 2	0,92522	4	2 3 1 2 1 2 2 0 2 2 2 1 1 2 3 1 2 3 1 2 1 1 0 3 3	0,53754
			5	2 2 0 3 3 2 3 1 3 2	0,34384

Table 3. Reliability of level d=1,2,3,4,5 for networks G_1 and G_2.

5. Conclusion

This paper deals with the problem of two-terminal network reliability. Here, we are concentrating on the problem of improvement of such systems. More concretely, it is analysed how to improve the network to get as greater reliability as possible with minimal cost. In that purpose, we propose some of

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links to be changed with new links that work with greater level. A method for selection of those components is given.

On the other side, we want to use the knowledge of the minimal path vectors of the old network to find the minimal path vectors of the improved one. For that reason, some relations between minimal path vectors of those two networks are analysed and some useful properties are found. Using these properties, the minimal path vectors for the improved network can be found more quickly.

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