Generalized Functions Online Workshop

May 10th, 2024

Book of Abstracts

Elena Cordero, 9:45 – 10:10

Wigner Analysis of Fourier Integral Operators

We perform a Wigner analysis of Fourier integral operators (FIOs), whose main examples are Schrödinger propagators arising from quadratic Hamiltonians with bounded perturbations. The perturbation is given by a pseudodifferential operator $\sigma(x,D)$ with symbol in the Hörmander class $S^0_{\{0,0\}}$. We compute and study the Wigner kernel of these operators, that fall in a more general abstract class of FIOs named FIO(S), with $S \in S_p(d,R)$ the symplectic matrix representing the classical symplectic map. We shall show the algebra and the Wiener's property of this class. The algebra will be the fundamental tool to represent the Wigner kernel of the Schrödinger propagator for every $t \in \mathbb{R}$, also in the caustic points. This outcome underlines the validity of the Wigner analysis for the study of Schrödinger equations.

Olena Atlasiuk, 10:10 – 10:35

The solvability of inhomogeneous boundary-value problems in Sobolev spaces

We develop a general theory of solvability of linear inhomogeneous boundary-value problems for systems of ordinary differential equations of arbitrary order in Sobolev spaces. Boundary conditions are allowed to be overdetermined or underdetermined. They may contain derivatives, of the unknown vector-valued function, whose integer or fractional orders exceed the order of the differential equation. Similar problems arise naturally in various applications. The theory introduces the notion of a rectangular number characteristic matrix of the problem. The index and Fredholm numbers of this matrix coincide, respectively, with the index and Fredholm numbers of the inhomogeneous boundary-value problem. Unlike the index, the Fredholm numbers (i.e., the dimensions of the problem kernel and co-kernel) are unstable even with respect to small (in the norm) finite-dimensional perturbations. We give examples in which the characteristic matrix can be explicitly found. We also prove a limit theorem for a sequence of characteristic matrices. Specifically, it follows from this theorem that the Fredholm numbers of the problems under investigation are semicontinuous in the strong operator topology.

Such a property ceases to be valid in the general case.

Antonella Nastasi, 10:35 – 11:00

Regularity for functionals with non standard growth conditions

The talk shall focus on a class of double phase integrals characterized by nonstandard growth conditions, which constitute an important sub-field of the calculus of variations.

Variational methods are powerful tools in investigating the behaviour and regularity properties of minimizers and, more generally, quasiminimizers that minimize the energy integral up to a multiplicative constant. We study regularity theory, specifically local and global higher integrability, for quasiminimizers of a double phase integral with (p, q)-growth. The proofs follow a variational approach in the setting of metric measure spaces with a doubling measure and a Poincaré inequality. The main feature of the study is an intrinsic approach to double phase Sobolev-Poincaré inequalities. The results are part of a joint paper with Juha Kinnunen (Aalto University) and Cintia Pacchiano Camacho (Calgary University).

Jasmina Veta Buralieva, 11:10 – 11:35

Asymptotic analysis of distributions through some integral transforms and frames

Asymptotic analysis of distributions is a very attractive research field that took attention of different authors. One way to consider the asymptotic behavior of distributions is through generalized integral transforms and frames. Some Abelian- and Tauberian-type results from Buralieva (Funct. Anal. Appl. 57 (1), 38-51, 2023) relating the asymptotic behavior of Stockwell and wavelet transform to the asymptotic behavior of distributions are presented. Also, Abelian- and Tauberian-type results for the quasiasymptotic behavior of tempered distributions through localized frames from Buralieva, Stoeva, Saneva and Atanasova (Women in Analysis and PDE, ed. by M. Chatzakou, M. Ruzhansky, D. Stoeva, 2024) are given.

Jerielle Malonzo, 11:35 – 12:00

Finite Oblique Dual Fusion Frames

A frame for a finite-dimensional Hilbert space H is a collection of vectors that spans H. A fusion frame for H, which is an extension of a frame, is a collection of subspaces of H whose union spans H. Given a signal f in H, fusion frames provide reconstruction formulas via its so-called duals. In some applications, such as in wireless sensor networks, obtaining a consistent reconstruction of a signal requires us to do its analysis and synthesis in different subspaces. For such applications, we use oblique dual fusion frames. We explore finite oblique dual fusion frames with focus on the optimal oblique duals for erasures. We will show that there is always an oblique dual fusion frame that is optimal for erasures, with respect to the mean square error.

Sarita Singh, 12:00 – 12:25

An analyticity approach solution & general discussion about the solution of Navier-Stokes Equations in Fluid Mechanics

The Navier-Stokes equations in fluid mechanics are the most general description of a fluid's mechanical behaviour. Solving these equations requires applying some approximation to reduce their complexity. Numerical methods are primarily used in engineered systems because analytical solutions to the Navier-Stokes equations do not exist.

Among all the different fields of physics and engineering, one of the most mathematically demanding areas is fluid mechanics. The Navier-Stokes equations in fluid mechanics are the foundational equations governing fluid flow and the internal forces that drive fluid motion. Due to their complexity, it is natural to wonder how they can be solved.

The reality is that no analytical solutions exist to the Navier-Stokes equations in their most general form. In other words, you can only get to some kind of analytical solution in certain approximate situations, and the results may not ever be realized in an actual system. More geometrically complex systems will require a numerical approach to get some kind of a solution, which is accomplished with CFD simulations.

Panel discussion on the Challenges in PhD studies

13:40-14:40

In this panel we will discuss what challenges PhD students and PhD advisors face during the studies.

First we will exchange information about the procedures for PhD studies in different countries, the study-periods, requirements for the completion of a PhD degree, gender issues, etc., and then we will share experience on challenging points.

The panel will be led by Doz. Dr. Diana Stoeva, University of Vienna.

Davorka Radaković, 15:15 – 15:30

Why are female pupils losing interest in STEM fields in high school?

Identifying the reasons why female pupils lost interest in STEM fields in high school has long been of interest to both: researchers and educators. There are significant gender differences in choices for STEM occupations. This can be significantly noticed in professions related to informatics and computer science.

We examine the first- and second- year gifted mathematicians in Mathematical Grammar School. Regardless of extensive coverage of STEM subjects in this special high school, a small number of female pupils decide to participate in competitions in STEM subjects. We present the reasons given by female pupils why they do not participate in competitions even though they have equal knowledge.

Therefore, we advise encouraging more female pupils to participate in STEM competitions. We think that this study can provide a useful contribution to equalize the participation of male and female pupils in STEM competitions.

Irina Melnikova, 15:30 – 15:55

Stochastic models of financial mathematics: transition from discrete to continuous infinite-dimensional models

Stochastic models are models taking into account random disturbances. Such models allow to describe a wide class of processes arising in various fields of natural science, economics and social phenomena. Recently, great interest in the problems of financial mathematics has led to significant advances in the study of discrete and continuous models in the finite-dimensional case and somewhat less significant advances in the infinite-dimensional case.

The talk is devoted to connections, firstly, between discrete models of financial mathematics (specifically, n-period binomial) and finite-dimensional models in continuous time. They are models with continuous randomness of Brownian motion type, obtained as n tends to infinity in binomial models, and jump-like ones of Poisson processes type. Specifically, we established analogues in the formulas obtained for models of underlying assets and for models of options that belong to one of the most important classes of derivatives. Secondly, the connections between finite-dimensional and infinite-dimensional models are studied.

In the infinite-dimensional case, along with models of the prices of underlying assets and options under the influence of random disturbances infinite-dimensional analogues such as Brownian motion and Poisson processes, we consider models of important economic processes of another type. They are zero-coupon bonds $P(t,T)$, where $t \geq 0$ is current time and $T > t$ is bond payment time.

In our case of constructing a stochastic equation for bond price, we first consider the stochastic Heath-Jarrow-Morton equation for the bank forward rate $f(t)$ at the current time $t \leq T$ for a fixed T, then we obtain an infinite-dimensional equation for the bank forward rate $f(t,T)$ for variable t and T. Finally, thanks to the known connection between the bank forward rate and the bond price, we obtain an infinite-dimensional equation for the bond price $P(t,T)$.

In contrast to other infinite-dimensional models, interesting features of the design of such model for bond prices are shown. When constructing, for example, stochastic models of population size $u(t,x)$, where $t \geq 0$ is time and $x \geq 0$ is age, or of some substance diffusion oscillations $u(t,x)$, where t is time and x is spatial coordinate, both described in the deterministic case by two-dimensional partial differential equations for their $u(t,x)$, one studies the design of stochastic perturbations. Stochastic perturbations, as shown in these models, can be described by a Brownian sheet and corresponding infinite-dimensional stochastic equations for $u(t,x)$ are obtained for any fixed $t \geq 0$ with values in the space of functions depending on the spatial variable x. For the bond price infinite-dimensional stochastic equation is obtained from the finite-dimensional stochastic equation for the forward rate.

Stefan Tošić, 15:55 – 16:20

A fractional stochastic model for aerosol transmission of fluid droplets and virus exposure in closed space

Considering fractional Brownian motion and fractional white noise as a generalized stochastic processes in the framework of white noise analysis, we use them to model aerosol transmission of fluid droplets and virus exposure in closed space. The model is based on an airflow produced during coughing or sneezing governed by the incompressible Navier-Stokes equation, leading to the expulsion of contaminated aerosols that diffuse in a closed room and are subjected to random movements due to collision with other particles in the air. The proposed model involves stochastic components to grasp the uncertain nature of the aerosol diffusion and fractional derivatives to grasp the possibilities of a sub-diffusion or super-diffusion effect due to various physical conditions in the room. We prove existence and uniqueness of the solution to the proposed model, supported by numerical simulations and experiments.

Bakhyt Alipova, 16:20 – 16:45

Generalized functions formulation by solution of the boundary value problem of thermoelastodynamics of half-plane

The theory of generalized functions is used for the analytical solution for arbitrary surface forces and heat flow in the boundary value problem (BVP) of dynamics of a thermoelastic half-space under periodic on time surface forces and heat flows. Such kind of model could be named as the model of coupled thermoelasticity. The Green's tensor for each BVP (surface forces, heat flow, displacements, temperature) is constructed utilizing Fourier transformation. To solve this BVPs, generalized functions theory, tensor and differential algebra, operator method and integral transformations were used.