ANALYTICAL ESTIMATION OF OPTIMAL PV PANEL TILT BASED ON CLEAR-SKY IRRADIANCE MODEL

***Elena Golubovska*[1], *Biljana Citkuseva Dimitrovska*[2], *Roman Golubovski*[3]**

[1]Skopje R.Macedonia, email: *elena.golubovska@gmail.com*

[2]Faculty of Electrical Engineering University Goce Delcev Stip R.Macedonia, email: *biljana.citkuseva@ugd.edu.mk*

[3]Faculty of Natural Sciences and Mathematics University Ss. Cyril and Methodious Skopje R.Macedonia, email: *roman@pmf.ukim.mk*

Abstract

PV panel tilt and sun tracking is crucial aspect for PV conversion efficiency. We propose an analytical methodology for estimation of optimal PV panel tilt based on calculation of the sun position and application of a clear-sky solar irradiance model. Our model outputs three angles referencing a geolocation and the moment of interest: the incidence angle θ, the sun altitude α and the sun azimuth z. The irradiance model estimates the solar irradiation on a geolocation that can be used for PV conversion estimation based on specified tilt β. The moment PV power is used for calculation of the daily energy production, and the optimal β is identified in the tilt range of $0°$ to $90°$. Seasonal division of the year is performed and optimal seasonal tilt is estimated based on the maximal produced seasonal energy tested with every corresponding β. The methodology is tested on four typical seasonal models - 12 months, 4 three-month quarters, 2 half-year seasons and single optimal annual fixed β. Preliminary simulations produce promising results consistent with the practical engineering implementations.

Key words

PV panel tilt, optimal PV panel inclination, PV conversion efficiency, sun position model, clear-sky solar irradiance model.

Introduction

Renewable energies are subject to continuous research for their sustainability opposite to the depletive and hazardous character of the fossil fuels and nuclear fission. Solar energy is obviously the most sustainable form, independent of other circumstances until it reaches the atmosphere and acceptably degraded while propagating through it. Solar irradiation is efficiently used by photovoltaic (PV) conversion which is the cheapest electrical energy production technology compared to the rest. PV technologies are also affordable on the household level making them globally popular today. The widespread market sustains a growing PV production industry that continuously increases the PV conversion efficiency and lowers the costs.

However, besides the improved material performances, planners of PV plants also tackle installation efficiency issues for maximizing energy production against lower costs. Among other things they aim to optimal latitude (L) placement as well as optimal panel tilt (β) for sun incidence angle (θ) closest to the zenith possible, and longer during daylight possible. The incidence angle θ can be maintained optimal by horizontal azimuth tracking, but the panel azimuth is usually fixed toward local noon (1200h). The panel tilt optimization is subject to vertical inclination adjustment strategies from fixed tilt throughout the year to daily tracking (involving use of computers equipped with sensors and actuators introducing additional costs as hardware, cabling, maintenance and energy consumption), depending on the economic circumstances.

This paper proposes analytical methodology for estimation of optimal PV panel tilt based on estimated sun position defined with the incidence angle θ, the sun altitude α and its azimuth z, and application of a clear-sky solar irradiance model. The sun location is determined [1] against specified geolocation for a specified date and time. The solar irradiance model [2] calculates the maximal possible incoming solar irradiatiation that can be used for PV conversion estimation considering the latitude placement and sun position, as well as the panel tilt β. This allows for moment power estimation and possible energy production over specified period. The methodology allows tracking optimal β on daily basis (the fixed tilt for which maximal daily energy can be produced), or for arbitrary defined seasons (the fixed tilt for which maximal seasonal energy can be produced). This approach is tested on four typical seasonal models - 12 months, 4 three-month quarters, 2 half-year seasons and single optimal annual fixed β.

1. Sun position model

The current sun position model calculates the incidence angle θ that the sunrays fall under on a specific geolocation (latitude and longitude) in a specific moment of the year (date and time) - the angle between the sunray falling on that location and its perpendicular vertical line, as well as the seasonal sun altitude α and the daily azimuth z.

In order to calculate these three essential angles, additional specifics regarding the earth's rotations need to be considered. Figure 1 shows the earths annual (365.25 days) rotation around the sun, as well as the fixed declination δ of earth's axis (23.45°) which oscillates with respect to the sun, producing on earth solar declination angle between ±23.45° depending on the moment in the year.

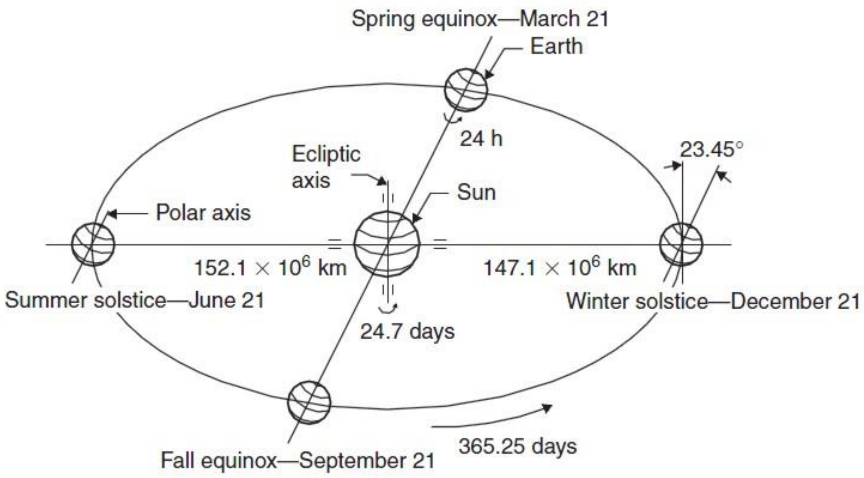


Fig. 1. Annual motion of the earth around the sun
Source: ?????

There are four fiducial dates during the year - 21 Jan (the summer solstice) which is the longest (summer) day on the north hemisphere; 21 Dec (the winter solstice) which is the shortest (winter) day on the north hemisphere; and 21 Mar and 21 Sep (the two equinoxes) with equal duration of their day and night. The daily declination δ for day N (of the 365 in a year) according to ASHRAE (the American Society of Heating, Refrigerating and Air-Conditioning Engineers) can be calculated with the expression 1.1:

$$\delta = 23.45° \, sin\left[\frac{360°}{365}(N - 81)\right] \qquad [°] \tag{1.1}$$

The specific moment is defined by the input date and time, expressed as the day N [1~365] in the year and the local standard time (LST) expressed as local minute time (LMT) [min] in that day. The moment needs to be converted to a current angle with respect to a reference meridian. This means that the specified moment of time needs to be converted from LST to the local apparent solar time (AST), and than to the hour angle h representing that moment.

In the local noon LST should be exactly 1200h (midday), and so should AST correspond to the solar zenith (1200h). However, during the annual rotation around the sun earth's path varies, so does its speed around the sun. Due to some specifics, the rotational speed around its own axis also varies. These variations imply two corrections required for acceptable LST to AST conversion. First one is the "equation of time" (EoT) which considers the eccentricity of the earth's orbit around the sun, and for day N is determined with the expression 1.2:

$$EoT = 9.87\sin(2B) - 7.53\cos(B) - 1.5\sin(B) \qquad [min] \tag{1.2}$$

$$B = (N-1)\frac{360°}{364} \qquad [°]$$

The second correction is the "longitudinal correction" of LST, given in expression 1.3 which represent current time with respect to a global time zone (T_GMT) or a standard meridian (SM) of the 24 defined for zoning the globe. It takes 4min for the sun to traverse 1°, and LST is "constant" in the watch for whole 15° (representing an hour from 24 zones). The correction is intended to cancel out the running difference between the local longitude (LL) and the SM. Additionally can the daylight saving (DS) be considered, expressed in [min] (being 0 when ignored, and 60 otherwise).

$$AST = LST + EoT \pm 4(SL - LL) - DS \qquad [°] \tag{1.3}$$

Now, the current hour angle h considering the current AST is given in the expression 1.4:

$$h = (AST - 12)\cdot 15° \qquad [°] \tag{1.4}$$

Having h correspond to the specified date and time, it can be put in the context of the corresponding latitude L, solar declination δ, local zenith Φ, sun altitude α and sun azimuth z. Figure 2 and figure 3 depict the context in which the angle parameters correlate.

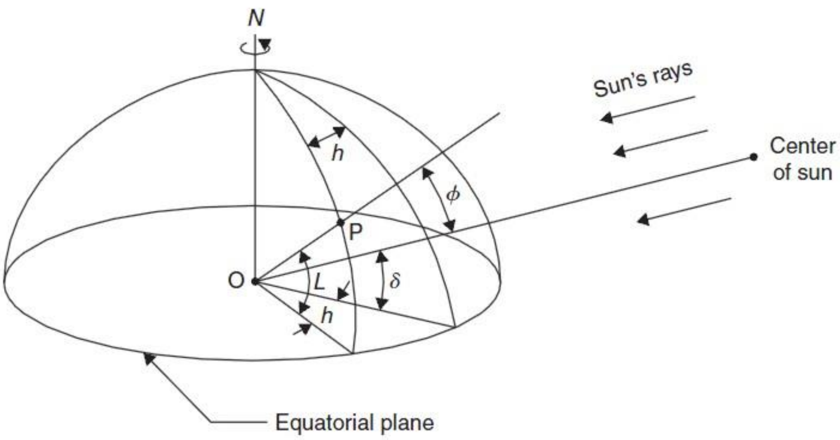


Fig. 2. Definition of latitude, hour angle, and solar declination
Source: ?????

Sun altitude α oscillates with seasonal dynamics and can be correlated trigonometrically with the local latitude L, the solar declination δ and the hour angle h in expression 1.5:

$$\alpha = sin^{-1}[sin(L)sin(\delta) + cos(L)cos(\delta)cos(h)] \qquad [°] \tag{1.5}$$

Sun azimuth z oscillates with daily dynamics and can be correlated trigonometrically with the sun altitude α, the solar declination δ and the hour angle h in expression 1.6:

$$z = sin^{-1}\left[\frac{cos(\delta)sin(h)}{cos(\alpha)}\right] \qquad [°] \tag{1.6}$$

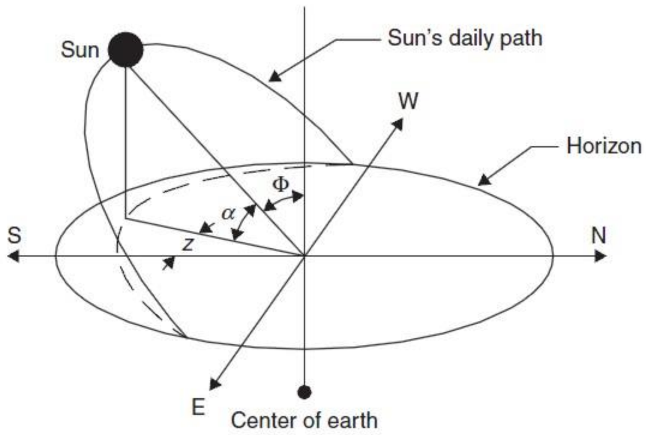


Fig. 3. Solar angles defining its position on the sky along its daily path
Source: ?????

Figure 4 defines the trigonometrical context of inclined plane.

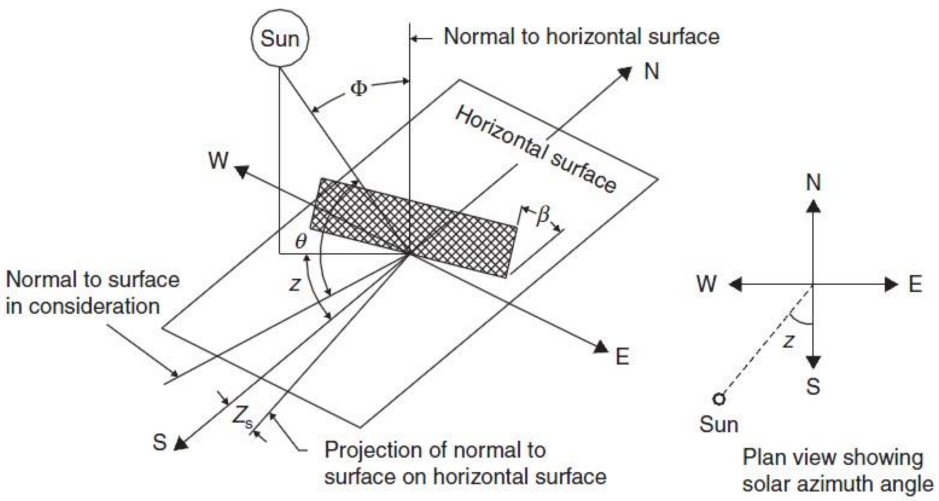


Fig. 4. Solar incidence angle for a non-horizontal surface
Source: ?????

The incident angle θ equals the zenith angle Φ if the plane is not tilted, but for panel with tilt β > 0° and own azimuth fixed toward local noon (0°), expression 1.7 gives the cos(θ) required by the PV conversion model.

$$\begin{aligned} cos(\theta) = {} & sin(L)sin(\delta)cos(\beta) - cos(L)sin(\delta)sin(\beta)cos(Z) + \\ & + cos(L)cos(\delta)cos(h)cos(\beta) + \\ & + sin(L)cos(\delta)cos(h)sin(\beta)cos(Z) + cos(\delta)sin(h)sin(\beta)sin(Z) \end{aligned} \tag{1.7}$$

2. Energy conversion model

The used PV energy conversion model is of "clear-sky" type [2]. The clear-sky solar irradiation model assumes ideal meteorological conditions, where the irradiation of the location of interest is unobstructed with clouds or other atmospheric circumstances. Although not real, such ideal context is optimal for comparative analysis.

The extraterrestrial solar radiation arrives at the outer border of earth's atmosphere in almost constant intensity, defined as the solar constant G_{SC}. The solar constant is the quantity of solar radiation arriving perpendicular at the atmosphere, with very small variations among different analytical estimations and satellite measurements. The value used in this paper is 1367 [W/m^2] as proposed by Iqbal (1983).

However, sun's radiation declines intrinsically with an annual rate of 0.02% (insignificant and can be disregarded), and varies due to earth's variable distance from the sun which results in ± 3.3% which is significant and must be taken into consideration. This results in the effective arrived radiation at the atmosphere G_{ATM}, expressed for day N of the year according to Spencer (1971) with the following formula:

$$G_{ATM} = G_{SC}\left[1 + 0.033\, cos\left(\frac{360°}{365}N\right)\right] \qquad [W/m^2] \tag{2.1}$$

The direct radiation (beam radiation) G_{DIR} is the amount of irradiation propagated at the location of interest through the atmosphere without dissipation. Diffuse radiation (sky radiation) G_{DIF} is the dissipated irradiation arriving at the location of interest from the surrounding space. The total irradiation G is the amount measured at the location of interest.

The energy model is expected to provide analytical estimation of the PV conversion power, considering the direct and diffuse irradiation, and ignoring the reflection component. The PV conversion efficiency η is assumed up to 20% as the commercial PV technologies currently offer. The expression for the PV converted power at the location of interest is:

$$P = \eta\,[(G_{DIR} + G_{DIF})\, S_{DIR} + G_{DIF}\, S_{DIF}] \qquad [W] \tag{2.2}$$

G_{DIR} and G_{DIF} result from the interaction of G_{ATM} with the atmosphere (its vapor molecules and micro particles) while propagating and dissipating through it. This interaction is defined as atmospheric transmittance τ. The transmittance of direct irradiation τ_{DIR} is proposed by Hottel (1976) in the following expression:

$$\tau_{DIR} = a_0\, r_0 + a_1\, r_1\, e^{-\frac{k \cdot r_k}{cos(\Phi)}} \tag{2.3}$$

Where Φ is the zenith angle (90° - α), and a_0, a_1 and k are the atmospheric parameters of a clear sky with visibility up to 23 km, and for altitudes (A) of up to 2.5 km, given in expression 2.4:

$$a_0 = 0.4237 - 0.00821\,(6 - A)^2 \tag{2.4}$$

$$a_1 = 0.5055 - 0.00595\,(6.5 - A)^2$$

$$k = 0.2711 - 0.01858\,(2 - A)^2$$

And where r_0, r_1 and r_k are corrective climate factors declared in table 1:

Table 1 Corrective climate factors

Климатски тип	r_0	r_1	r_k
Tropical latitudes (0° ≤ L < 23.45°)	0.95	0.98	1.02
Mid latitudes (23.45° ≤ L < 66.55°) in summer	0.97	0.99	1.02
Mid latitudes (23.45° ≤ L < 66.55°) in winter	1.03	1.01	1.00
Polar latitudes (66.55° ≤ L ≤ 90°) in summer (durring daylight)	0.99	0.99	1.01

According to the same model the transmittance of diffuse radiation τ_{DIF} is given in expression 2.5:

$$\tau_{DIF} = 0.271 - 0.294\ \tau_{DIR} \qquad (2.5)$$

Knowing both transmittances τ_{DIR} and τ_{DIF} allows calculation of the direct and diffuse irradiation that fall perpendicular at the location of interest $G_{DIR(n)}$ and $G_{DIF(n)}$ as given in expression 2.6:

$$G_{DIR(n)} = \tau_{DIR}\ G_{ATM} \qquad [W/m^2] \qquad (2.6)$$

$$G_{DIF(n)} = \tau_{DIF}\ G_{ATM} \qquad [W/m^2]$$

Knowing the tilt β, and having calculated cos(θ) it is now possible to correct both irradiation components in their final form:

$$G_{DIR} = G_{DIR(n)}\ cos(\theta) = G_{ATM}\ \tau_{DIR}\ cos(\theta) \qquad [W/m^2] \qquad (2.7)$$

$$G_{DIF} = G_{DIF(n)}\ \frac{1 + cos(\beta)}{2} = G_{ATM}\ \tau_{DIF}\ \frac{1 + cos(\beta)}{2} \qquad [W/m^2] \qquad (2.8)$$

In expression 2.5, it is obvious that the diffuse component is neglective compared to the direct, so the final expression for the PV conversion power is:

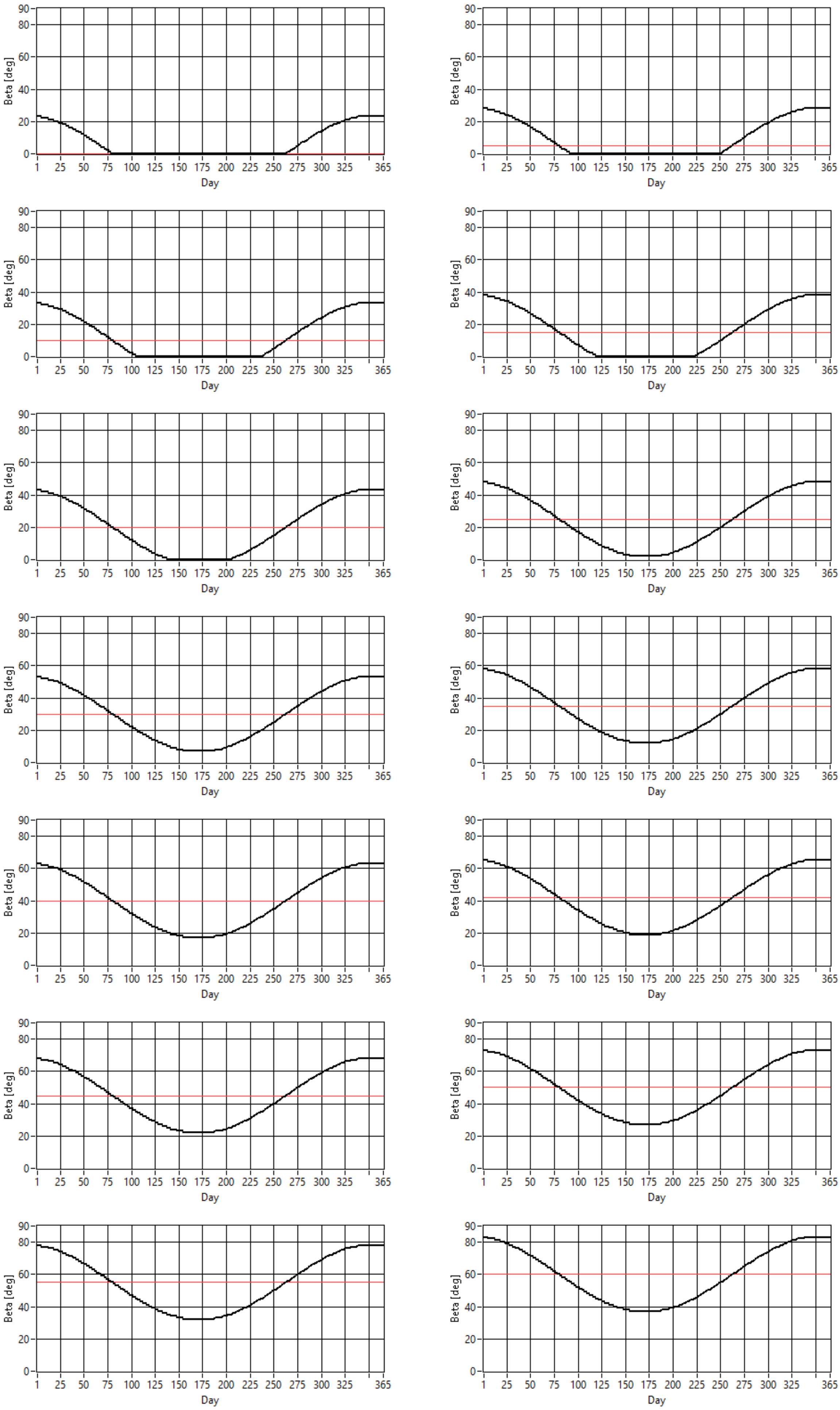
$$P = \eta\,(G_{DIR}\ S_{DIR} + G_{DIF}\ S_{DIF}) \qquad [W] \qquad (2.9)$$

Tracing expression 2.9 backwards, it is clear that PV conversion power P at specified geolocation can be estimated with the clear-sky model and the sun position model for a specified moment in the year by just knowing date and time.

3. Optimal tilt calculation

Optimal panel inclination is the tilt β for which the panel produces maximal daily energy DE_{MAX}, calculated by integrating its power P in 15min intervals during the daylight of specified date (N). The algorithm for optimal tilt determination checks (calculates) DE against "all" β values in the range of 0° to 90° with step of 1°. The tilt that produces DE_{MAX} is the optimal daily tilt β_{OD}.

Figure 5 provides the optimal β curves for "all" latitudes in the north hemisphere from 0° (the equator) to 90° (the north pole) with step 5°. Every graph corresponds to a latitude labeled with red horizontal line. Added is latitude of Skopje (42°).


Beta [deg]
0 20 40 60 80 90
Day
1 25 50 75 100 125 150 175 200 225 250 275 300 325 365

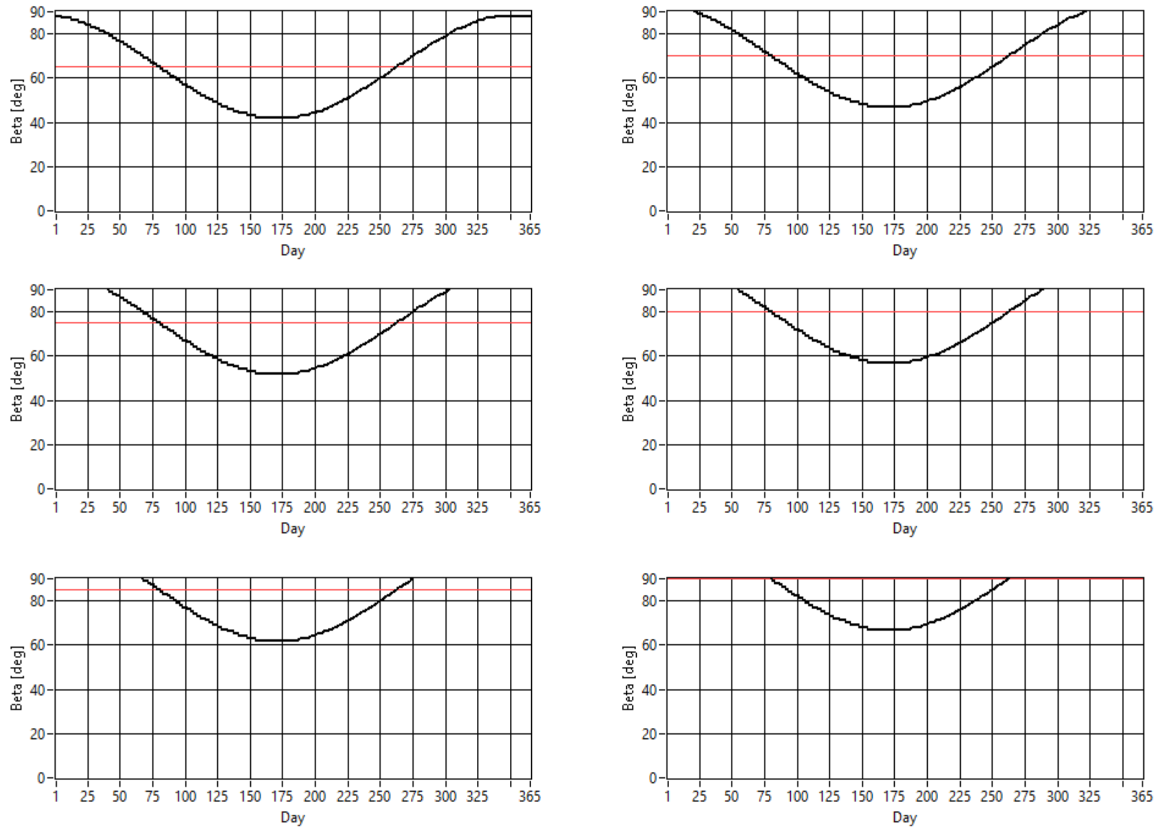


Fig. 5. Annual tracking of daily optimal tilt for all latitudes from 0° to 90° with step 5° (including 42°)

It is obvious that the tropical curves (0° to 23.45°) are "cut below" at 0° since during summer in the north hemisphere, these latitudes are closer to zenith than the equator, and the panels are optimally laid on the ground and cannot tilt down any more. In the mid latitudes (from 23.45° to 66.55°) obvious is the optimal β oscillation of ± 23.45° due to the corresponding oscillation of the solar declination δ. The polar latitudes (above 66.55°) are in a daylight half the year also due to δ.

Daily tracking (especially for large PV plants) requires use of computerized equipment (for automatic tilting) that may not always be economically feasible, so the most widespread strategy is seasonal optimization. Being able to calculate optimal tilt β_{OD} for daily tracking allows determination of seasonal optimal tilt β_{OS}. After an arbitrary season is defined with starting and ending date, cumulative seasonal energy SE is calculated for the whole season for every β_{OD} of the embraced dates. When SE_{MAX} is identified its corresponding β_{OD} is the optimal seasonal tilt β_{OS}.

The methodology is tested on 4 scenarios:

1. Monthly tilting - 12 seasons with optimal tilt

2. Four seasons - defined to have least possible β_{OD} variation among the embraced days
   S#1 → 5 Nov ~ 4 Feb (winter, 92 days)
   S#2 → 5 Feb ~ 6 May (spring, 91 days)
   S#3 → 7 May ~ 5 Aug (summer, 92 days)
   S#4 → 6 Aug ~ 4 Nov (autumn, 90 days)

3. Two seasons - defined to have least possible β_{OD} variation among the embraced days
   H#1 → 21 Sep ~ 20 Mar (winter, 181 days, 21 Dec - centered)
   H#2 → 21 Mar ~ 20 Sep (summer, 184 days, 21 Jun - centered)

4. Fixed annual tilt

Table 2 shows the results of the simulation of the 4 scenarios:

Table 2 Seasonal tilt optimization

Lat	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	S#1	S#2	S#3	S#4	H#1	H#2	365
0°	23	18	4	0	0	0	0	0	0	14	22	23	23	0	0	0	21	0	1
5°	28	23	8	0	0	0	0	0	2	19	27	28	28	5	0	5	26	0	5
10°	33	28	13	0	0	0	0	0	7	24	32	33	33	10	0	10	31	0	10
15°	38	33	18	1	0	0	0	0	12	29	37	38	38	15	0	15	35	0	14
20°	43	38	23	6	0	0	0	2	17	34	42	43	43	20	0	20	40	0	19
25°	48	43	28	11	3	2	2	7	25	38	47	48	48	26	2	25	45	3	25
30°	53	48	33	16	8	7	7	12	30	43	52	53	53	31	7	30	49	7	30
35°	58	52	38	21	13	12	12	17	35	48	57	58	58	36	12	35	54	12	35
40°	63	57	43	26	18	17	17	22	40	53	62	63	63	41	17	40	58	17	40
45°	68	62	48	31	23	22	22	27	45	58	67	68	68	46	22	45	63	22	45
50°	73	66	53	36	28	27	27	32	50	62	72	73	73	51	27	50	67	27	50
55°	78	71	58	41	33	32	32	37	55	67	77	78	78	56	32	55	71	32	55
60°	83	76	63	46	38	37	37	42	60	72	81	83	83	61	37	60	75	37	60
65°	87	80	68	51	43	42	42	47	65	76	85	88	87	66	42	65	78	42	65
70°	89	84	72	55	48	47	47	52	67	81	88	100	89	65	47	65	80	47	54
75°		87	77	60	53	52	52	57	71	85				68	52	68	82	52	56
80°		89	81	65	58	57	57	62	76	87				70	57	70	85	57	58
85°			84	70	63	62	62	67	81	90				72	62	72	88	62	62
90°			87	75	68	67	67	72	83					74	67	75		67	67

The monthly tilting (scenario #1) for all latitudes is graphically presented on figure 6:

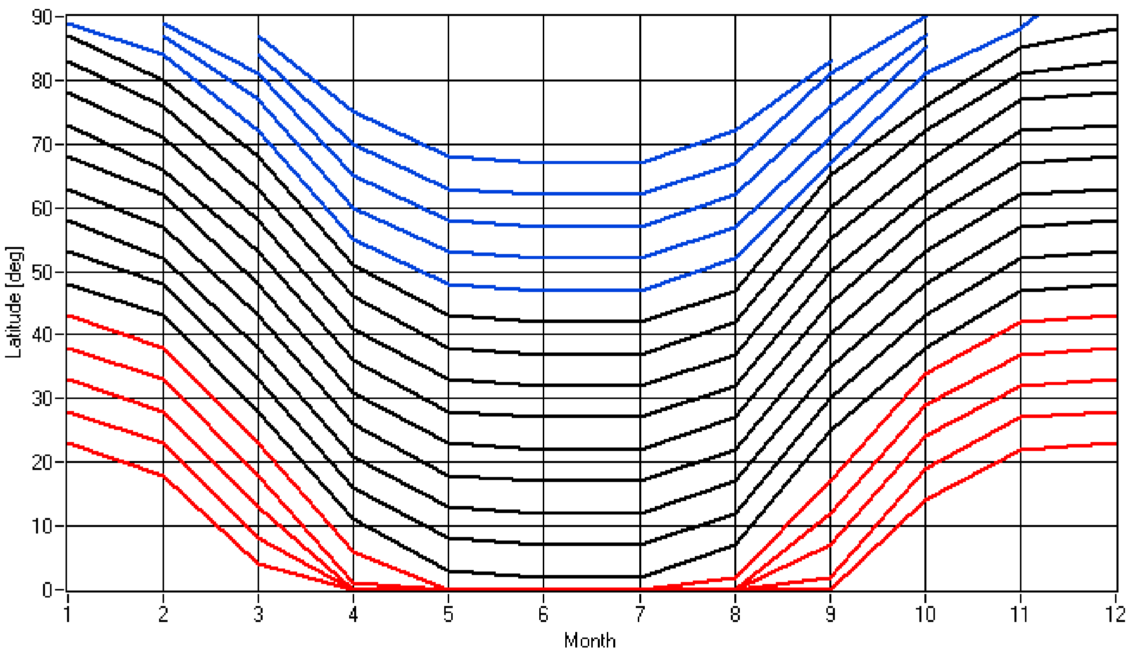


Fig. 6. Annual tracking of daily

The mid latitudes β_{OS} curves show same oscillation of ± 23.45° corresponding to the annual solar declination (δ) change. It is also obvious that fixed annual β_{OS} for all tropical and mid latitudes equals the corresponding latitude which confirms the standard engineering recommendation and practice.

Conclusions

The proposed concept for analytical estimation of optimal PV panel tilt based on sun position and clear-sky irradiance models provides a reliable methodology for daily tracking and arbitrary seasonal tilt optimization. The simulation results are consistent with the common engineering practice. The algorithm is precise and easy to implement, thus providing a "cheap" and straight-forward to use tool for that purpose.

References

[1] Kalogirou, S. A.: Solar Energy Engineering Process and Systems. (Elsevier, 2009).

[2] Duffie, J. A., Beckman, W. A.: Solar Engineering of Thermal Processes, Fourth Edition. (John Wiley and Sons, 2013).

[3] Mehleri, E.D., Zervas, P.L., Sarimveis, H., Palyvos, J.A., Markatos, N.C.: “Determination of the optimal tilt angle and orientation for solar photovoltaic arrays”. *Renewable Energy*, 35 (11), (Elsevier, 2010), pp. 2468-2475.

[4] Tang, R., Wu, T.: “Optimal tilt-angles for solar collectors used in China”. *Applied Energy*, 79 (3), (Elsevier, 2004), pp. 239-248.

[5] Yakup, M., Malik, A.Q.: “Optimum tilt angle and orientation for solar collector in Brunei Darussalam”. *Renewable Energy*, 24 (2), (Elsevier 2001), pp. 223-234.