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OPTIMAL TAXATION POLICY, MONETARY POLICY AND STATE-CONTINGENT DEBT, TIME INCONSISTENCY IN RAMSEY PROBLEM, TAX SMOOTHING, NON-CRRA PREFERENCES AND TAXATION IN LQ ECONOMY

Dushko Josheski[1*], Natasha Miteva[2], Tatjana Boshkov[3]
[1]Associate professor Faculty of Tourism and Business Logistics, Goce Delcev University, Stip, North Macedonia; e-mail: dusko.josevski@ugd.edu.mk ; ORCID: 0000-0002-7771-7910
[3]Assistant professor, Faculty of Tourism and Business Logistics, Goce Delcev University, Stip, North Macedonia, natasa.miteva@ugd.edu.mk ; ORCID: 0009-0004-8082-6458
[2]Associate professor, Goce Delcev University of Stip, Faculty of tourism and business logistics, R.North Macedonia, e-mail: tatjana.dzaleva@ugd.edu.mk ;ORCID: 0000-0001-8821-9223

**Corresponding author: dusko.josevski@ugd.edu.mk*

Abstract

This paper illustrates optimal fiscal and monetary policies with state-contingent debt as in Lucas,Stokey (1983) and the issue of time inconsistency in the Ramsey problem, tax smoothing as in Barro (1979) and Tax smoothing and Ramsey time inconsistency and non-CRRA preferences and taxation in LQ economy. Results show how the government lowers the interest rate by raising consumption. In the case of fall of consumption (in case of shock), labor supply increases during this two-time period tax rate increases for six periods, government consumption and output increase for two periods. Results differ from the results for LQ economy. When a state variable is negative, optimal tax is positive (obviously state variable here can be interest rate), and when there is positive state variable optimal tax rate becomes negative In LQ economy interest rate and inflation rate respond differently to technology and government consumption shocks respectively.

Keywords: *optimal fiscal policy, optimal monetary policy, state- contingent debt, time inconsistency, Ramsey problem, LQ economy*

JEL codes: E00,E62,H21

Introduction

A fundamental question in modern macroeconomics is how should the fiscal and monetary policy be conducted over the business cycle[1]? And In the tradition of neoclassical economics on optimal fiscal policy of Ramsey (1927), Barro (1979), and Lucas and Stokey (1983),it has been emphasized that, when taxation is distortionary, societal welfare is being maximized if the government smoothes taxes across different period of time and different realizations of uncertainty. If government can trade Arrow-Debreu securities (state-contingent debt) perfect smoothing across all dates and states is possible, implying that the optimal tax rate is invariant ,see Lucas and Stokey (1983), Chari, Christiano,Kehoe (1991), and Benigno, Woodford (2003). The optimal fiscal policy follows Ramsey (1927) who considers choosing optimal tax structure only when distorting taxes are available, and the business cycle theory stems from Kydland, Prescott (1982), and Long,Plosser (1983), who analyzed quantitative role of shocks to the technology and government consumption in generating fluctuations to output and unemployment. So, an optimal policy in this sense is one which maximizes the

[1] Major conclusions in this area of research suggests: tax rates on labor are constant over business cycle, zero-capital taxes, Friedman rule (see Friedman (1969)) is optimal here nominal interest rates are around zero, monetary policy is countercyclical with respect to technology shocks and procyclical with respect to government consumption.

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welfare of representative agent subject to the constraint that the resulting outcomes constitute competitive equilibrium. In terms of fiscal policy optimal taxes should smooth distortion of taxes across time, which in turn means running surplus in good times and deficit in bad times. Smoothing properties of taxes means that taxes on labor and consumption should be constant. While taxes on capital should be close to zero on average, which is reminiscent to Judd (1985) and Chamley (1986) result, see also Angeletos, G.-M. (2003). Benigno and Woodford (2003),seek to offer an integrated analysis of optimal fiscal and monetary policy building on two branches of the literature. The first is dynamic optimal taxation stemming from Lucas,Stokey (1983), Dynamic taxation most famous example in the literature are: Diamond-Mirrlees (1978); Albanesi-Sleet (2006), Shimer-Werning (2008), Ales-Maziero (2008), Golosov-Troshkin-Tsyvinsky (2011).A sizeable literature in NDPF(New Dynamic Public Finance) studies optimal taxation in dynamic settings, see also(Golosov, Kocherlakota, and Tsyvinski (2003), Golosov,Tsyvinski, and Werning (2006), Kocherlakota (2010). The second part of the literature is on optimal monetary stabilization policy, see Goodfriend and King (1997), Rotemberg and Woodford (1997) and Woodford (2000).The approach by Benigno and Woodford (2003), models the price stickiness as per Calvo (1983)[2].Both previous areas of literature are assuming benevolent government which seeks to stabilize the response of economic outcomes to exogenous shocks with a combination of fiscal and monetary policies chosen once and for all at some previous date. In the optimal taxation literature fiscal shocks are fluctuations in government expenditure, and this rules out lump-sum taxes such as in Ramsey (1927).Distortionary taxes create wedge between marginal rates of transformation and marginal rates of substitution[3],and government policy becomes a source of frictions. The monetary stabilization literature considers environments where frictions are present even without government policy. These frictions are due to nominal rigidities and imperfect competition in product or labor market. The corresponding wedges reduce the level of economic activity and may be subject to stochastic fluctuations, known as cost-push shocks[4]. The government's only fiscal policy instrument is a lump-sum tax. Lucas and Stokey show that it is optimal to respond to fiscal shocks by appropriately setting the state contingent returns on government debt. Taxes and real returns on government debt inherited serial correlation structure of underlying shocks, and taxes are smooth, in the sense of having a small variance relative to fiscal shocks, see Albanesi, (2003). Chari, Christiano, and Kehoe (1991), Chari et al.(1995) extend the analysis to monetary economies with risk-free debt and show that it is optimal to use state-contingent inflation as a fiscal shock absorber. They find that the standard
deviation of optimal taxes is close to zero, while real returns on government debt are highly volatile for calibrated examples. In the monetary stabilization literature, rigidities in nominal

[2] This price stickiness has been used in papers with micro foundations, see Goodfriend and King(1997); Clarida et al.(1999); Woodford, (2003).

[3] p_x, p_y are prices before taxation τ is tax rate. Now $p_{x(aftertax)} = (1+\tau)p_x$, slope of budget constraint after tax is: $slope\ of\ budget\ constraint = -\frac{p_{x(aftertax)}}{p_y} = -\frac{(1+\tau)p_x}{p_y}$. $MRT = \frac{MC_x}{MC_y}$; $MC_{aftertax} = \frac{MC_{aftertax}}{MC_y} = \frac{(1+\tau)MC_x}{MC_y}$.Now the wedge is :$tax\ wedge = \frac{MRT_{aftertax}}{MRS} = \frac{\frac{(1+\tau)MC_x}{MC_y}}{unchanged\ MRS}$

[4] SRAS is given as: $p = p_e + \alpha(Y - Y^*); p = p_e + \alpha(Y - Y^*) + \Delta SRAS$. Now, in the short run, the economy might not immediately adjust to the shock, so the output level might temporarily remain below the potential output. This causes $Y < Y^*$, resulting in a positive output gap. Since $AD = AS \Rightarrow Y = Y^* - \beta(p - p_e)$, where β represents the responsiveness of aggregate demand to changes in the price level. The cost-push shock increases production costs, leading to a higher price level. We can solve for the new equilibrium price level by substituting the new SRAS curve into the equilibrium equation:$p = p_e + \alpha(Y - Y^*) + \Delta SRAS$; $Y = Y^* - \beta(p - p_e) + \Delta SRAS$.

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prices and wages imply that innovations in inflation reduce the average markups and increase equilibrium output. The presence of nominal rigidities imply that inflation generates relative price distortions. The resulting trade-off between inflation and output stabilization implies that the volatility and persistence of optimal inflation will depend on the stochastic properties of the cost-push disturbances and on the degree of nominal rigidities, see Albanesi, (2003). In this paper we will revie competitive equilibrium with distorting taxes first,implmentability constraint, the issue of time inconsistency in Ramsey problem, model specification with CRRA , Recursive formulation of Ramsey problem, intertemporal delegation and two Bellman equations example with one period anticipated war, Tax smoothing and Ramsey time inconsistency and non-CRRA preferences and taxation in LQ economy.

Competitive equilibrium with distorting taxes

First here we will describe economy such as in Lucas and Stokey (1983).This model revisits classic issue of how to pay for a war. Now let's turn out explanation to a competitive equilibrium with distorting taxes. At time $t \geq 0$ some random variable s_t belongs to time invariant set :

equation 1

$$\mathcal{S} = [1,2, \dots, S]$$

For a history $s^t = [s_t, s_{t-1}, \dots, s_0], \forall t \geq 0$ of an endogenous state s_t has joint PDF $\pi_t(s^t)$.Assuption here is that government purchases $g_t(s^t) \forall t \geq 0$ depend on s^t.Now, let $c_t(s^t), n_t(s^t), l_t(s^t)$ denote consumption, leisure and labor supply at history s^t and date t . Now,

equation 2

$$n_t(s^t) + l_t(s^t) = 1$$

And:

equation 3

$$c_t(s^t) + g_t(s^t) = n_t(s^t)$$

Representative household preferences over $\{c_t(s^t), l_t(s^t)\}_0^\infty$ are given as:

equation 4

$$\sum_{t=0}^{\infty} \sum_{s_t^{\prime}} \beta^t \pi_t(s^t) u[c_t(s^t), l_t(s^t)]$$

Now the government imposes flat tax rate $\tau_t(s^t)$ on labor income at time t and history s^t . There are complete markets in one period Arrow securities[5]. Where one unit of Arrow

[5] See Arrow (1951) and Arrow,Debreu (1954),who proved that competitive equilibrium in Arrow-Debreu economy is Pareto optimal and discovered class of convex Arrow-Debreu economies for which competitive equilibria always exist. The Arrow-Debreu security is a theoretical concept used in general equilibrium theory in economics. It represents a contingent claim on consumption in a specific state of the world. In equilibrium, the sum of the prices of all Arrow-Debreu securities must equal one. This reflects the fact that all possible

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security issued at time t at history s^t and promising to pay one unit of time $t+1$ consumption in state s_{t+1} costs :

equation 5

$$p_{t+1}(s_{t+1}|s^t)$$

The government has a sequence of budget constraints whose time $t \geq 0$ component is given as:

equation 6

$$g_t(s^t) = \tau_t(s^t)n_t(s^t) + \sum_{s_{t+1}} p_{t+1}(s_{t+1}|s^t)b_{t+1}(s_{t+1}|s^t) - b_t(s_t|s^{t-1})$$

Where: $p_{t+1}(s_{t+1}|s^t)$ is competitive equilibrium price of one unit of consumption at date $t+1$ in state s_{t+1} at date t and history s^t ,and $b_t(s_t|s^{t-1})$ is government debt falling due at time t ,and history s^t . Now the representative household has a sequence of budget constraints whose time $t \geq 0$ component is :

equation 7

$$c_t(s^t) + \sum_{s_{t+1}} p_t(s_{t+1}|s^t)b_{t+1}(s_{t+1}|s^t) = [1 - \tau_t(s^t)]n_t(s^t) + b_t(s_t|s^{t-1}), \forall t \geq 0$$

A government policy is exogenous sequence $\{g(s_t)\}_0^\infty$and a tax rate sequence $\{\tau_t(s^t)\}_{t=0}^\infty$ and a government debt sequence: $\{b_t(s_t|s^{t-1})\}_0{}^\infty$. And the price system is a sequence of Arrow securities:

equation 8

$$\{p_{t+1}(s_{t+1}|s^t)\}_0^\infty$$

In the Arrow-Debreu version of price system we have that $q_t^0(s^t)$ is the price at $t=0$ and the following recursion relates Arrow-Debreu prices $\{q_t^0(s^t)\}_0^\infty$to Arrow security prices $\{p_{t+1}(s_{t+1}|s^t)\}_0^\infty$:

equation 9

$$q_t^0(s^{t+1}) = p_{t+1}(s_{t+1}|s^t)q_t^0(s^t) \;\; s.t. q_0^0\,(s^0) = 1$$

Implementability constraint

By sequential solution of the budget constraint :

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states of the world must be accounted for. So: $\sum_{i=1}^N p_i = 1$,where N are states of the world. And payoff is :$D = \begin{cases} 1 \rightarrow \exists S_i \\ 0 \rightarrow \nexists S_i \end{cases}$

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equation 10

$$c_t(s^t) + \sum_{s_{t+1}} p_t(s_{t+1}|s^t) b_{t+1}(s_{t+1}|s^t) = [1 - \tau_t(s^t)] n_t(s^t) + b_t(s_t|s^{t-1}), \forall t \geq 0$$

We can obtain household present budget constraint ;

equation 11

$$\sum_{t=0}^{\infty} \sum_{s'} q_t^0(s^t) c_t(s^t) = \sum_{t=0}^{\infty} \sum_{s'} q_t^0(s^t) [1 - \tau_t(s_t')] n_t(s^t) + b_0$$

FOC's for household problem for $l_t(s^t)$ and $b_t(s_t|s^{t-1})$ imply that :

equation 12

$$\left(1 - \tau_t(s^t)\right) = \frac{u_l(s^t)}{u_c(s^t)}$$

And :

equation 13

$$p_{t+1}(s_{t+1}|s^t) = \beta^t \pi(s_{t+1}|s^t) \frac{u_c(s^{t+1})}{u_c(s^t)}$$

Where $\pi(s_{t+1}|s^t)$ is the PDF of s_{t+1} conditional on history s^t .Previous equation implies that:

equation 14

$$q_t^0(s^t) = \beta^t \pi_t(s^t) \frac{u_c(s^t)}{u_c(s^0)}$$

Using the FOC's of $\left(1 - \tau_t(s^t)\right) = \frac{u_l(s^t)}{u_c(s^t)}$ and $p_{t+1}(s_{t+1}|s^t) = \beta^t \pi(s_{t+1}|s^t) \frac{u_c(s^{t+1})}{u_c(s^t)}$ in order to eliminate taxes from $\sum_{t=0}^{\infty} \sum_{s'} q_t^0(s^t) c_t(s^t) = \sum_{t=0}^{\infty} \sum_{s'} q_t^0(s^t) [1 - \tau_t(s_t')] n_t(s^t) + b_0$,we can derive implementability condition:

equation 15

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi_t(s^t) [u_c(s^t) c_t(s^t) - u_t(s^t) n_t(s^t)] - u_c(s^0) b_0 = 0$$

The Ramsey problem is to choose a feasible allocation that maximizes:

equation 16

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi_t(s^t) u\left(c_t(s^t)\right), 1 - n_t(s^t)$$

Next, we will define pseudo utility function :

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equation 17

$$V[c_t(s^t), n_t(s^t), \Phi] = u[c_t(s^t), 1 - n_t(s^t)] + \Phi[u_c(s^t)c_t(s^t) - u_l(s^t)n_t(s^t)]$$

Lagrangian is of the form:

equation 18

$$\mathcal{L} = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi_t(s^t)\{V[c_t(s^t), n_t(s^t), \Phi] + \theta_t(s^t)[n_t(s^t) - c_t(s^t) - g_t(s_t)]\} - \Phi u_c(0)b_0$$

Where $\{\theta_t(s^t); \forall s^t\}_{t\geq 0}$ is a sequence of Lagrange multipliers on the feasible condition, and given initial debt b_0,we want to maximize $\mathcal{L}$ with respect to $[c_t(s^t), n_t(s^t); \forall s^t]_{t\geq 0}$.FOC's for Ramsey problem for period $t \geq 1; t = 0$ are given as:

equation 19

$$c_t(s^t): (1 + \Phi)u_c(s^t) + \Phi[u_{cc}(s^t)c_t(s^t) - u_{lc}(s^t)n_t(s^t)] - \theta_t(s^t) = 0\,; t \geq 1$$
$$n_t(s^t): -(1 + \Phi)u_l(s^t) + \Phi[u_{cl}(s^t)c_t(s^t) - u_{ll}(s^t)n_t(s^t)] + \theta_t(s^t) = 0\,; t \geq 1$$

And :

equation 20

$$c_0(s^0, b_0): (1 + \Phi)u_c(s^0, b_0) + \Phi[u_{cc}(s^0, b_0)c_0(s^0, b_0) - u_{lc}(s^0, b_0)n_0(s^0, b_0)] - \theta(s^0, b_0)$$
$$-\Phi u_{cc}(s^0, b_0)b_0 = 0$$
$$n_0(s^0, b_0): -(1 + \Phi)u_l(s^0, b_0) + \Phi[u_{cl}(s^0, b_0)c_0(s^0, b_0) - u_{ll}(s^0, b_0)n_0(s^0, b_0)] - \theta(s^0, b_0)$$
$$+\Phi u_{cl}(s^0, b_0)b_0 = 0$$

If we suppress time subscript and the index, s^t we obtain:

equation 21

$$(1 + \Phi)u_c(c, 1 - c - g) + \Phi[cu_{cc}(c, 1 - c - g) - (c + g)u_{lc}(c, 1 - c - g)]$$
$$= (1 + \Phi)u_l(c, 1 - c - g) + \Phi[c_{ucl}(c, 1 - c - g) - (c + g)u_{ll}(c, 1 - c - g)]$$

Proposition 1: If government purchases are equal after two histories s^t and $\tilde{s}^\tau$ for $t, \tau \geq 0$ i.e. if:

equation 22

$$g_t(s^t) = g^\tau(\tilde{s}^\tau)$$

Then it follows from $(1 + \Phi)u_c(c, 1 - c - g) + \Phi[cu_{cc}(c, 1 - c - g) - (c + g)u_{lc}(c, 1 - c - g)] = (1 + \Phi)u_l(c, 1 - c - g) + \Phi[c_{ucl}(c, 1 - c - g) - (c + g)u_{ll}(c, 1 - c - g)]$ that the Ramsey choices of consumption and leisure $c_t(s^t), l_t(s^t)$ and $c_j(\tilde{s}^\tau), l_j(\tilde{s}^\tau)$ are identical. For Ramsey see Ramsey (1927).

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Further specialization and determining the Lagrange multiplier

Transition matrix of s which is governed by finite state Markov chain[6] is given as:

equation 23

$$\Pi(s'|s) = Prob(s_{t+1} = s'|s_t = s)$$

We also, assume that government purchases g are an exact time-invariant function of $g(s)$ of s.The household FOC's imply that:

equation 24

$$\left(1 - \tau_t(s^t)\right) = \frac{u_l(s^t)}{u_c(s^t)}$$

And the implied one period Arrow securities prices:
equation 25

$$p_{t+1}(s_{t+1}|s') = \beta\Pi(s_{t+1}|s_t)\frac{u_l(s^{t+1})}{u_c(s^t)}$$

The household budget constraint by combining previous gives:

equation 26

$$u_c(s^t)[n_t(s^t) - g_t(s^t)] + \beta \sum_{s_{t+1}} \Pi\,(s_{t+1}|s')b_{t+1}(s_{t+1}|s') = u_l(s^t)n_t(s^t) + u_c(s^t)b_t(s_t|s^{t-1})$$

We define:
equation 27

$$x_t(s^t) = u_c(s^t)b_t(s_t|s^{t-1})$$

From Ramsey allocation we learned $c_t(s^t), n_t(s^t), b_t(s_t|s^{t-1})$ and henceforth $x_t(s^t\ )$ are each functions of s^t only, and are independent of history s^{t-1} for $t \geq 1$.Now

equation 28

$$u_c(s)[n_t(s) - g_t(s)] + \beta \sum_{s_{t+1}} \Pi\,(s'|s)x'(s') = u_l(s)n(s) + x(s)$$

If we let $\vec{n}, \vec{g}, \vec{x}$ denote $S \times 1$ vectors whose elements are the respective n, g, x values when $s = i$ and let $\Pi = 1$ then:
equation 29

$$\vec{u}_c(\vec{n} - \vec{g}) + \beta\Pi\vec{x} = \vec{u}_l\vec{n} + \vec{x}$$

---

[6] A Markov chain is collection of random variables $\{X_t\}$ (where the index t runs through 0, 1, ...) having the property that, given the present, the future is conditionally independent of the past.In other words,$P\left(x_n = a_{i_n}\middle|x_{n-1} = a_{i_{n-1}}, \dots \dots, x_1 = aa_{i_1}\right) = P)x_n = a_{i_n}\middle|x_{n-1} = a_{i_{n-1}}\big)$,see Papoulis, A. (1984).

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This is a system of linear equations S in $S \times 1$ vector x whose solution is :

equation 30

$$\vec{x} = (I - \beta\Pi)^{-1}[\vec{u}_c(\vec{n} - \vec{g}) - \vec{u_l}\,\vec{n}]$$

The issue of time inconsistency

Theorem 1: Ramsey plan is inconsistent

Proof:
Let $\{\tau_t(s^t)\}_0^\infty$,$\{b_{t+1}(s_{t+1}|s^t)\}_0^\infty$ be at time 0, state s_0 Ramsey plan. Then $\{\tau_j(s^j)\}_{j=t}^\infty$,$\{b_{t+1}(s_{j+1}|s^j)\}_{j=t}^\infty$ is a time t ,history s^t continuation of a time 0,state s_0 Ramsey plan. At time t history s^t Ramsey plan is Ramsey plan that starts from initial conditions $s^t, b_t(s_t|s^{t-1})$.This means that Ramsey plan is inconsistent.■

Model specification with CRRA utility

Aiyagari, Marcet, Sargent, and Seppälä (2002), extended Lucas-Stokey model and modified one period utility

equation 31

$$u(c, n) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{n^{1+\gamma}}{1+\gamma}$$

Where $\sigma > 0, \gamma > 0$.And we assume $c_t + g_t = n_t$,we also eliminate Lukas-Stokey restriction that $l_t + n_t \leq 1$. Nd assumption for labor is :$n_t \in [0, +\infty]$.Now the following replacements are being made:

equation 32

$$\begin{gathered} u_l(c,l) \sim -u_n(c,n) \\ u_c(c,l) \sim u_c(c,n) \\ u_{l,l}(c,l) \sim u_{nn}(c,n) \\ u_{c,c}(c,l) \sim u_{cc}(c,n) \\ u_{c,l}(c,l) \sim 0 \end{gathered}$$

Now previous becomes:

equation 33

$$\begin{gathered} (1+\Phi)[u_c(c_0) + u_n(c+g)] + \Phi[cu_{cc}(c) + (c+g)u_{nn}(c+g)] = 0 \\ (1+\Phi)[u_c(c_0) + u_n(c_0+g_0)] + \Phi[c_0u_{cc}(c_0) + (c_0+g_0)u_{nn}(c_0+g_0)] - \Phi u_{cc}(c_0)b_0 = 0 \end{gathered}$$

In addition, at time $t = 0$ budget constraint is satisfied at c_0 and initial government debt b_0:

equation 34

$$b_0 + g_0 = \tau_0(c_0 + g_0) + \beta\sum_{s=1}^{S} \Pi(s|s_0)\frac{u_c(s)}{u_{c,0}}b_1(s)$$

In previous equation:

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equation 35

$$\tau_0 = 1 - \frac{u_{l,0}}{u_{c,0}}$$

Recursive formulation of Ramsey problem, intertemporal delegation and two Bellman equations

In the Lucas-Stokey specification:

equation 36

$$x_t(s^t) = u_c(s^t)b_t(s_t|s^{t-1})$$

Is a forward looking variable. But also $x_t(s^t)$ is a natural candidate for state variable in a recursive formulation of the Ramsey problem, one that records history dependence also Is backward looking. A key step in representing Ramsey plan recursively is to regard the marginal utility scaled government debts $x_t(s^t) = u_c(s^t)b_t(s_t|s^{t-1})$ as predetermined quantities that continuation Ramsey planners at times $t \geq 1$ are obliged to attain. That is how household make choices that imply that:

equation 37

$$u_c(s^t)b_t(s_t|s^{t-1}) = x_t(s^t)$$

After s_t has been realized at time $t \geq 1$the state variables confronting the time t continuation Ramsey planer is (x_t, s_t)

- ✓ Let $(V(x,s)$ be the value of continuation Ramsey plan at $x_t = x, s_t = s, t \geq 1$
- ✓ Let $W(b,s)$ be the value of Ramsey plan at time $t = 0, b_0 = b, s_0 = s$

The Bellman equation for a time $t \geq 1$ continuation Ramsey problem is
equation 38

$$V(x,s) = \max_{n,(x',(s'))} u(n - g(s), 1 - n) + \beta \sum_{s' \in S} \Pi(s'|s)V(x', s')$$

Where:
equation 39

$$x = u_c\big(n - g(s)\big) - u_l n \beta \sum_{s' \in S} \Pi(s'|s)x'(s')$$

Associated with a value function $V(x,s)$ that solves Bellman equation are $S + 1$time invariant policy functions:
equation 40

$$n_t = f(x,s), t \geq 1$$
$$x_{t+1}(s_{t+1}) = h(s_{t+1}; x_t, s_t), s_{t+1} \in \mathcal{S}, t \geq 1$$

The Bellman equation of the time 0 Ramsey planner is :

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equation 41

$$W(b_0, s_0) = \max_{n_0,(x',(s'))} (n_0 - g_0, 1 - n_0) + \beta \sum_{s_1 \in S} \Pi(s_1|s_0) V(x'(s_1), s_1)$$

Where maximization over n_0 and the $\mathcal{S}$ elements of $x'(s_1)$ is subject to time 0 implementabiity constraint:
equation 42

$$u_{c,0b_0} = u_{c,0}(n_0 - g_0) - u_{l,0}n_0 + \beta \sum_{s_1 \in S} \Pi(s_1|s_0) x'(s_1)$$

Associated function $W(b_0, n_0)$ that solves Bellman equation are $S + 1$ time0 policy functions:

equation 43

$$n_0 = f_0(b_0, s_0)$$
$$x_1(s_1) = h_0(s_1; b_0, s_0)$$

Now for the FOC's: At time $t \geq 1$ constrained maximization problem on the right-side pf the continuation Ramsey planner's Bellman equation are:
equation 44

$$\beta \Pi(s'|s) V_x(x', s') - \beta \Pi(s'|s) \Phi_1 = 0 \; for \; x'(s')$$
$$(1 + \Phi_1)(u_c - u_l) + \Phi_1 \left[n(u_{ll} - u_{lc}) + \left(n - g(s)\right)(u_{cc} - u_{lc}) \right] = 0 \; for \; n$$

For time $t = 0$ on the right side of Ramsey planner's Bellman equation FOC's are:
equation 45

$$V_x(x(s_1), s_1) = \Phi_0, for \; x(s_1), s_1 \in \mathcal{S}$$
$$(1 + \Phi_0)\left(u_{c,0} - u_{n,0}\right) + \Phi_0 \left[n_0 \left(u_{ll,0} - u_{lc,0}\right) + \left(n_0 - g(s_0)\right)\left(u_{cc,0} - u_{lc,0}\right) \right] - \Phi_0 \left(U_{cc,0} - u_{cl,0}\right) b_0 \; = 0$$

These equations:
equation 46

$$V_x(x', s') = V_x(x, s) = \Phi_1(x, s)$$
$$V_x(x(s_1), s_1) = \Phi_0$$

Imply that $\Phi_0 = \Phi_1$

Example : Anticipated one period war

Th example will illustrate how Ramsey planner manages risk. Government expenditures are known for all period except one when there will be war.For $t < 3; t > 3$ we assme $g_t = g_l = 0.1$.Now, at $t = 3$ war occurs with probability 0.5.If there is a war:
equation 47

$$g_3 = g_h = 0.2$$

If there is no war:

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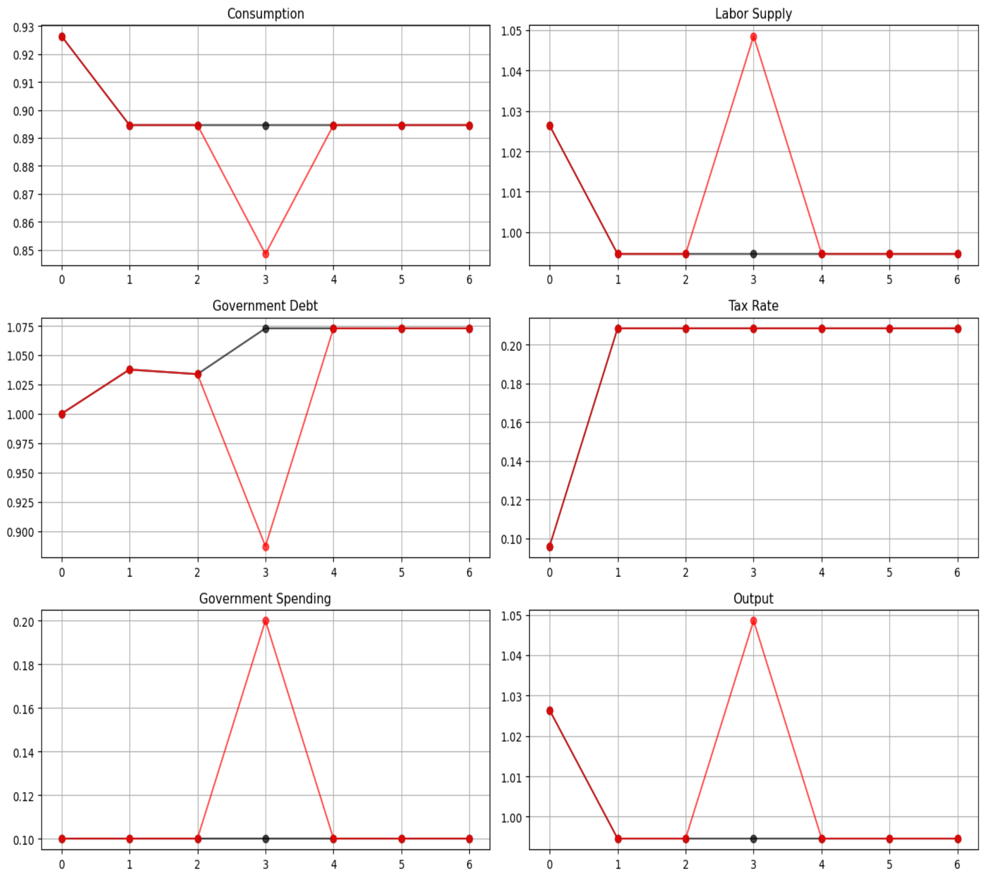
equation 48

$$g_3 = g_l = 0.1$$

Now we can define pairs:$(0, g_l), (1, g_l), (2, g_l), (3, g_l), (3, g_h), (t \geq 4, g_l)$ and $s \in (1, .., 6)$

The transition matrix is :$\Pi = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$.Government expenditures are:$g = \begin{pmatrix} 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0.2 \\ 0.1 \end{pmatrix}$.Representative agent utility function is CRRA: $u(c, n) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{n^{1+\gamma}}{1+\gamma}$

Figure 1



Source : author’s own calculations based on a code available at:
https://github.com/QuantEcon/quantecon-notebooks-python/blob/master/opt_tax_recur.ipynb

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Example : Tax smoothing and Ramsey time inconsistency and non-CRRA preferences

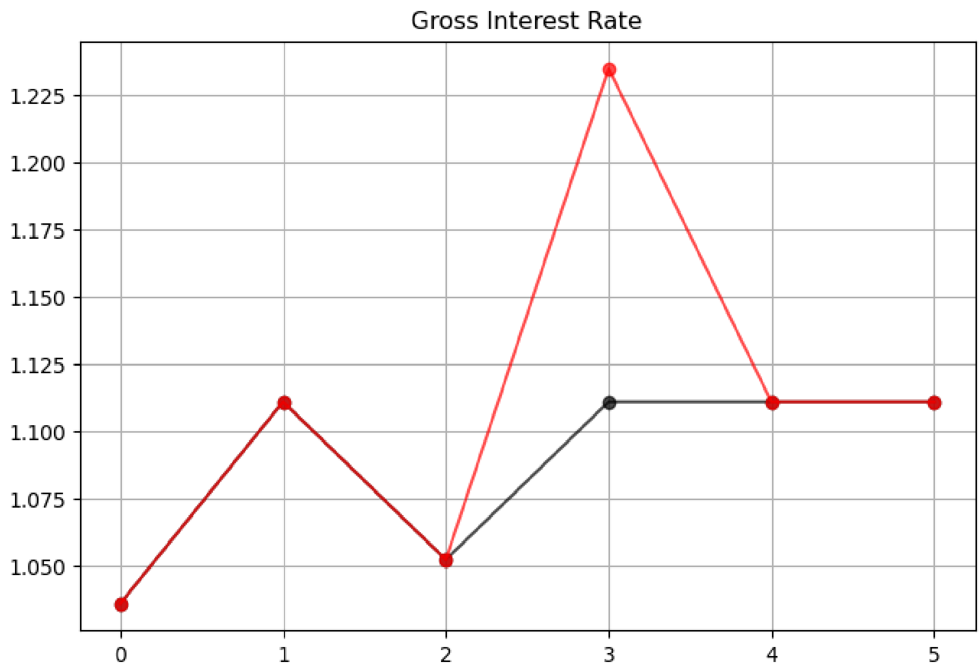
In the context of Robert Barro's seminal paper "On the Determination of the Public Debt" published in 1979 (see Barro 1979), tax smoothing refers to the idea that governments should adjust tax rates gradually to smooth out fluctuations in government spending over time, rather than making frequent changes in tax policy in response to short-term fluctuations in revenue or expenditure. According to the tax-smoothing hypothesis[7], the government sets the budget surplus equal to expected changes in government expenditure, see Adler (2006).In this example tax rate is constant $\forall t \geq 1$, $t \neq 3$ it is a consequence of g_t being same in all those dates. Under one period utility functions, the time $t = 3$ tax rate could be higher or lower for dates $t \geq 1, t \neq 3$ .Tax rate is same for low or high g_t . The value of gross interest rate for risk free loans between $t \to t + 1$ equals:

equation 49

$$R_t = \frac{u_{c,t}}{\beta E_t[u_{c,t+1}]}$$

A tax policy that makes time $t = 0$ consumption be higher than time $t = 1$ consumption evidently decreases the risk-free rate one-period interest rate, R_t , at $t = 0$. Lowering the time $t = 0$ risk-free interest rate makes time $t = 0$ consumption goods cheaper relative to consumption goods at later dates, thereby lowering the value $u_{c,0}b_0$ of initial government debt b_0. The following plot illustrates how the government lowers the interest rate at $t = 0$ by raising consumption.

Figure 2 The government lowers the interest rate at $t = 0$ by raising consumption



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https://github.com/QuantEcon/quantecon-notebooks-python/blob/master/opt_tax_recur.ipynb

[7] When expenditure is expected to increase, the government runs a budget surplus, and when expenditure is expected to fall, the government runs a budget deficit

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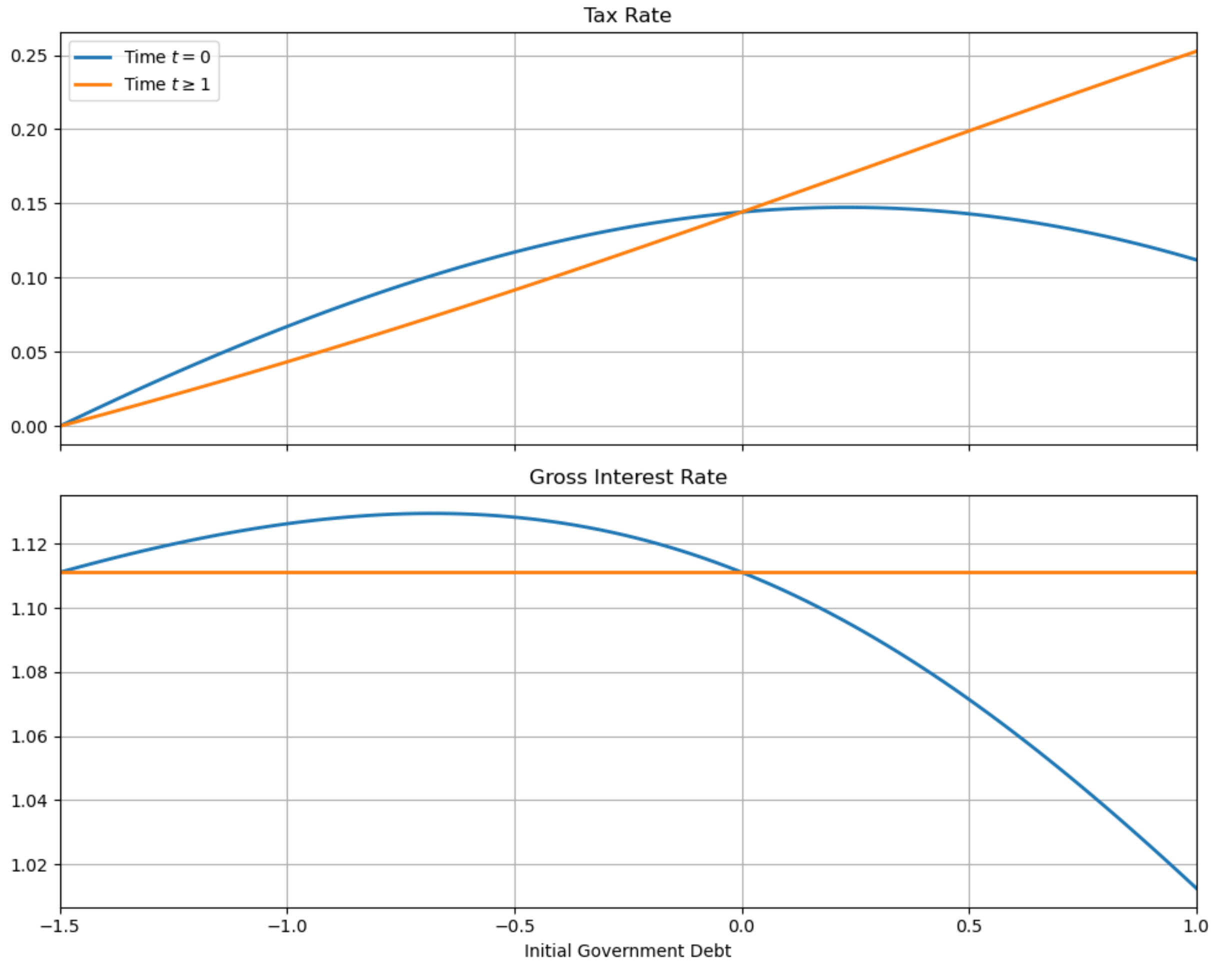
The Ramsey tax rate at $t=0 \neq t=1$ .To explore what is going on here, let's simplify things by removing the possibility of war at time $t=3$.The Ramsey problem then includes no randomness because $g_t = g_l, \forall t$ .The figure below will plot the Ramsey tax rates and gross interest rates at time $t=0$ and time $t \geq 1$ as functions of the initial government debt by using the sequential allocation solution and a CRRA utility function defined above.

Figure 3 Ramsey tax rates and gross interest rates



Source : author's own calculations based on a code available at:
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Previous figure indicates that if the government enters with positive debt, it sets a tax rate at $t=0$ that is less than all later tax rates. By setting a lower tax rate at $t=0$, the government raises consumption, which reduces the value $u_{c,0}b_0$ of its initial debt. It does this but $\uparrow c_0$,$\downarrow u_{c,0}$.Conversely, if $b_0 < 0$, Ramsey planner will set the tax rate at $\tau(t=0) \gg \forall \tau(t+n)$. One side effect of lowering time$t=0$ consumption is that it lowers the one period interest rate at time $c(t=0) \ll c(t+n)$. There are only two values of initial government debt at which the tax rate is constant $\forall\ t \geq 0$. The first one is $b_0 = 0$, the government cannot use $t=0$ tax rate to alter the value of initial debt. The second occurs when the government enters with sufficiently large assets that the Ramsey planner can achieve first best and sets

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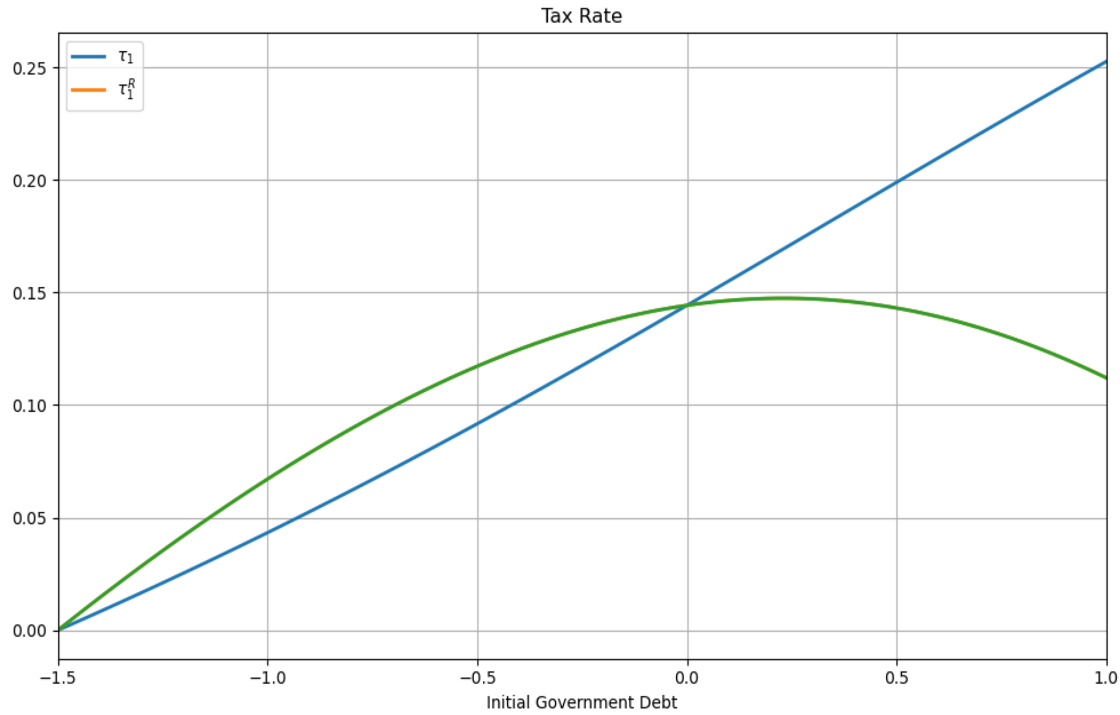
$\tau t = 0 \quad \forall t$ .It is only for these two values of initial government debt that the Ramsey plan is time consistent. Or, except for these two values of initial government debt, a continuation of a Ramsey plan is not a Ramsey plan. Let's consider a Ramsey planner who starts with an initial government debt b_1
associated with one of the Ramsey plans computed above. We will name τR_1 the time $t = 0$ tax rate chosen by the Ramsey planner confronting this value for initial government debt government. The figure below shows both the tax rate at $t = 1$ chosen by the original Ramsey planner and what a new Ramsey planner would choose for its time $t = 0$ tax rate.

Figure 4 tax rate at t=1 chosen by the original Ramsey planner and a choice by a new Ramsey planner for its time t=0 tax rate



Source : author's own calculations based on a code available at:
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The tax rates in the previous figure are equal for only two values of initial government debt. Tax smoothing was a consequence of assumption of CRRA preferences. In the context of Ramsey model low σ [8],

equation 50

$$\frac{c_1}{c_2} = \left(\frac{p_1}{p_2}\right)^{\sigma}$$

[8] $\sigma$is a measure of the strength of the substitution effect that a change in relative prices induces.

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means a strong preference for avoiding inequality between generations in excess of what follows from the discounting in the utility function. That all income elasticities are equal to one makes it possible to have balanced growth paths also when there is productivity growth. The first order condition Euler equation in the Ramsey model without natural growth is given as:

equation 51

$$\frac{u'(c_t)}{u'(c_{t+1})} = \beta[1 + f'(k_{t+1}]$$

With CRRA utility previous will become:

equation 52

$$\left(\frac{c_{t+1}}{c_t}\right)^{\theta} = \beta[1 + f'(k_{t+1})]$$

Consumption growth rate becomes:

equation 53

$$\frac{c_{t+1}}{c_t} = \left[\beta[1 + f'(k_{t+1}]\right]^{\frac{1}{\theta}} = \left(\frac{1 + f'(k_{t+1})}{1 + \rho}\right)^{\sigma}$$

Marginal rate of substitution (MRT) is given as:

equation 54

$$\frac{u'(c_1)}{u'(c_2)} = \frac{c_1^{-\theta}}{c_2^{-\theta}} = \left(\frac{c_2}{c_1}\right)^{\theta}$$

For CRRA preferences we can see relations like $U_{cc}c = -\sigma U_c; U_{nn}n = \gamma U_n$ to derive following:

equation 55

$$\frac{(1 + (1 - \sigma)\Phi)U_c}{(1 + (1 - \gamma)\Phi)U_n} = 1$$

The previous equation implies that the tax rate is constant. Sometimes for other preferences tax rate may not be constant, for example if the utility function is:

equation 56

$$(1 + (1 - \sigma)\Phi)U_c(1 + (1 - \gamma)\Phi)U_n = 1$$

In the next plot, it is shown that as should be expected, the recursive and sequential solutions produce almost identical allocations. Unlike outcomes with CRRA preferences, the tax rate is not perfectly smoothed. Instead, the government raises the tax rate when g_t is high.

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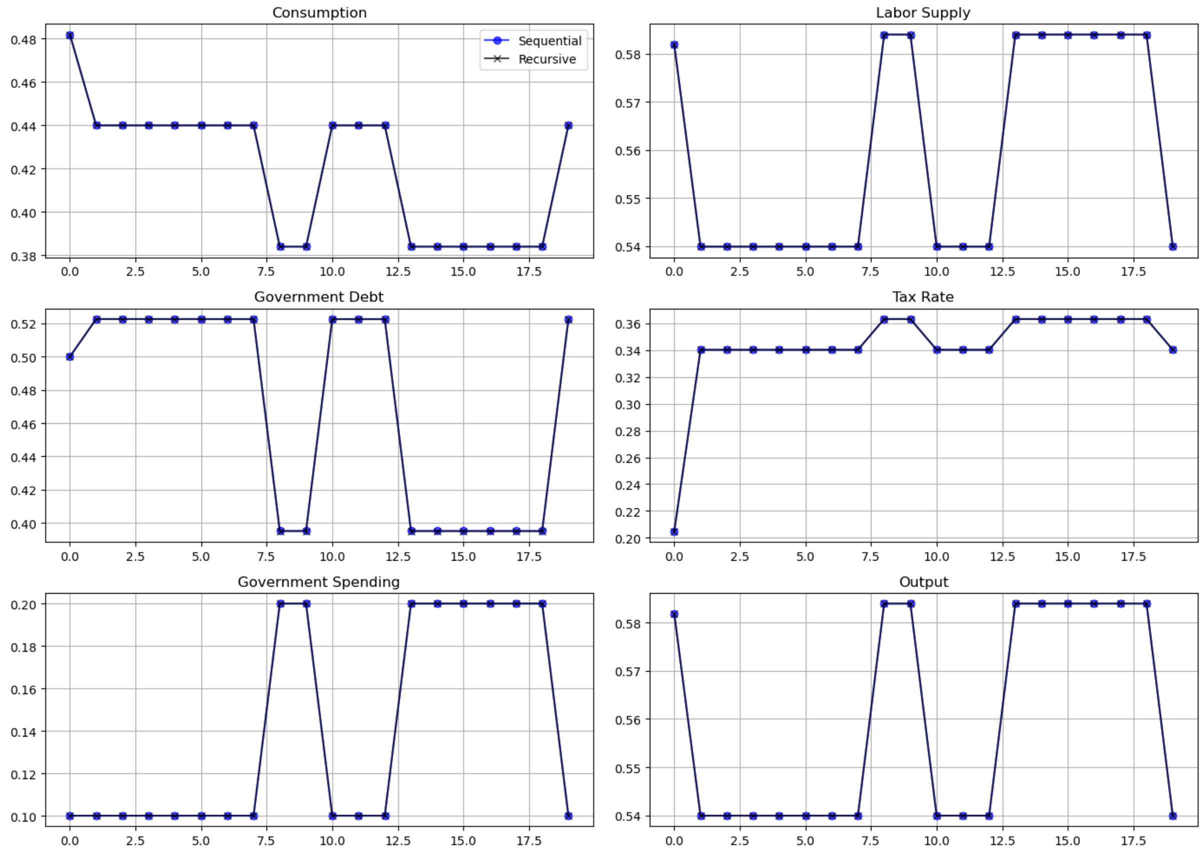
Figure 5 Recursive and sequential solutions for consumption, labour supply, government debt, tax rate, government spending and output



Source : author's own calculations based on a code available at: https://github.com/QuantEcon/quantecon-notebooks-python/blob/master/opt_tax_recur.ipynb

Taxation in LQ (linear quadratic) economy

Barro (1979), has formalized the idea that taxes should be smooth by saying that they should be a martingale, regardless of the stochastic process for government expenditures, see Sargent,Velde (1998). Barro (1979) model is about government that borrows and lends to help it minimize an intertemporal measure of distortions caused by taxes[9].Hansen, Sargent, Roberds (1991) use the following linear quadratic model to formalize Barro's findings. The government chooses a rule for taxes to maximize the criterion:

equation 57

$$\max -E\sum_{t=0}^{\infty}\beta^t\mathcal{T}^2$$

s.t. initial condition $\mathcal{B}_0$:

[9] Barro's 1979 model looks a lot like a consumption-smoothing model

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equation 58

$$\gamma(\mathcal{L})g_t = \rho(\mathcal{L})w_t$$
$$\mathcal{B}_{t+1} = \mathcal{R}(\mathcal{B}_t + g_t - \mathcal{T}_t)$$

Where $\mathcal{T}_t, g_t, \mathscr{b}_t$ denote tax collections, government expenditures, and the stock of risk free government debt, and where $\mathcal{R}_t$ is a risk free interest rate and $\beta \in (0,1)$ is a discount factor and $\gamma(\mathcal{L}), \rho(\mathcal{L})$ are stable one sided polynomials in nonnegative powers of the lag operator $\mathcal{L}$ and w_t is aa scalar martingale[10] difference sequence adapted to its own history. If we make assumption $\mathcal{R}\beta = 1$ the solution of this problem for taxes that satisfies this condition $E\sum_{t=0}^{\infty}\beta^t\mathcal{T}^2 < +\infty$ is a rule for taxes:

equation 59

$$\mathcal{T}_t - \mathcal{T}_{t-1} = \left[\frac{(1-\beta)\rho(\beta)}{\gamma(\beta)}\right]w_t$$

The second equation can be written as:
equation 60

$$\pi_{t+1} = \mathcal{B}_{t+1} - \mathcal{R}[\mathcal{B}_t - (\mathcal{T}_t - g_t)] = 0$$

Where π_{t+1} can be interpreted as payoff of government debt in excess of risk-free rate.The cumulative excess payoff to government creditors will be:
equation 61

$$\Pi_t = \sum_{s=1}^{t}\pi_s = 0$$

So in this economy households maximize:
equation 62

$$-\mathbb{E}\frac{1}{2}\sum_{t=0}^{\infty}\beta^t[(c_t - \mathscr{b}_t)^2 + \ell_t^2]$$

s.t.:
equation 63

$$\mathbb{E}\frac{1}{2}\sum_{t=0}^{\infty}\beta^t p_0^t(d_t + (1-\tau_t)\ell_t + s_t - c_t)] = 0$$

[10] A sequence of random numbers $X_0, X_1, ..$ with finite means and conditional expectation of $X_{n+1}|X_0, \dots, X_n = X_n$ i.e., $\langle X_{n+1}|X_0, \dots, X_n\rangle = X_n$

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Where d_t is an endowment process, p_0^t is Arrow-Debreu price at $t = 0, x^t = t + j$ contingent goods, τ_t is a flat-tax rate on labor income, s_t is aa promised coupon payment on debt issued by the government, and $[\ell_t, c_t]$ is labor-consumption path. Now if μ is Lagrangian multiplier to $\mathbb{E}\frac{1}{2}\sum_{t=0}^{\infty}\beta^t p_0^t(d_t + (1-\tau_t)\ell_t + s_t - c_t)] = 0$, the FOC's are given as:

equation 64

$$p_0^t = \frac{c_t - b_t}{\mu}$$
$$\ell_t = \frac{c_t - b_t}{1 - \tau_t}$$

Now, if $\mu = b_0 - c_0$ we can write these conditions as:
equation 65

$$p_0^t = \frac{b_t - c_t}{b_0 - c_0}$$
$$\ell_t = 1 - \frac{\ell_t}{b_t - c_t}$$

Government budget constraint is:
equation 66

$$\mathbb{E}\sum_{t=1}^{\infty}\beta^t p_t^0(s_t + g_t - \tau_t\ell_t) = 0$$

If we substitute $\begin{aligned} p_0^t &= \frac{b_t - c_t}{b_0 - c_0} \\ \ell_t &= 1 - \frac{\ell_t}{b_t - c_t}\end{aligned}$ in government budget constraint we will get :

equation 67

$$\mathbb{E}\sum_{t=1}^{\infty}\beta^t[(b_t - c_t)(s_t + g_t - \ell_t) + \ell_t^2] = 0$$

Associated Lagrangian with previous is given as:

equation 68

$$\mathcal{L} = \mathbb{E}\left\{\frac{1}{2}[(c_t - b_t)^2 + \ell_t^2] + \lambda[(b_t - c_t)(\ell_t - s_t - g_t) - \ell_t^2] + \mu_t(d_t + \ell_t - c_t - g_t)]\right\}$$

The FOC's associated with c_t, ℓ_t are given as:

equation 69

$$-(c_t - b_t) + \lambda[-\ell_t + (g_t + s_t)] = \mu_t$$
$$\ell_t - \lambda[(b_t - c_t) - 2\ell_t] = \mu_t$$

Now if :

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equation 70

$$\begin{aligned} v &:= \frac{\lambda}{1+2\lambda} \\ \bar{\ell}_t &:= \frac{b_t - d_t + gt}{2} \\ \bar{c}_t &:= \frac{b_t + d_t - gt}{2} \\ m_t &:= \frac{b_t + d_t - st}{2} \end{aligned}$$

Now one can show that (knowing previous) that:
equation 71

$$\begin{aligned} \ell_t &= \bar{\ell}_t - v m_t \\ c_t &= \bar{c}_t - v m_t \end{aligned}$$

This term $(b_t - c_t)(s_t + g_t - \ell_t) + \ell_t^2$, since $\bar{\ell} = b - \bar{c}$ can be rewritten as:

equation 72

$$(b_t - \bar{c}_t)(g_t + s_t) + 2m_t^2(v^2 - v)$$

If we reinsert this into $\mathbb{E}\ \sum_{t=1}^{\infty} \beta^t [(b_t - c_t)(s_t + g_t - \ell_t) + \ell_t^2] = 0$ we get:
equation 73

$$\mathbb{E}\left\{\sum_{t=0}^{\infty} \beta^t (b_t - \bar{c}_t)(g_t + s_t)\right\} + (v^2 - v)\mathbb{E}\left\{\sum_{t=0}^{\infty} \beta^t 2m_t^2\right\} = 0$$

Let us consider quadratic term v in previous.The two geometric sum are :
equation 74

$$\begin{aligned} b_0 &:= \mathbb{E}\left\{\sum_{t=0}^{\infty} \beta^t (b_t - \bar{c}_t)(g_t + s_t)\right\} \\ a + 0 &:= \mathbb{E}\left\{\sum_{t=0}^{\infty} \beta^t 2m_t^2\right\} \end{aligned}$$

The problem for solving will be reduced to:

equation 75

$$b_0 + a_0(v^2 - v) = 0$$

Since $4b_0 < a_0$, $\exists\ v \in \left(0, \frac{1}{2}\right)$, and unique corresponding $\lambda > 0$ For this variable $(b_t - \bar{c}_t)(g_t + s_t)$ inside the summation can be expressed as:

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equation 76

$$\frac{1}{2}x_t'\left(S_b - S_d + S_g\right)'\left(S_g + S_s\right)x_t$$

Where endowments, government expenditure, the preference shock processes, and promised coupon payments on initial government debt s_t are exogenous and given by:

equation 77

$$\begin{gathered} d_t = S_d x_t \\ g_t = S_g x_t \\ b_t = S_b x_t \\ s_t = S_s x_t \end{gathered}$$

Where S_d, S_g, S_b, S_s are primitives [11]and $\{x_t\}$ is exogenous stochastic process taking values in $\mathbb{R}^k$.For the second expectation the random variable $2m_t^2$ can be written as:

equation 78

$$\frac{1}{2}x_t'\left(S_b - S_d + S_g\right)'(S_b - S_d - S_s)x_t$$

What follows that both objects of interest are special cases of the expression:
equation 79

$$q(x_0) = \mathbb{E}\sum_{t=0}^{\infty}\beta^t x_t' H x_t$$

where H is a matrix conformable to x_t and x_t' is the transpose of column vector x_t .Now about finite state Markov case.

Definition: A finite Markov chain is a memoryless homogeneous discrete stochastic process with a finite number of states[12].
.

If we suppose that x_t is the discrete Markov process described as above and $x_t = \{x^1, .., x^N\} \subset \mathbb{R}^k$.Now let $h: \mathbb{R}^k \to \mathbb{R}$ be a given function, and we wish to evaluate:

---

[11] A nonnegative square matrix $A = \left(a_{ij}\right)$ is said to be a if $\exists k$ such that $A^k \gg 0$,i.e. $\exists k, \forall i, j \gg 0$.Sufficient condition for matrix to be primitive matrix is to be nonnegative, irreducible matrix with a positive element on the main diagonal.

[12] A discrete stochastic process is a discrete system in which transitions occur randomly according to some probability distribution. The process is memoryless if the probability of an $i \to j$ transition does not depend on the history of the process. Or: $(\forall i, j, u_0, \dots, u_{t-1} \in V)(P\left(X_{t+j} = j \middle| X_t = i, X_{t-1} = u_{t-1}, \dots, X_0 = u_0\right) = P(X_{t+1} = j | X_t = i\}$.In a addition if the transition probability $p_{ij} = P(X_{t+1} = j | X_t = i\}$ does not depend on the time t , we call the process homogenous.

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equation 80

$$q(x_0) = \mathbb{E}\sum_{t=0}^{\infty}\beta^t h(x_t); x_0 = x^j$$

And we know that $h(x_t) = x_t' H x_t$, now:

equation 81

$$q(x_0) = \mathbb{E}\sum_{t=0}^{\infty}\beta^t (P^t h)(j); x_0 = x^j$$

About the other variables :

equation 82

$$p_{t+j}^t = \frac{\mathscr{b}_{t+j} - c_{t+j}}{\mathscr{b}_t - c_t}$$

As the scaled Arrow-Debreu time t of history contingent claim on one unit consumption at time $t+j$. Present value of government obligations outstanding at time$t$ can be written as:

equation 83

$$\mathcal{B}_t = \mathbb{E}\sum_{j=0}^{\infty}\beta^j \frac{(\mathscr{b}_{t+j} - c_{t+j})(\ell_{t+j} - \mathscr{g}_{t+j}) - \ell_{t+j}^2}{\mathscr{b}_t - c_t}$$

Using the equation :

equation 84

$$p_{t+j}^t = p_{t+1}^t p_{t+j}^{t+1}$$

We can write :

equation 85

$$\mathcal{B}_t = (\tau_t \ell_t - \mathscr{g}_t) + E_t \sum_{j=1}^{\infty} p_{t+j}^t \big(\tau_{(t+j)} \ell_{t+j} - \mathscr{g}_{t+j}\big)$$

We will define:

equation 86

$$\mathcal{R}_t^{-1} \coloneqq \mathbb{E}_t \beta^j p_{t+1}^t$$

$\mathcal{R}_t$ is a risk free interest rate $t \to t+1$. Now about martingale sequence. We can describe two object as below:

equation 87

$$\pi_{t+1} \coloneqq \mathcal{B}_{t+1} - \mathcal{R}_t[\mathcal{B}_t - (\tau_t \ell_t - \mathscr{g}_t]$$

$$\Pi_t \coloneqq \sum_{s=0}^{t}\pi_t$$

By using previous expressions, we can obtain:

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equation 88

$$\pi_{t+1} = \mathcal{B}_{t+1} - \frac{1}{\beta E_t p_{t+1}^t}[\beta E_t p_{t+1}^t \mathcal{B}_{t+1}]$$
$$\pi_{t+1} = \mathcal{B}_{t+1} - \tilde{E}_t \mathcal{B}_{t+1}$$

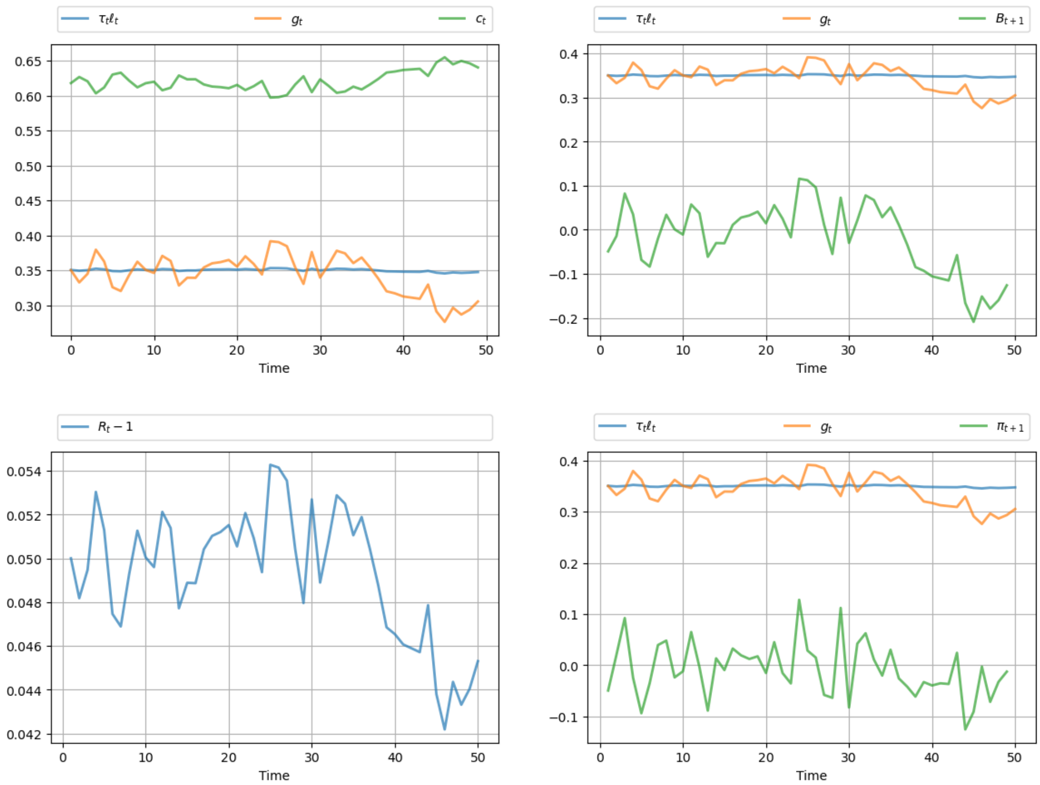
Where $\tilde{E}_t$ is a conditional mathematical expectation taken with respect to a one-step transition density that has been formed by multiplying the original transition density with the likelihood ratio.

equation 89

$$m_{t+1}^t = \frac{p_{t+1}^t}{E_t p_{t+1}^t}$$
$$\tilde{E}_t \pi_{t+1} = \tilde{E} \mathcal{B}_{t+1} - \tilde{E}_t \mathcal{B}_{t+1} = 0$$

Next, we will show graphically continuous case of optimal taxation in LQ economy.

Figure 6 Continuous case of optimal taxation in LQ economy



Source: author's calculations based on a code available at: https://github.com/QuantEcon/quantecon-notebooks-python/blob/master/lqramsey.ipynb

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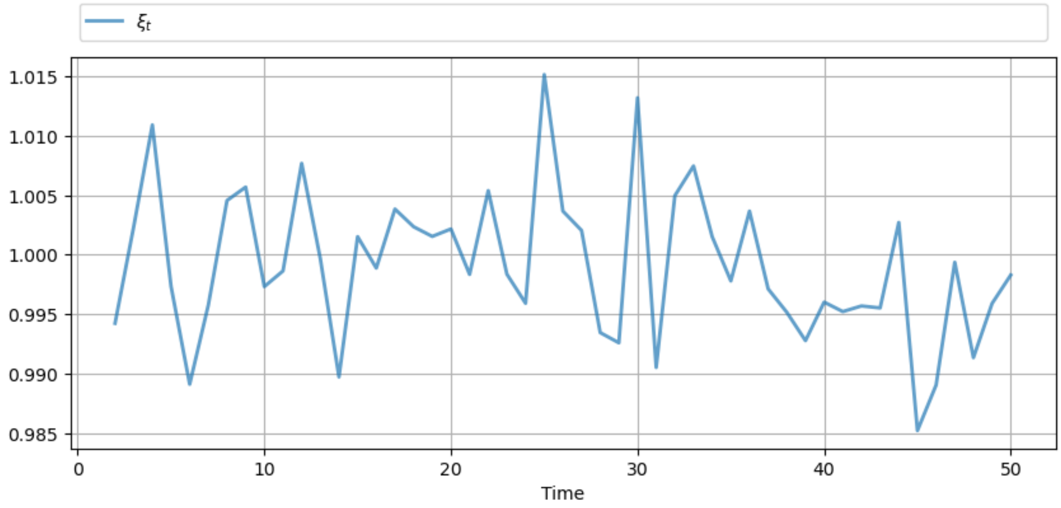
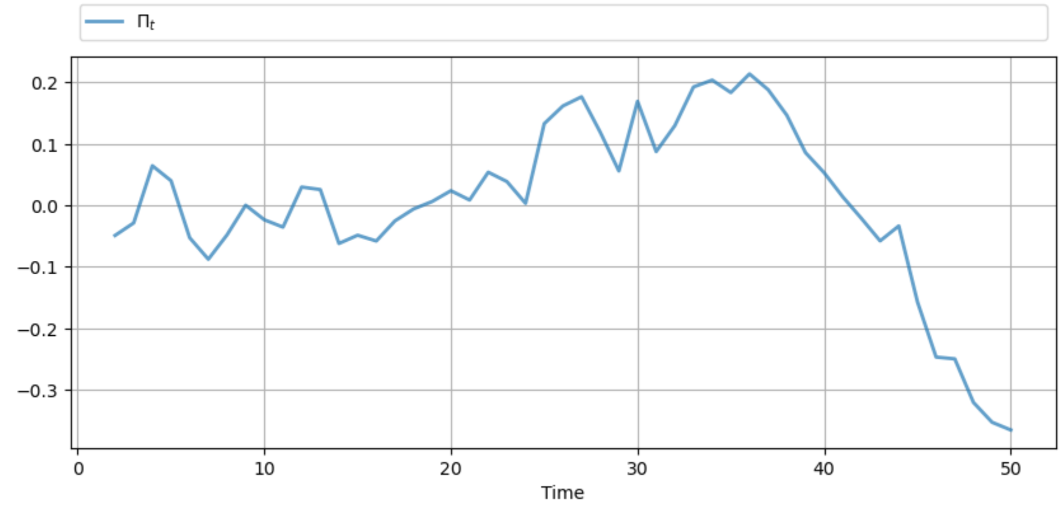
Figure 7 Continuous case of optimal taxation in LQ economy (contd.)





Source: author's calculations based on a code available at: https://github.com/QuantEcon/quantecon-notebooks-python/blob/master/lqramsey.ipynb

In this example: $\beta = \frac{1}{1.05}$; $b_t = 2.135$; $s_t = d_t = 0\ \forall t$.Government spending evolves according to:
equation 90

$$g_{t+1} - \mu_g = \rho(g_t - \mu_g) + C_g w_{g,t+1}$$

With now $\rho = 0.7, \mu_g = 0.35; C_g = \mu_g\sqrt{1 - \frac{\rho^2}{10}}$.

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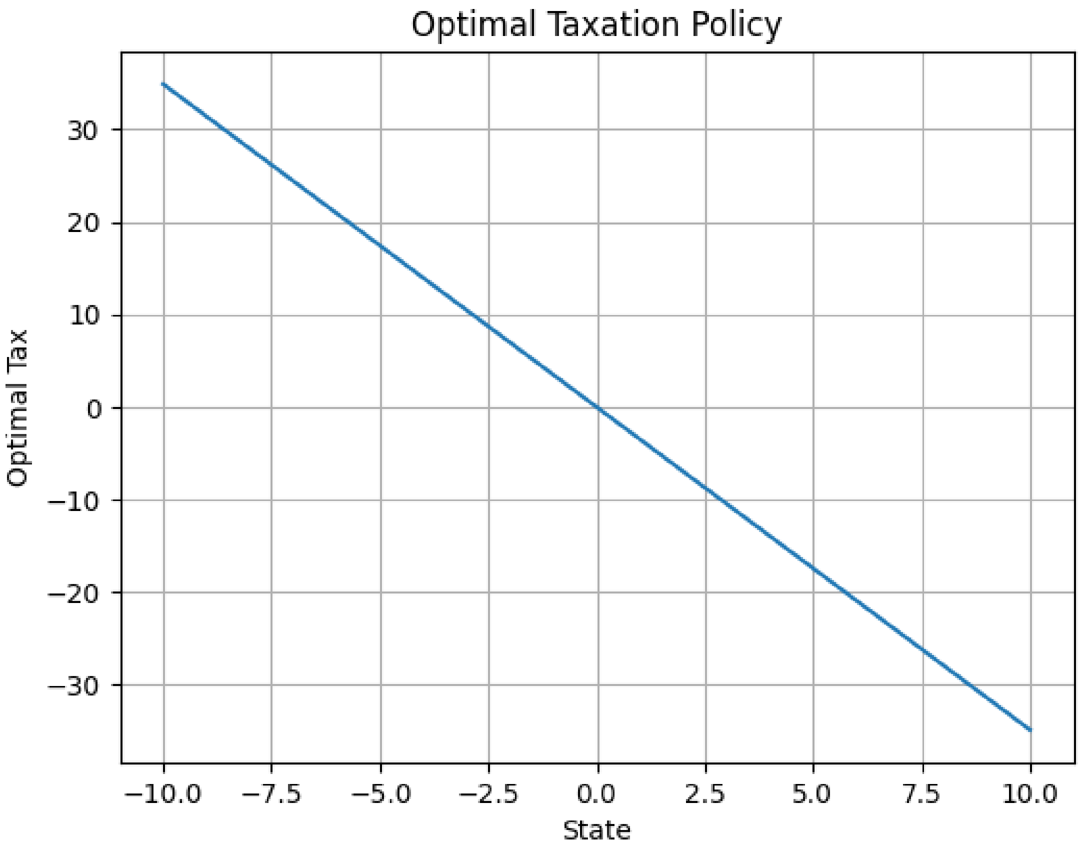
Figure 8 Optimal taxation policy in LQ economy



Source: Authors own calculations

The previous example uses Riccati equation. This equation in general form is given as:

equation 91

$$y_x' = f(x)y^2 + g * x)y + h(x)$$

Given a particular solution $y_0 = y_0(x)$ of the Riccati equation, the general solution can be given as:

equation 92

$$y = y_0(x) + \Phi(x)\left[C - \int f(x)\Phi(x)dx\right]^{-1}$$

$$\Phi(x) = \exp\left[\int [2f(x)y_0(x) + g(x)]dx\right]$$

Where C is an arbitrary constant. The solution $y_0(x)$ corresponds $C = \infty$.Now for the substitution we have:

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equation 93

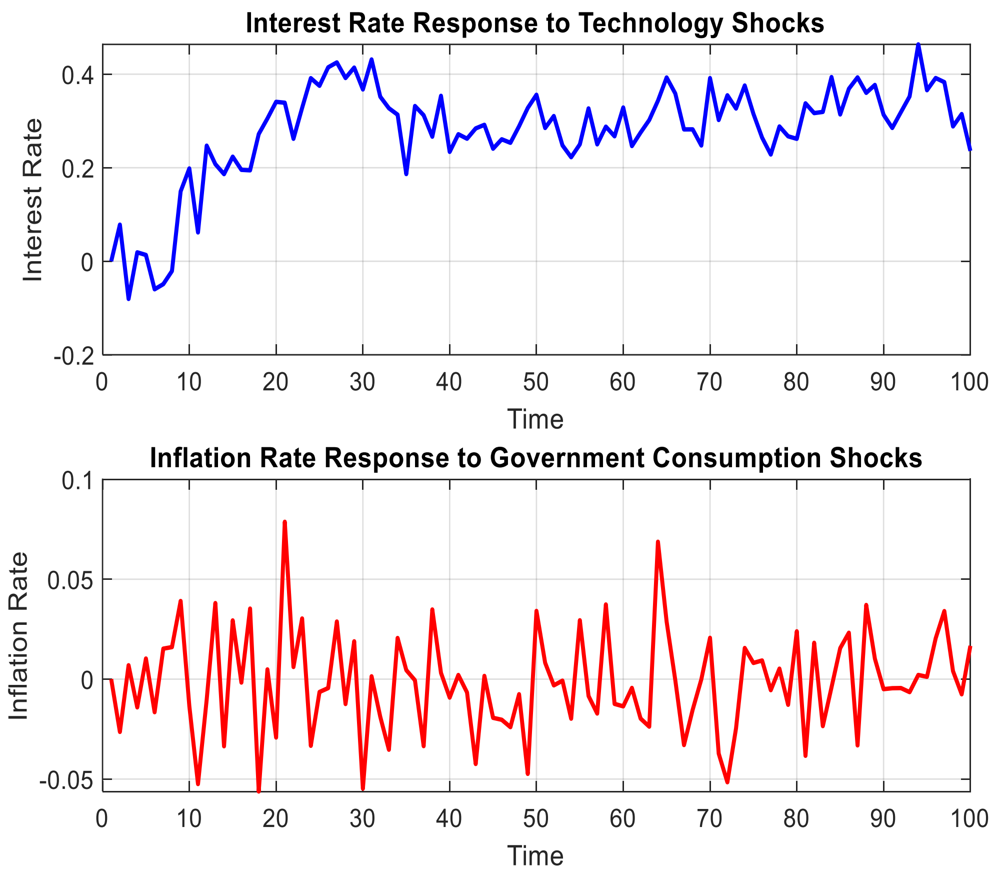
$$u(x) = \exp\left[-\int f(x)y(x)dx\right]$$

Reduces the general Riccati equation to a second order linear equation:
equation 94

$$f(x)u''_{xx} - [f'_x(x) + f(x)g(x)]u'_x + f^2(x)h(x)u = 0$$

Which is easier to solve than general Riccati equation[13].

Figure 9 Plot Optimal fiscal and monetary policy with labor taxes and consumption taxes constant over business cycle ,capital taxes round zero, Friedman rule optimal ,and monetary policy is countercyclical with respect to technology shocks and procyclical with respect to government consumption



Source: Authors own calculations

[13] See Murphy, G. M., (1960), Reid, W. T.(1972), Polyanin, A. D. and Zaitsev, V. F.(2003).

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Conclusions

This paper reviewed competitive equilibrium with distorting taxes and this section proposed that If government purchases are equal after two histories then it follows that the Ramsey choices of consumption and leisure $c_t(s^t), l_t(s^t)$ and $c_j(\tilde{s}^\tau), l_j(\tilde{s}^\tau)$ are identical. But there is issue of time inconsistency in Ramsey plan i.e. Ramsey plan is inconsistent. A key step in representing Ramsey plan recursively is to regard the marginal utility scaled government debts $x_t(s^t) = u_c(s^t)b_t(s_t|s^{t-1})$ as predetermined quantities that continuation Ramsey planners at times $t \geq 1$ are obliged to attain. Consumption and government debt follow a similar pattern in the shock in this model, but not before and after shock. Same can be written for government spending, output, and labor supply during the shock but not before and after the shock. In the tax smoothing model, a tax policy at initial time makes initial consumption higher than following period consumption but decreases risk free interest rate which makes initial consumption goods cheaper relative to consumption goods at later dates. Ramsey tax rates and gross interest rates show that tax rate at $t = 0$ is higher than the tax rate at $t = 1$ but sometimes later they are qual and later tax rate is higher than the initial afterwards. Gross interest rate rate at $t = 0$ is higher than the gross interest rate rate at $t = 1$ but sometimes later they are equal and later gross rate is higher than the initial afterwards. Tax rate at $t = 1$ chosen by the original Ramsey planner and a choice by a new Ramsey planner for its time $t = 0$ tax rate are only equal when government debt is zero while previously when we have negative initial government debt original Ramsey planner tax rate is lower than the new Ramsey planner tax rate and only when government debt becomes positive original Ramsey panner tax rate is higher than the tax rate by the new Ramsey planner. Recursive and sequential solutions for consumption, labor supply, government debt, tax rate, government spending and output show that sequential and recursive formulation change for different periods of time while at times being the same for more periods and then change suddenly. The continuous case of optimal taxation in LQ economy shows that labor taxes are cointegrated with consumption and government consumption i.e. they share common stochastic trend. While present value of government obligations is cointegrated with the gross-interest rate from previous period and Π_{t+1} inflation for future period is cointegrated with current government consumption and labor taxes. When a state variable is negative, optimal tax is positive (obviously state variable here can be interest rate), and when there is positive state variable optimal tax rate becomes negative. In LQ economy interest rate and inflation rate respond differently to technology and government consumption shocks respectively

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