MONETARY POLICY AND UNEMPLOYMENT :LITERATURE REVIEW AND COMPUTATIONAL EXAMPLES

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Abstract

This paper will review current stance on New- Keynesian monetary policy models with unemployment as well as search-matching models,also one particular model will be investigated that divides unemployment rate in steady-state on rationing (without presence of search frictions) and frictional unemployment.Review of computational examples following McCall (1970) and DMP model will provide policy conclusions from these models. Central role of unemployment in the policy debate ensured that this variable will not be conspicuously absent from the new generation of models that have become the workhorse for the analysis of monetary policy.

Keywords: monetary policy,unemployment,search-matching model,rationing nemployment

JEL code:E24,E32,J64

1.Introduction

Real business cycles (RBC) models imply fluctuations in employment,but they do not account for unemployment as such. Incidentally, it is worth pointing out that standard RBC models share the shortcomings of two paradigms: they neither can explain involuntary unemployment nor have any role for monetary policy, see Gali (2010). For that, we need to introduce unemployment in the model and some notion of friction in the labor market, and one particularly popular approach is based on search frictions. In a standard labor market model firms can hire as much labor as they want at prevailing wage while workers can find employment at prevailing wage. None of these assumptions are true in the search model. A classic example of DSGE model is the Real Business Cycle (RBC) model associated with Kydland; Prescott (1982) and Long and Plosser (1983). There unemployed workers need to find jobs, firms with vacancies need to find workers, these activities take time and resources. Christiano et al.(2018) writes that RBC models crumbled because of the three assumptions: micro data that cast doubt on the key assumptions of the model such as: perfect credit and insurance market, frictionless labor market ("in which fluctuations in hours worked reflect movements along a given labor supply curve or optimal movements of agents in and out of the labor force"…);second these models did not take into account volatility in hours worked, the equity premium (the difference between the return on a stock and the return on a bond); the low co-movement of real wages and hours worked see Christiano and Eichenbaum (1992); King and Rebelo (1999). New Keynesian DSGE models have been built on the basis of these RBC models to allow nominal frictions, in labor and goods markets. The DSGE model proposed by Christiano, Eichenbaum, and Evans (2005) and later estimated by Smets and Wouters (2003) using Bayesian techniques, is currently considered to be a benchmark richly specified DSGE model for a closed economy, see Kolasa et al. (2012)[1]. New- Keynesian DSGE models such as Yun (1996), Clarida, Galí, and Gertler (1999), and Woodford (2003), it is said to satisfy Fisherian and anti-Fisherian property. Fisherian property satisfies that *permanent* changes in monetary policy induce roughly on-to-one changes in inflation and nominal interest rate (neutrality of money); and anti-Fisherian property states that *transitory* changes in monetary policy induce movements in nominal interest rates and inflation of the opposite sign. DSGE models have been subject to negative scrutiny

[1] These models are called Friedmanite DSGE models, they assume that monetary policy has no effect on real variables such as: output and real interest rate in the long run. But because of the sticky prices and wages, monetary policy matters in the short run. Thus, these models do embody fundamental view of the Friedman (1968), seminal Presidential Address to the American Economic Association.

recently by New-Keynesian or Neo-Keynesian economists such as Blanchard (2018) and Stiglitz (2018). For example, Blanchard (2018) takes a negative opinion on the assumption on which these models are built. First, aggregate demand is derived as consumption demand for infinitely long lived and foresighted consumers. Blanchard (2018) continues to argue that its implications for the degree of foresight (through the value of discount factor) and the role of interest rate. And, price adjustment is characterized by a forward-looking inflation, which does not capture the fundamental inertia of inflation. The idea that a theory of unemployment[2] can fruitfully be built on the assumption that trade in the labor market is an economic activity was first explored by a number of authors in the late 1960s, in what became known as search theory. The most influential papers in this tradition were Alchian (1969), Phelps (1968), and Mortensen (1970); they were collected with other contributions in the same spirit in the Phelps volume (Phelps et al. (1970)). The driving behind this research came from Phelps's (1967) and Friedman's (1968) reappraisal of the Phillips curve (see Phillips (1958)) and the natural rate approach to which this led. Early search theory assumed the existence of a distribution of wage offers for identical jobs; unemployment arose in equilibrium because workers rejected low-wage jobs[3].Unemployment in capitalist economies is mostly involuntary. John Maynard Keynes in his The General Theory of Employment, Interest and Money (1936), argued that capitalist economy could poses equilibria that are characterized by the persistent involuntary unemployment[4], see also, Akerlof,Yellen (1987).According to Taylor (2008) ..” The most common and analytically useful definition of involuntary unemployment is based on the labor supply curve: if workers are off the labor supply curve – so that there is an excess supply of labor at the current real wage – then, by definition, there is involuntary unemployment. The amount of involuntary unemployment is equal to the amount of excess labor supply.” Broadly speaking two types of models exist which can explain short run swings in the unemployment rate. First. are models grounded in Keynesian fashion(tradition) that include wage-price stickiness “in the face of variations in (nominal) aggregate demand” see Calvo (1982) and Barro, Grossman (1971), Taylor (1979)[5].Second type models, assume that there are no institutional barriers to perfect price wage flexibility see for example Azariadis (1975), Baily (1974), Salop (1979),and monetary shocks see: Lucas ,Rapping (1969), Lucas (1972),Lucas (1975), Blinder ,Fischer (1978). From a more recent literature, the unemployment rate and the labor force participation rate have been discussed prominently in the light of the Great Recession (2007-2009). One shortcoming of standard monetary dynamic stochastic general equilibrium (DSGE) models is that “they are silent about these important variables”, see Cristiano et al. (2021). The Diamond-Mortensen-Pissarides (DMP) search and matching approach of unemployment represents a leading framework and has been integrated into monetary models by a number of authors, see for example : Blanchard and Gali (2010), Campolmi , Gnocchi (2016) , Christiano, Ilut, Motto and Rostagno (2008), Christoffel, and Kuester (2008), Gertler, Sala and Trigari (2009), Krause,Lopez-Salido ,Lubik (2008) etc. According to Gali (2010), Cheron ,Langot (2000) “were the first to bring together nominal rigidities and labor market frictions, showing how the resulting framework could generate both a Beveridge curve (a negative correlation between vacancies and unemployment) and a Phillips curve (a negative correlation between inflation and unemployment) in the presence of both technology and monetary shocks”. Afterwards Walsh (2003), Walsh (2005) and Trigari (2009) analyzed the impact of labor market frictions into the basic New Keynesian model with sticky prices but flexible wages, with a focus on the size and persistence of the effects of monetary policy shocks. Later models relaxed the assumptions of flexible wages, and introduced forms of nominal and real wage rigidity, see Trigari (2006) and Christoffel ,Linzert (2005).

[2] Unemployment to an average person is an involuntary idleness (Andolfatto,2006 in The New Palgrave Dictionary of Economics, 2nd edition, 2008). This is inconsistent in the way in which unemployment is in fact defined and measured. Because according to International Labor Organization (ILO) conventions, which are followed by most of the nation's labor force surveys, unemployment relates to those individuals that are unemployed but are actively searching job. Those unemployed who are not actively included in search are classified as non-participants.

[3] This aspect of the theory was criticized both on logical grounds see Rothschild (1973) and on empirical grounds (Tobin (1972); Barron (1975)). An equilibrium model that met Rothschild's criticisms, but with a trivial role for workers looking for alternative jobs, was first presented in Lucas and Prescott (1974).

[4] For Keynes worker is involuntary unemployed if the market wage for his labor exceeds his shadow wage, which is a wage at which a worker would be indifferent between not accepting and accepting job offer, see Hahn (1987).

[5] “These models can exhibit “genuine” or “involuntary” unemployment because at equilibrium there may exist an excess supply of labor”. see Calvo (1982)

Later, the focus has shifted toward normative analysis, and the implications of labor market frictions and unemployment for the design of monetary policy (see Blanchard,Gali (2010) in a model with real wage rigidities, Thomas (2008) in a model with nominal wage rigidities, provides explanation of optimal monetary policy in the context of New Keynesian model with labor market frictions. One can also take a look at Arseneau ,Chugh (2008) in a model with flexible prices and quadratic costs of nominal wage adjustment. Large literature pinpoints on the importance on matching frictions and job rationing[6] as a source of unemployment, Michaillat (2012)[7].As, it is said in that paper that previous search-and-matching models of unemployment, either with bargained wages see Pissarides (2000) , or rigid wages such as in Hall (2005a), converge asymptotically[8]to full employment which makes the "inadequate to study recessionary unemployment" , see Michaillat (2012). Rationing unemployment measures the shortage of jobs in the absence of matching frictions. In a model where output is produced with capital and labor if the production function is of the usual Cobb-Douglass type [9] and if the model features diminishing marginal product of labor and for a given wage firms hire workers until marginal product of labor falls to equal the real wage. This is rationing unemployment, see Clymo (2020). Frictional unemployment is attributed to matching frictions. Or to hire workers firms post vacancies at flow costs, which are filled when firms match with worker. Afterwards, workers produce their MPL (marginal product of labor) for the firm in each period, worker is being paid until the match ends. The cost of hiring is paid upfront while the firm's benefit (MPL-wage), is spread over time. If the firms discount future more heavily, they will discount the future benefits of hiring more relative to the cost of hiring, which are paid current. Previous will reduce the incentives to hire, and hence raises equilibrium unemployment, and it is a core mechanism of unemployment see in Hall (2017) and Kehoe et al.(2019).Second paper shows how variations in the discount rate are a driving force for unemployment fluctuations. The model describes a labor market under the influence of volatile financial impulses and fluctuations in productivity that arise outside the model. There is no assumption that these influences are exogenous. Instead, the model shows what happens on the labor market when discounts for risky payoffs rise substantially. Blanchard, Gali (2007a) construed model with labor market frictions, real wage rigidities, and staggered price setting. That model thus integrated Diamond-Mortensen-Pissarides search-model of labor market flows. Labor market frictions, real wage rigidities, and staggered price setting are all needed to explain movements in unemployment, the effects of change in productivity in economy, and the role of monetary policy in shaping those effects. Unemployment is more persistent in "sclerotic "labor market, and strict inflation targeting in a presence of real wage rigidities does not deliver best monetary policy results. As in Blanchard, Gali (2007b) , the best monetary policy comes with accommodation of inflation and comes with some persistence in unemployment. The degree of labor market tightness $\theta_t = \frac{v_t}{u_t} = \frac{\text{job vacancies}}{\text{unemployment rate}}$ is the central element in an economy of a country, and a tighter labor market could induce inflation issues by raising marginal costs, which becomes issue of the relationship between unemployment and labor market tightness. This paper will review examples of inflation dynamics under optimal monetary policy and discretionary monetary policy in a liquidity trap, unemployment fluctuations in a discrete time search model, Labor market with precautionary savings with a reference to HJB equation (Hamilton-Jacobi-Bellman), Optimal Public Expenditure with Inefficient Unemployment, McCall job search model, DMP model of job search, Gali (2010) Monetary policy and unemployment, and rationing unemployment and frictions unemployment in DSGE framework.

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[6] Models of job ration include efficiency wage models, Solow (1979), Akerlof gift-exchange model (1982), insider-outsider models such as Lindbeck ; Snower (1988),and social norm models Akerlof (1980).

[7] This survey in modeling the matching frictions used the literature and it imposed a vacancy posting costs, see: Pissarides (1985); Mortensen ,Pissarides (1994); Pissarides (2000); Shimer (2005); Hall (2005a).As for the wage schedule in such an environment: The marginal product of labor always exceeds the flow value of unemployment, normalized to zero, so there are always mutual gains from matching. But there is no compelling theory of wage determination there. In other models such as those as: Hall (2005b); Shimer (2012), the labor market rapidly converges to an equilibrium in which inflows to and outflows from employment are large.

[8] $\{X_n; n = 1,2 \ldots \infty\} \to X \,; \forall \epsilon, \delta > 0 \,; \exists N \; s.t. \forall \; n \geq N; \Pr\{|X_n - X| > \delta\} < \epsilon; \forall \delta > 0, \lim_{n\to\infty} \Pr\{|X_n - X| > \delta\} = 0; \operatorname{plim} X_n = X.$

[9] $y = zk^{\alpha}l^{1-\alpha}$ where z is TFP total factor productivity and the wage is equal to marginal product of labor:$(1-\alpha)zk^{\alpha}l^{-\alpha} = w$

2.Inflation dynamics under optimal monetary policy and Discretionary monetary policy in a liquidity trap

First, we introduce loss function for the monetary authority

equation 1

$$\mathcal{L} = \frac{1}{2}(\hat{x}_t^2 + \lambda\hat{\pi}_t^2), \lambda > 0$$

Loss function is s.t. NK Phillips curve:

equation 2

$$\hat{\pi}_t = \beta\mathbb{E}\{\hat{\pi}_{t+1}\} + k\hat{x}_t + u_t$$

Where cost push shocks follow AR(1) process and $\hat{x}_t$ is the output gap:

equation 3

$$u_{t+1} = \phi u_t + \epsilon_{t+1}; o \leq \phi \leq 1$$

monetary authority's weight on inflation is $\lambda = 1$,with zero mean and varinace is σ_u^2.Monetary authority will minimize:

equation 4

$$\min \mathcal{L} = \frac{1}{2}(\hat{x}_t^2 + \lambda(k\hat{x}_t + \xi_t)^2)$$

In previous $\xi_t \equiv \beta\mathbb{E}_t\{\hat{\pi}_{t+1}\} + u_t$.FOC for this problem is : $\hat{x}_t = -k\lambda\hat{\pi}_t$, which when plugged into NK Phillips curve gives:

equation 5

$$\hat{\pi} = \frac{\beta}{1 + k^2\lambda}\mathbb{E}_t\{\hat{\pi}_{t+1}\} + \frac{1}{(1 + k^2\lambda)} + u_t$$

This is a single stochastic diff.equation in $\hat{\pi}_t$ taking as given the process for the cost push shocks u_t . Now if $\hat{\pi}_t = \psi_{\pi u} u_t$ so $\mathbb{E}_t\{\hat{\pi}_{t+1}\} = \psi_{\pi u}\phi u_t$ so that pluging into SDE gives:

equation 6

$$\psi_{\pi u} u_t = \frac{\beta}{1 + k^2\lambda}\psi_{\pi u}\phi u_t + \frac{1}{(1 + k^2\lambda)} u_t$$

We have restriction that :

equation 7

$$\psi_{\pi u} = \frac{\beta}{1 + k^2\lambda}\psi_{\pi u}\phi u_t + \frac{1}{(1 + k^2\lambda)}$$

And so :

$$\psi_{\pi u} = \frac{1}{(1 - \phi\beta) + k^2\lambda}$$

Since discretionary policy is $\hat{x}_t = -k\lambda\hat{\pi}_t$ we have:

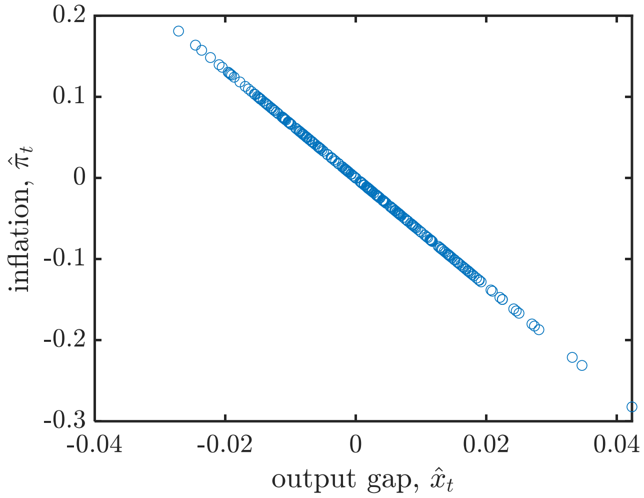
equation 8

$$\psi_{xu} = -k\lambda\psi_{\pi u} = -\frac{k\lambda}{(1-\phi\beta)+k^2\lambda}$$

Cost push shock $u_t > 0$ increases inflation $\hat{\pi}_t > 0$ and reduces the output gap $\hat{x}_t < 0$,Graphicall shown in inflation output gap graph.

inflation dynamics under optimal monetary policy

Figure 1inflation dynamics under optimal monetary policy



Source :Authors calculations based on a code by Chris Edmond available at: http://www.chrisedmond.net/hons2019.html

Discretionary monetary policy in a liquidity trap. Consider a continuous-time new Keynesian model with dynamic IS curve

equation 9

$$\dot{x}(t) = \frac{1}{\sigma}\big(i(t) - \pi(t) - r^n(t)\big)$$

And Phillips curve:

equation 10

$$\dot{\pi}(t) = \rho\pi(t) - kx(t)$$

Here $x(t)$ is the lo output gap,$\pi(t)$ is the instantaneous inflation rate , $i(t)$ is the nominal interest rate,and $r^n(t)$ is the natural rate.Natural rate follows:

equation 11

$$r^n(t) = \begin{cases} \underline{r} & t \in [0,T) \\ \bar{r} & t \in [T,\infty) \end{cases} \qquad \underline{r} < 0 < \bar{r}$$

For a given period $T > 0$ or horizon $\underline{r} < 0$ so that zero lower bound on $i(t)$ is binding for $\forall t \in [0,T)$ .Now lets suppose that $r^n(t) = \bar{r} > 0$ so that it is possible to implement $i(t) = \bar{r}$.So after liquidity trap i.e. $\forall t \in [T,\infty)$ MP implements $i(t) = \bar{r}$ via sufficiently interactive interest rate rule:

equation 12

$$i(t) = \bar{r} + \phi_\pi \pi(t), \phi_\pi > 1$$

In the absence of shocks we have $\pi(t), x(t) = (0,0)\ \forall t \in [T, \infty)$ and in particular $\pi(T0, x(T) = (0,0)$. During the liquidity trap we have $r^n(t) = \underline{r} < 0$ and $i(t) = 0$ so that evolution in time of output gap is:

equation 13

$$\dot{x}(t) = -\frac{1}{\sigma}(\pi(t) + \underline{r})$$

$\dot{x}(t) > 0$ whenever $\pi(t) + \underline{r} < 0$ i.e. $\pi(t) < -\underline{r}$. On the next graph $\dot{x}(t) = 0$ isocline is vertical line at $\pi < -\underline{r}$ and divide π, x plane into two halves one the left at $\pi < -\underline{r}$ with relatively low inflation where the output gap is increasing and to the right $\pi(t) < -\underline{r}$ with relatively high inflation where the output gap is decreasing.NK Phillips curve is $\dot{\pi}(t) = \rho\pi(t) - kx(t)$ so that $\dot{\pi}(t) > 0$ whenever $\rho\pi(t) - kx(t) > 0$ i.e. $x(t) < \left(\frac{\rho}{k}\right)\pi(t)$.In terms of graph $\dot{\pi}(t) = 0$ is a isocline a straight line $x = \left(\frac{\rho}{k}\right)\pi$ and divides π, x phase plane into two halves, one above the line $x = \left(\frac{\rho}{k}\right)\pi$ where inflation is decreasing and the other below the line $x = \left(\frac{\rho}{k}\right)\pi$ where inflation is increasing.The intersection of the two lines introduces pseudo-steady state $(\bar{\pi}, \bar{x}) = \left(-\underline{r}, -\left(\frac{\rho}{k}\right)\underline{r}\right)$ where neither inflation nor output gap are changing. This pseudo state vanishes when liquidity trap is over i.e. when $r^n(t) = \bar{r}$.If we differentiate NK Phillips curve we get:

equation 14

$$\ddot{\pi} = \rho\dot{\pi}(t) - k\dot{x}(t)$$

If we substitute in the IS curve $\dot{x}(t) = \frac{1}{\sigma}\left(i(t) - \pi(t) - r^n(t)\right)$ we get:

equation 15

$$\ddot{\pi} - \rho\dot{\pi}(t) - \frac{k}{\sigma}\left(\pi(t) + \underline{r}\right) = \frac{k}{\sigma}\underline{r}$$

The roots of previous are given by the quadratic:

equation 16

$$r^2 - \rho r - \frac{k}{\sigma} = 0$$

Since $r_1 + r_2 = \rho > 0$ and determinant $r_1 r_2 = -\left(\frac{k}{\sigma}\right)$ one root is positive and one is negative. Solutions of the homogenous equation are given as:

equation 17

$$\pi(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t}$$

Since $\bar{\pi} = -\underline{r} > 0$ [10],so that the general solution has the form of :

$$\pi(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t} + \bar{\pi}$$

And r_1, r_2 are the roots of the quadratic and $\bar{\pi} = -\underline{r} > 0$,c_1, c_2 are two constants to be determined.General solution impies that:

[10] Since $\pi(t) = \bar{\pi}$; $\ddot{\pi} = \dot{\pi}(t) = 0$ or $0 - \rho 0 - \frac{k}{\sigma}\bar{\pi} = \frac{k}{\sigma}\underline{r}$

equation 18

$$\pi(0) = c_1 + c_2 + \bar{\pi}$$

Terminal conditions were $\pi(T) = x(T) = 0$ so now:

equation 19

$$\pi(T) = c_1 e^{r_1 t} + c_2 e^{r_2 t} + \bar{\pi} = 0$$

When $t = T$ we have $\dot{\pi}(T) = c_1 r_1 e^{r_1 t} + c_2 r_2 e^{r_2 t} = 0$ this sonstitutes equation with two unknowns or a system represented as:

equation 20

$$\begin{pmatrix} e^{r_1 T} & e^{r_2 T} \\ r_1 e^{r_1 T} & r_2 e^{r_2 T} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + \begin{pmatrix} \bar{\pi} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Alebra gives:

equation 21

$$c_1 = \frac{r_2}{r_1 - r_2} e^{-r_1 T} \bar{\pi}$$
$$c_2 = -\frac{r_2}{r_1 - r_2} e^{-r_2 T} \bar{\pi}$$

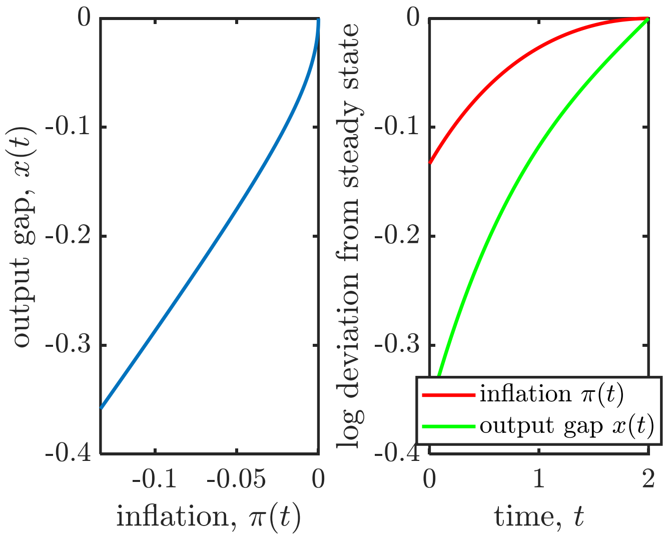
If we plug these back into inflation equation[11]:

equation 22

$$\pi(t) = \bar{\pi} - \frac{1}{r_1 - r_2} \left[r_1 e^{r_2 (T-t)} - r_2 e^{-r_1 (T-t)} \right] \bar{\pi}$$

The initial output gap from the new Keynesian Phillips curve and our solution for inflation are given as:$x(0) = \frac{\rho\pi(0) - \dot{\pi}(0)}{k}$

Figure 2 discretionary monetary policy in a liquidity trap



[11] So that indeed $\pi(t) = 0$ as $t \to T$

3.Unemployment fluctuations in a discrete time search model

Time here is $t = 0,1,2, \ldots$ and the Cobb-Douglas matching function is given as:

equation 23

$$F(u_t, v_t) = \bar{m} u_t^{1-a} v_t^{a} \,; \bar{m} > 0, 0 < a < 1$$

Where $u_t$denotes unemployment rate and v_t is the vacancy rate. Labor market tightness is given as:

equation 24

$$\theta_t = \frac{v_t}{u_t}$$

Firms productivity $z_t > 0$ follows exogenous stochastic process with long term mean $\bar{z} > 0$,he worker receives wage w_t and the firms profits are $z_t - w_t$ .Job matches are exogenously destroyed at time t with probability $\delta \in (0.1)$.Evolution of the aggregate unemployment rate is :

equation 25

$$u_{t+1} - u_t = \delta(1 - u_t) - f(\theta_t) u_t$$

The value to a firm for a vacancy V_t and filled job J_t is:

equation 26

$$V_t = -k\,\bar{z} + \beta \mathbb{E}_t\{q(\theta_t) J_{t+1} + (1 - q(\theta_t) V_{t+1}\}$$
$$J_t = z_t - w_t + \beta \mathbb{E}_t\{\delta V_{t+1} + (1 - \delta) J_{t+1}\}$$

U_t, W_t denote workers value of unemployment and employment[12].

equation 27

$$U_t = b\bar{z} + \beta \mathbb{E}_t\{f(\theta_t) W_{t+1} + (1 - f(\theta_t) U_{t+1}\}$$
$$W_t = w_t + \beta \mathbb{E}_t\{\delta U_{t+1} + (1 - \delta) W_{t+1}\}$$

And for Nash bargaining in order to determine the workers wage:$W - U = \frac{\lambda}{1-\lambda}(J - V)$. The first Bellman equation for filled jobs if $V = \neq 0$ is :

equation 28

$$J = \frac{\bar{z} - w + \beta\delta V}{1 - \beta(1 - \delta)} = \frac{\bar{z} - w}{1 - \beta(1 - \delta)} = \frac{k\bar{z}}{\beta q(\theta)}$$

Rearranging for wage we get :

equation 29

$$w = \bar{z} - \frac{1 - \beta(1 - \delta)}{\beta} \frac{k\bar{z}}{q(\theta)}$$

Since discount rate $r \equiv \frac{1}{\beta} - 1$ we get :

equation 30

$$w = \bar{z} - (r + \delta) \frac{k\bar{z}}{q(\theta)}$$

[12] In non-stochastic case $U = b\bar{z} + \beta\{f(\theta_t) W_{t+1} + (1 - f(\theta_t) U\}$ $W_t = w + \beta\{\delta U_{t+1} + (1 - \delta) W\}$

On the workers side from things from Nash bargaining we get :$W - U = \frac{\lambda}{1-\lambda}(J - V) = \frac{\lambda}{1-\lambda}\frac{k\bar{z}}{q(\theta)}$. The Bellman's equation for unemployment gives:

equation 31

$$(1-\beta)U = b\bar{z} + \beta f(\theta)\frac{\lambda}{1-\lambda}J = b\bar{z} + \frac{\lambda}{1-\lambda}\frac{k\bar{z}}{q(\theta)}$$

Since $fq(\theta) = \theta q(\theta)$ [13]from Bellman equation we get Bellman eq. for unemployment:

equation 32

$$W - U = \frac{w + \beta\delta U}{1 - \beta(1-\delta)}$$

By using Nash-Bargaining we get:

equation 33

$$\frac{w - (1-\beta)U}{1-\beta(1-\delta)} = \frac{\lambda}{1-\lambda}J = \frac{\lambda}{1-\lambda}\frac{\bar{z} - w}{1-\beta(1-\delta)} \Rightarrow w - (1-\beta)U = \frac{\lambda}{1-\lambda}(\bar{z} - w)$$

Since $(1-\beta)U = b\bar{z} + \frac{\lambda}{1-\lambda}\frac{k\bar{z}}{q(\theta)}$ we get :

equation 34

$$w = (1-\lambda)b\bar{z} + \lambda(1 + k\theta)\bar{z}$$

We can back out unemployment from Beveridge curve:

equation 35

$$\bar{u} = \frac{\delta}{\delta + f(\bar{\theta})}$$

Since $\bar{v} = \bar{\theta}\bar{u}$ we can write the intersection of marginal productivity condition and the wage curve as:

equation 36

$$\frac{\overline{w}}{z} = 1 - (r+\delta)\frac{k}{q(\theta)} = (1-\lambda)b + \lambda(1 + k\bar{\theta})$$

$\theta = \frac{v}{u}$ is a market tightness, and for the firms probability of filling a vacancy is given as:$\frac{m(u,v)}{v} = m\left(\frac{1}{\theta}, 1\right) \equiv q(\theta)$, and $q'(\theta) < 0$; and for the workers probability of finding a job is: $\frac{m(u,v)}{v} = m(1,\theta) \equiv \theta q(\theta)$.There flowing applies : $\lim_{\theta\to 0}[\theta q(\theta)] = \lim_{\theta\to\infty} q(\theta) = 0$ and $\lim_{\theta\to\infty}[\theta q(\theta)] = \lim_{\theta\to\infty} q(\theta) = +\infty$.In the following example MTAB simulation and Dynare used following parameters: α = 0.5; // match elasticity on unemployment, β = 1/1.02; // time discount factor; λ = 0.5; // labor's weight in Nash bargaining $\bar{z}$ = 0; // long-run productivity *log* level, $\overline{m}$ = 0.5; // match efficiency, b = 0.4; // unemployment benefits ,δ = 0.04; // job destruction rate; κ = 0.28; // vacancy cost V = 0; // free entry condition. Results are given in the following graph:

[13] $f(\theta)$ is a job finding probability

Figure 3 discretionary monetary policy in a liquidity trap

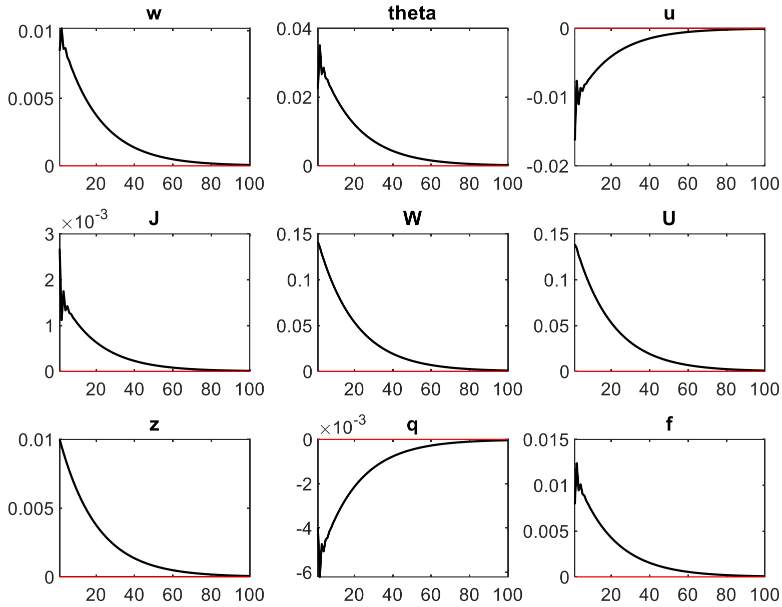


Figure 4 Convergence of productivity, unemployment rate, wages, labor market tightness and vacancy rate to steady-state

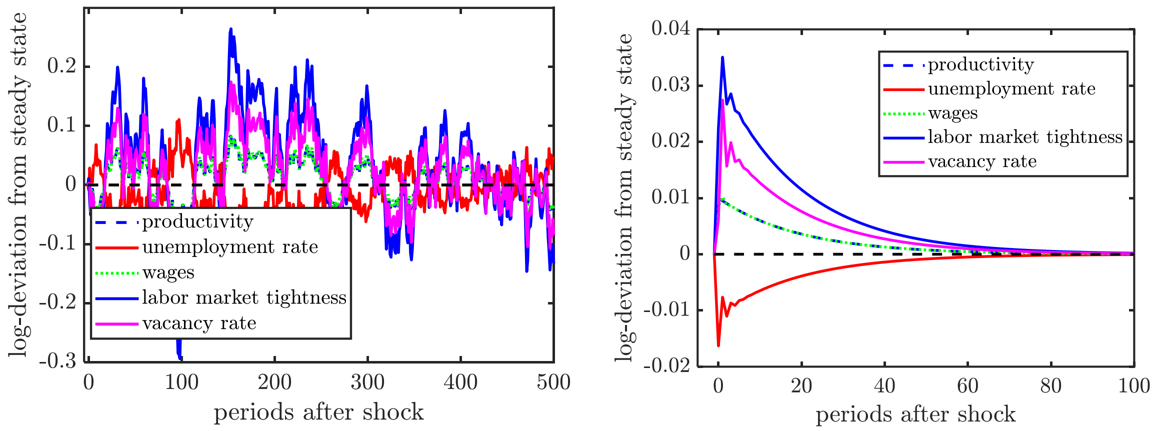


Table 1 Steady-state results

w	-0.0166961	Wage
θ	1.99672	Market tightness
u	-3.95806	Unemployment rate
J	0.281477	Value of filled job
W	49.5926	Worker's value of employment
U	49.3111	Worker's value of unemployment
z	0	productivity
q	0.0145399	The vacant job costs per unit time
f	0.719907	Probability of finding job

4.Labor market with precautionary savings with a reference to HJB equation

This part follows code written by Krusell, Mukoyama, Sahin (2010),in a continuous-time setting using a finite-difference method as explained in Achdou et al.(2022) : two functions v_1, v_2 at I discrete points in the space dimension a_i , $i = 1, \dots, I$. Equispaced grids are denoted by Δa_ias the distance by the grid points, and shot hand notation used is $v_{i,j} \equiv v_j(a_i)$ and so on. Backward difference approximation is given as:

equation 37

$$\begin{cases} v_j'(a_i) \approx \dfrac{v_{i+1,j} - v_{i,j}}{\Delta a} \equiv v_{i,j,F}' \\ v_j'(a_i) \approx \dfrac{v_{i+1,j} - v_{i-1,j}}{\Delta a} \equiv v_{i,j,B}' \end{cases}$$

Now for the Kolmogorov Forward (Fokker-Planck[14]) equation we have folowing: let x be a scalar diffusion :$dx = \mu(x)dt + \sigma(x)dW, x(0) = x_0$.Let's suppose that we are interested in the evolution of the distribution of $x, f(x,t)$ and $\lim_{t\to\infty} f(x,t)$ .So, given an initial distribution $f(x,0) = f_0(x), f(x,t)$ satisfies PDE :

equation 38

$$\frac{\partial f(x,t)}{\partial t} = -\frac{\partial}{\partial x}[\mu(x)f(x,t)] + \frac{1}{2}\frac{\partial^2}{\partial x^2}[\sigma^2(x)f(x,t)]$$

Previous PDE is called "Kolmogorov Forward Equation" or "Fokker-Planck Equation".

Corollary 1: if a stationary equilibrium exists $\lim_{t\to\infty} f(x,t) = f(x)$, it satisfies ODE

equation 39

$$0 - \frac{d}{dx}[\mu(x)f(x)] + \frac{1}{2}\frac{d^2}{dx^2}[\sigma^2(x)f(x)]$$

The deterministic optimal control problem is given as:

equation 40

$$V(x_0) = \max_{u(t)_{t=0}^{\infty}} \int_0^\infty e^{-\rho t} h(x(t), u(t)dt \text{ s.t. } \dot{x}(t) = g\big(x(t)\big), u(t), u(t) \in U\,; t \geq 0, x(0) = x_0$$

In previous expression:$\rho \geq 0$ is the discount rate, $x \in X \subseteq \mathbb{R}^m$ is a state vector; $u \in U \subseteq \mathbb{R}^n$ is a control vector, and $h: X \times U \to R$. The value function of the generic optimal control problem satisfies the Hamilton-Jacobi-Bellman equation, i.e.:

equation 41

$$\rho V(x) = \max_{u\in U} h(x,u) + V'(x) \cdot g(x,u)$$

In the case with more than one state variable $m > 1, V'(x) \in \mathbb{R}^m$ is the gradient of the value function. Now for the derivation of the discrete-time Bellman eq. we have: time periods of length Δ ,discount factor $\beta(\Delta) = e^{-\rho\Delta}$, here we can note that $\lim_{\Delta\to\infty} \beta(\Delta) = 0$ and $\lim_{\Delta\to 0} \beta(\Delta) = 1$.Now that discrete Bellman equation is given as:

equation 42

$$V(k_t) = \max_{c_t} \Delta U(c_t) + e^{-\rho\Delta} V(k_{t+\Delta}) \text{ s.t. } k_{t+\Delta} = \Delta[F(k_t) - \delta k_t - c_t] + k_t$$

[14] See Fokker (1914), Planck (1917), Kolmogorov (1931).

In the multivariate case Kolmogorov Forward Equation is given as:

equation 43

$$\frac{\partial f(x,t)}{\partial t} = -\sum_{i=1}^{m} \frac{\partial}{\partial x_i}[\mu(x) f(x,t)] + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m} \frac{\partial^2}{\partial x^2}\left[\sigma_{ij}^2(x) f(x,t)\right]$$

Hamilton-Jacobi-Bellman equation in stochastic settings is given as:

equation 44

$V(x_0)) = \max_{u(t)_{t=0}^{\infty}} \mathbb{E}_0 \int_0^{\infty} e^{-\rho t} h\big(x(t), u(t)\big) dt$ s.t. $dx(t) = g\big(x(t), u(t)\big) dt + \sigma\big(x(t)\big) dW(t)$, $u(t) \in U$; $t \geq 0, x(0) = x_0$

In previous expression $x \in \mathbb{R}^m; u \in \mathbb{R}^n$. HJB equation without derivation is :

equation 45

$$\rho V(x) = \max_{u \in U} h(x,u) + V'(x) g(x,u) + \frac{1}{2} V''(x) \sigma^2(x)$$

HJB equation was a result of the theory of dynamic programing pioneered by Richard Bellman (namely Bellman(1954),Bellman(1957),Bellman, Dreyfus,(1959). Back to our model KMS, model depart from DMP model (Diamond-Mortensen-Pissarides ;Diamond(1982); Pissarides (1985); and Mortensen and Pissarides (1994)) in was that workers can insure themselves against job loss by accumulating assets. Alternatively, one might think about it as an Aiyagari (1994) model with endogenous job-finding rate. The main computational challenge compared to the standard models comes from wage setting. Heterogeneity in wealth creates heterogeneity in the value of unemployment.Labor market in this models is that vacant jobs and unemployed workers are matching randomly in every instant according to agregate matching function.And the job finding and job filling rates are given as:

equation 46

$$f_t = \frac{M(u_t, v_t)}{u_t} = M(1, \theta_t); q_t = \frac{M(u_t, v_t)}{v_t} = M(\theta_t^{-1}, 1)$$

Unemployment rate motion is given as:

equation 47

$$\dot{u} = \sigma(1 - u_t) - f_t u_t$$

Where σ as λ previously is a Poisson process. On the asset market, there are two assets capital and equity, which is a claim to firm's profit. Now, r is rate of return of capital, p is the price of equity, and d is a dividend it pays in every instant. No arbitrage rate of return on riskless assets is :$r_t - \delta = \frac{d_t + \dot{p}_t}{p_t}$. Assets of the worker are given as:

equation 48

$$a_t = s_t^K K_t + s_t^p p_t = s_t(K_t + p_t)$$

Where s_t is the share in the total wealth of the economy held by a worker (workers are heterogeneous), s^K, s^p are portfolio weights. Workers status is $s_t \in \{e, u\}$ employed, unemployed, they do not value leisure and are always on a search for a job. Unemployed are engaged in home production (they do not pay taxes and their home production is not financed by taxes) resulting in a flow of income h, while they earn wage $w(a_t)$ to be determined via Nash bargaining. The workers problem can be written as:

equation 49

$$W(a_0, s_0) = \max_{\{c_t\}_{t\geq 0}} \mathbb{E}_0 \int_0^{\infty} e^{-\rho t} u(c_t) dt\,;$$
$$\dot{a}_t = y(a_t, s_t) + (r_t - \delta)a_t - c_t$$
$$a_t \geq a$$
$$s_t \in \{e, u\}\; Poisson\; process\; PDF, f_t, \sigma$$
$$y_t(a_t, s_t) = \begin{cases} w(a_t) & s_t = e \\ h & s_t = u \end{cases}$$

Where y_t is income.In recursive form problem of employed worker can be summarized by a pair of HJB equation and state-constraint boundary condition:

equation 50

$$\rho W(a, e, t) = \max_e \begin{Bmatrix} u(c) + \partial_a W(a, e, t)[w(a, t) + (r_t - \delta)a - c] + \\ \sigma[W(a, u, t) - W(a, e, t)] + \partial_t W(a, e, t) \end{Bmatrix}$$
$$\partial_a W(a, e, t) \geq u'(w(a, t) + (r_t - \delta)\underline{a})$$

The problem of unemployed worker is analogously:

equation 51

$$\rho W(a, u, t) = \max_c \begin{Bmatrix} u(c) + \partial_a W(a, u, t)[h + (r_t - \delta)a - c] + \\ f_t[W(a, e, t) - W(a, u, t)] + \partial_t W(a, u, t) \end{Bmatrix}$$
$$\partial_a W(a, u, t) \geq u'(h + (r_t - \delta)\underline{a})$$

Consumption and savings policy function are given as HJB equations:

equation 52

$$c(a, s, t) = (u')^{-1}\big(\partial_a W(a, s, t)\big)$$
$$\dot{a}(a, s, t) = y(a, s, t) + (r_t - \delta)a - c(a, s, t)$$

The savings function is above mentioned Kolmogorov forward equation:

equation 53

$$\partial_t(a, e, t) = -\partial_a[\dot{a}(a, e, t)g(a, e, t)] + f_t g(a, u, t) - \sigma g(a, e, t)$$
$$\partial_t(a, u, t) = -\partial_a[\dot{a}(a, u, t)g(a, u, t)] + f_t g(a, e, t) - \sigma g(a, u, t)$$

Dynamics of the distribution of workers over individual states is $g(a, s)$ .About the firms as in standard DMP model time dependent HJB equations can be written as:

equation 54

$$(r_t - \delta)J(a, t) = \max_k \big\{ z_t F(k) - r_t k - w(a, t) + \partial_a J(a, t)\dot{a}(a, e, t) + \sigma\big(V(t) - J(a, t)\big) + \partial_t J(a, t) \big\}$$
$$(r_t - \delta)V(t) - -\xi + q_t \int_0^{\infty} J(a, t) \frac{g(a, u, t)}{u_t} da + \partial_t V(t)$$

Where ξ is a flow vacancy cost, and $\frac{g(a,u,t)}{u_t}$ I marginal distribution of unemployed over assets, and free entry implies that $V(t) \equiv 0$ and thus:

equation 55

$$(\sigma + r_t - \delta)J(a,t) = \max_k\{z_t F(k) - r_t k - w(a,t) + \partial_a J(a,t)\dot{a}(a,e,t) + \partial\_tJ(a,t)\}$$

$$\xi = q_t \int_0^{\infty} J(a,t)\frac{g(a,u,t)}{u_t}da$$

About the wage Nash bargaining implies:

equation 56

$$(1-\beta)[W(a,e) - W(a,u)] = \beta J(a)$$

Resulting wage[15] is given as:

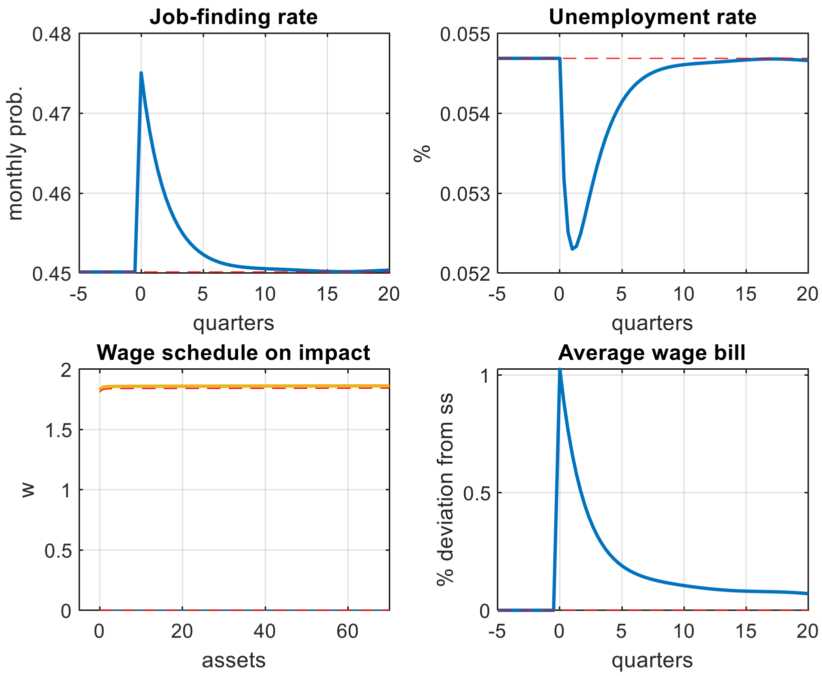
equation 57

$$w(a)\left[(1-\beta)\frac{\partial_a W(a,e)}{\rho+\sigma} + \beta\frac{1-\partial_a J(a)}{r-\delta+\sigma}\right] = \beta\frac{zF(k) - rk + \partial_a J(a)\,[(r-\delta)a - c_e]}{r-\delta+\sigma} -$$

$$(1-\beta\,)\frac{u(c_e) + \partial_a W[(r-\delta)a - c_e] - \rho W(a,u)}{r+\rho}$$

Next labor market with precautionary savings will be presented.

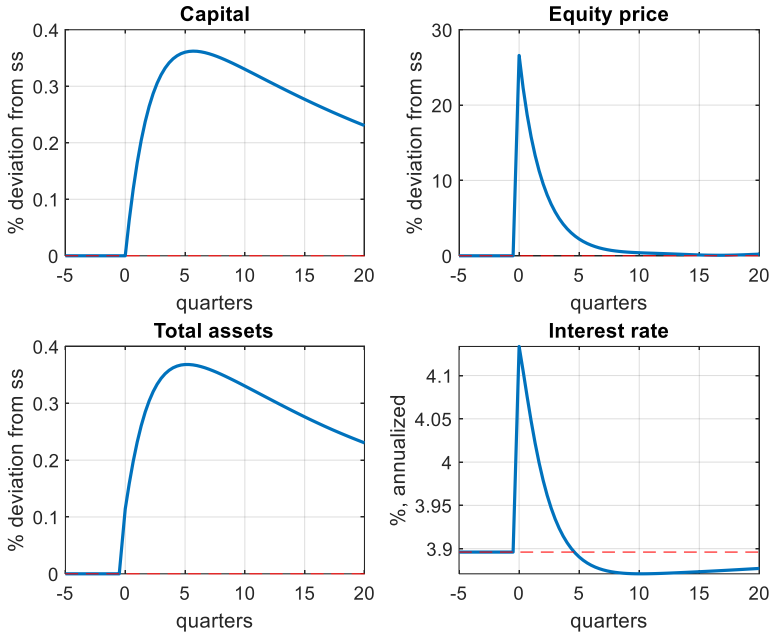
Figure 5 Impulse responses to a positive technology shock 1: labor market



Source: solved with Benjamin Moll codes https://benjaminmoll.com/codes/

[15] This wage is a result of an egalitarian bargaining. Otherwise, wage was implicitly given as: $w(a) = \arg\max_w[\widetilde{W}(w,a,e) - W(a,u)]^{\beta}[\tilde{J}(w,a)]^{1-\beta}$, with FOC: $\frac{\beta\partial_w\widetilde{W}}{W(a,e)-W(a,u)} + \frac{(1-\beta)\partial_w\tilde{J}}{J(a)} = 0$.Resulting wage is given by following: $W(a) = \beta\frac{zF(k)-rk+\partial_a J(a)\,[(r-\delta)a-c_e]}{1-\partial_a J(a)(1-\partial_w c_e)} - (1-\beta)\frac{u(c_e)+\partial_a W[(r-\delta)a-c_e]-\rho W(a,u)}{\partial_a W(a,e)}$

Figure 6 Impulse responses to a positive productivity shock 2: asset market



Source: solved with Benjamin Moll codes https://benjaminmoll.com/codes/

5.Optimal Public Expenditure with Inefficient Unemployment (Michaillat,Saez (2019))

Michaillat,Saez (2019) , propose a theory of optimal public expenditure when unemployment is inefficient.And the theory is based on matching model.Optimal expenditure deviates from Samuelson (1954) rule to reduce unemployment gap which is difference between current and efiicient rates of unemployment.Remember Samuelson rule for optimal provision of publis goods was:$\sum_{i=1}^{n} MRS = MRT$.$\sum_{i=1}^{n} MRS$ is a marginal rate of substitution [16].In this paper When unemployment is inefficiently high and the multiplier is positive, the formula yields the following results. First, optimal stimulus spending is positive and increasing in the unemployment gap. Second, optimal stimulus spending is zero for a zero multiplier, increasing in the multiplier for small multipliers, largest for a moderate multiplier, and decreasing in the multiplier beyond that. Third, Optimal stimulus spending is zero if extra public goods have no value, it becomes larger as the elasticity of substitution increases, and it completely fills the unemployment gap if extra public goods are as valuable as extra private goods.Deviation from Samuleson rule is "stimulus spending".

Definition 1. The unemployment multiplier and marginal rate of substitution between private and publis consumption are given as:

equation 58

$$m = -y\frac{du}{dg};\ MRS_{gc} = \frac{\partial U/\partial g}{\partial U/\ \partial c} > 0$$

Where $U(g, c)$ is instantaneous utility from public and private consumption. Steady state rate of unemployment is : $u(x) = \frac{s}{s+f(x)}$; where s is separation rate. The elasticity of substitution between

[16] $MRS_{xy} = \frac{MU_x}{MU_y} = \frac{p_x}{p_y}$

public and private consumption is given as: $\frac{1}{\epsilon} = -\frac{dln\,(MRS_{gc})}{dln\left(\frac{g}{c}\right)}$. Optimal expenditure in this model satisfies:

Lemma 1. Optimal public expenditure satisfies:

equation 59

$$1 = MRS_{gc} + \frac{\partial y}{\partial x} \cdot \frac{dx}{dg}$$

Where $1 = MRS_{gc}$ is the Samuleson term, and $\frac{\partial y}{\partial x} \cdot \frac{dx}{dg}$ is the correction. Optimal stimulus spending satisfies:

Lemma 2.

equation 60

$$\frac{\frac{g}{c} - \left(\frac{g}{c}\right)^*}{\left(\frac{g}{c}\right)^*} \sim z_0 \epsilon m \frac{u - u^*}{u^*}$$

Where ϵ, m are evaluated at $\left[\frac{g}{c}, u\right]$ and $z_0 = \frac{1}{(1-\eta)(1-u^*)^2}$.Also MRS_{gc} can be approximated as follows:

Lemma 3.

$$1 - MRS_{gc} \approx \frac{1}{\epsilon} \cdot \frac{\frac{g}{c} - \left(\frac{g}{c}\right)^*}{\left(\frac{g}{c}\right)^*}$$

Definition 2. The unemployment multiplier m, the empirical unemployment multiplier M, and the output multiplier dY/dG are given as:

equation 61

$$m = \frac{(1-u) \cdot M}{1 - \frac{G}{Y} \cdot \frac{\eta}{1-\eta} \cdot \frac{\tau}{u} \cdot M}$$

$$M = \frac{dY}{dG}$$

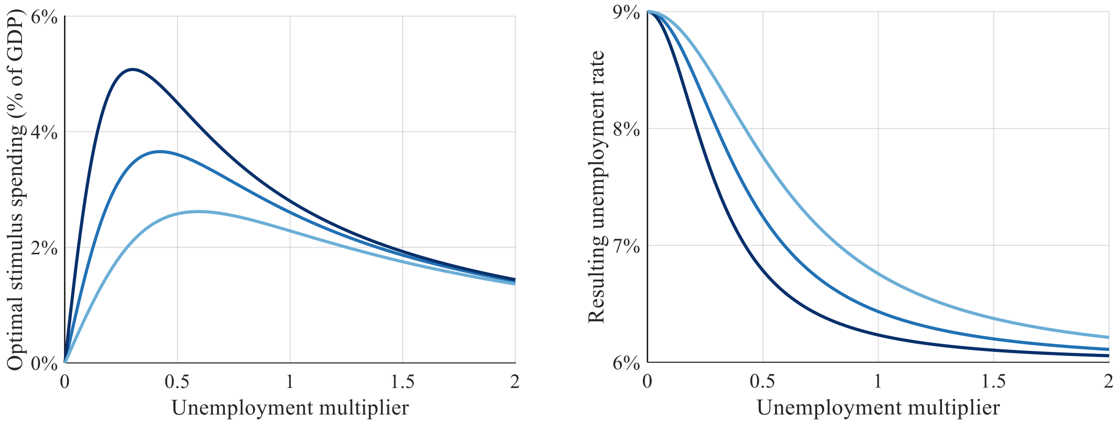
Where $\tau = \frac{\rho s}{q(x) - \rho s}$ or this is the wedge between consumption and expenditure caused by matching. Where $\rho > 0$ are vacancy costs,$\eta \in (0,1)$ is the matching elasticity function, and a Cobb-Dougrals matching function here is defined as:

equation 62

$$h(t) = \omega v(t)^{1-\eta} [k - Y(t)]^{\eta}$$

where ω is a matching efficiency function,k is the aggregate capacity of the economy, $Y(t)$ is the output. Next by MATLAB simulation graphically are reproduced results for the unemployment multiplier.

Figure 7 Unemployment multiplier and optimal stimulus spending (% GDP) and unemployment multiplier and resulting unemployment rate



Source: Author's calculation based on code posted by Michaillat, Saez (2019) available at : https://github.com/pmichaillat/stimulus-spending

Optimal stimulus is measured as share of GDP,$\left(\frac{G}{Y}-\left(\frac{G}{Y}\right)^*\right)$, optimal stimulus spending is computed for following proposition:

Proposition 1. $\frac{\frac{g}{c}-\left(\frac{g}{c}\right)^*}{\left(\frac{g}{c}\right)^*} \approx \frac{z_0\epsilon m}{1+z_1 z_0 \epsilon m^2}\cdot\frac{u_0-u^*}{u^*}$ and $z_1 = \frac{\left(\frac{g}{y}\right)^*\left(\frac{c}{y}\right)^*}{u^*}$ and under optimal policy unemployment rate is :

$$u \approx u^* + \frac{u_0 - u^*}{1 + z_1 z_0 \epsilon m^2}$$

Previous figure displays optimal stimulus spending in response to the shock, and the unemployment rate that would be reached after such spending. So higher unemployment multiplier is associated with lower unemployment rate and lower optimal stimulus spending. Optimal stimulus spending is zero for a zero multiplier, increasing in the multiplier for small multipliers, largest for a moderate multiplier, and decreasing in the multiplier beyond that.

6.McCall job search model

This is a verison of McCall (1970) model.Bellman equation for this model is:

equation 63

$$V(w) = \max\{W(w), U(w)\}$$
$$W(w) = w + \beta W(w)$$
$$U(w) = cw + \beta \sum_{w' \in S} P(w'|w, reject)V(w')$$

Where c indicates replacement rate of the current wage for the unemployed and $P(w'|w,x)$denotes the probability to transit from state $w' \to w$, given the action $x \in X \equiv \{accept, reject\}$. The transition probability $P(w'|w, reject) = 1/N$ and discrete space is $S = \{w_1, \dots, w_n\}$. The agent is infinitely lived and aims to maximize the expected discounted sum of earnings $\mathbb{E}\sum_{t=0}^{\infty}\beta^t y_t$ where $\beta \in (0,1)$ is a discount factor. Optimal value function is given as:

equation 64

$$v^*(s) = \max\left\{\frac{w(s)}{1-\beta}.c + \beta \sum_{s' \in S} v^*(s')q(s')\right\}$$

Optimal policy $\sigma(s)$ and reservation wage$\overline{w}$ can be written as follows:

equation 65

$$\sigma(s) \coloneqq \mathbf{1}\left\{\frac{w(s)}{1-\beta} \geq c + \beta \sum_{s' \in S} v^*(s')q(s')\right\}$$
$$\sigma(s) \coloneqq \mathbf{1}\{w(s) \geq \overline{w}\}$$
$$\overline{w} \coloneqq (1-\beta)\left\{c + \beta \sum_{s}^{\prime} v^*(s')q(s')\right\}$$

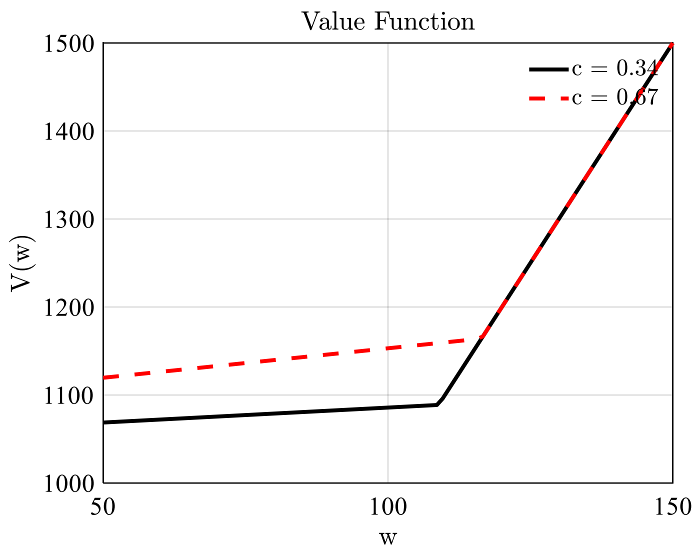
Where :

equation 66

$$v^*(i) = \max\left\{\frac{w(i)}{1-\beta}, c + \beta \sum_{1 \leq j \leq n} v^*(j)q(j)\right\}, for\ i = 1, \ldots, n$$

Next, McCall model of job search is graphically depicted.

Figure 8 McCall job search model and replacement rate c as a function from value function $V(w)$ mean preserving spread



Source : Authors' calculations based on a code available at: Andreas Muller research page at : https://sites.google.com/site/mrandreasmueller/resources

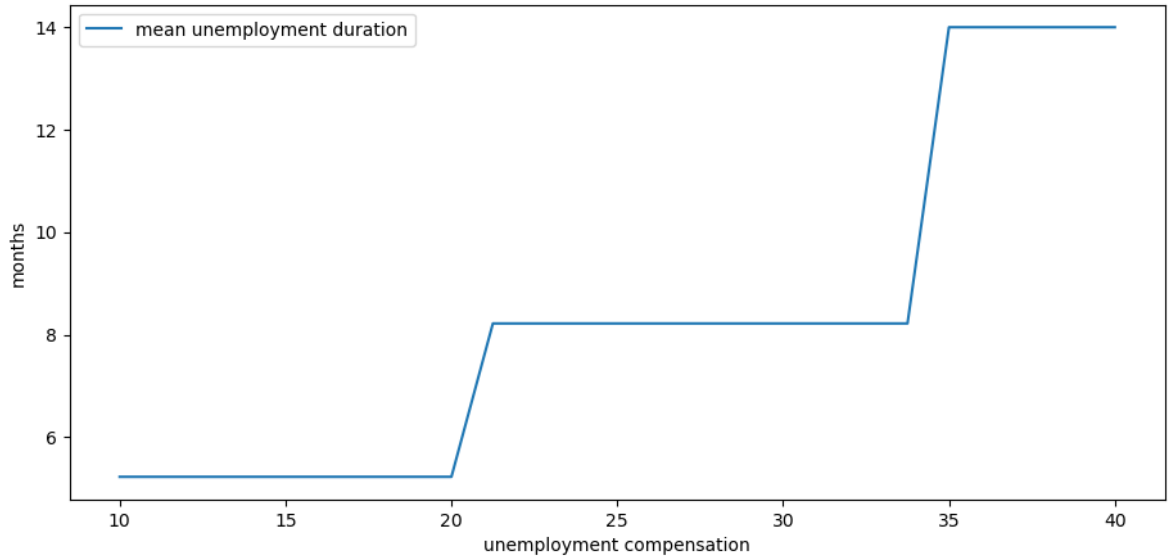
The $N \times N$ transition probability matrices read as:

equation 67

$$\Pi(accept) = \begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{pmatrix}; \Pi(reject) = \begin{pmatrix} 1/N & \cdots & 1/N \\ \vdots & \ddots & \vdots \\ 1/N & \cdots & 1/N \end{pmatrix}$$

Replacement rate is the ratio of net income while out of work (mainly unemployment benefits if unemployed or means-tested benefits if on social assistance) divided by net income while in work. So, for lower levels of wage grid higher replacement rate is associated with higher levels of valuation function and at certain threshold of wage replacement rates valuation function are equaling themselves.

Figure 9 McCall job search model unemployment compensation and mean unemployment duration



Source: Authors' calculation in Jupyter Pyhton notebook on a code based from Quantecon package available at: https://python.quantecon.org/mccall_model.html#:~:text=The%20McCall%20search%20model%20%5BMcC70,unemployment%20compensation

From the previous model one can see that higher levels of unemployment compensation are associated with higher mean unemployment duration.

7.DMP model of job search

DMP model (Diamond-Mortensen-Pissarides ;Diamond(1982); Pissarides (1985); and Mortensen and Pissarides (1994)) is a wrokhorse model for macroeconomists when analysing labor markets for 50 years or more[17]. The association between unemployment productivity and benefits in DMP framework is as follows. Constant returns matching function is $M(uL, vL)$ where uL-are unemployed, vL-are vacancies and $M(uL, vL) = vL \cdot M\left(\frac{u}{v}, 1\right)$ and $\theta \equiv \frac{v}{u}$ and the vacancy filling rate is:$q \equiv \frac{M}{vL} = M\left(\frac{u}{v}, 1\right) = M\left(\frac{1}{\theta}, 1\right) = q(\theta)$. And unemployed exit hazard is :$\theta q(\theta) = \frac{M}{uL}$, where $\theta q(\theta) \to 0$ as $\theta \to 0$ and $\theta q(\theta) \to \infty$ as $\theta \to \infty$. Value of job vacancy is: $J = \frac{c}{q(\theta)}$, $1/q(\theta)$ is the expected time to fill a vacancy, and c are the cost per period.Where $c = y - w$; where y is output and w are the wages, and if we know that $rV = -c + q(\theta)(J - V)$, where J-is the value of filled vacancy,and $V$is the value of unfiled vacancy.Now if we assume that $V = 0$ than $J = c/q(\theta)$.Now if we equate job creation $\theta q(\theta) \times uL$ and job destruction rate $\delta(1 - u)L$ we get equilibrium unemployment equation such as:

equation 68

$$u = \frac{\delta}{\delta + \theta q(\theta)} = \frac{\delta}{\delta + \left(\frac{v}{u}\right) q \left(\frac{v}{u}\right)}$$

Where $q(\theta) = \frac{(r+\delta)c}{y-w}$ so that we can write: $u = \frac{\delta}{\delta+\theta\frac{(r+\delta)c}{y-w}} \Rightarrow \frac{\delta(y-w)}{\delta(y-w)+\theta(r+\delta)c} = 1 + \frac{\delta(y-w)}{\theta(r+\delta)c}$.If we take logs from both sides:

[17] Some important standard textbook in macroeconomics that use DMP framework include: Carlin and Soskice (2006); Williamson (2013); Chugh (2015).

equation 69

$$\ln(u) = \ln\left(\frac{\delta(y-w)}{\theta(y-w)+\theta(r+\delta)c}\right) = \ln(\delta) + \ln(y-w) - ln\theta(y-w) - \ln(\theta) - ln(r+\delta) - \ln(c) = \ln(\delta) + \ln(y) - \ln(w) - \ln(\theta)\, y - \ln(\theta) - \ln(r+\delta) - \ln(c) = \ln(\delta) + \ln(y) - \ln(w) - \ln(\theta)\, y - \ln(\theta) - \ln(r+\delta) - \ln(y) + \ln(w) = \ln(\delta) - \ln(\theta)\, y - \ln\,(\theta) - \ln(r+\delta)$$

For the association benefits and unemployment, the solution might be straightforward, since the value of unemployment is $rU = b + y(\theta)[W-U]$, where w is intertemporal value of employment and u-is intertemporal value of unemployment and $rW = w - \theta(W-U)$, and b are unemployment benefits. And now from previous we know that following applies:

equation 70

$$W - U = \frac{\beta}{1-\beta}(J-V) \Leftrightarrow (r+\delta)(W-U) = (r+\delta)(J-V) \Leftrightarrow (r+\delta)\big(w - b + \theta q(\theta)(W-U)\big) = y - w$$

For a free entry we have $J = \frac{c}{q(\theta)}$, and $W - U = \frac{\beta}{1-\beta}\frac{c}{q(\theta)}$; and the wage equation now becomes: $w = (1-\beta)b + \beta(y + c\theta)$ where $\beta$is the bargaining power of labor. If $\beta = 1$ real wage is equal to productivity +average search costs $\frac{cv}{u}$. If $\beta = 0$ real wage is equal to unemployed income. Labor market equilibrium is established on the intersection between wage setting curve (labor supply curve) and free entry conditions (which is approximately equal to labor demand curve), and now:

equation 71

$$w = (1-\beta)b + \beta(y + c\theta)$$

$$(1-\beta)(y-b) = \frac{c}{q(\theta)}[\delta + r + \beta\theta q(\theta)$$

Or if we define unemployment to be supply minus demand for labor i.e $u = w - (1-\beta)(y-b)$ and if we simplify $u = w - (y - b - \beta y + \beta b) = w - y + b - \beta y - \beta b$ and :

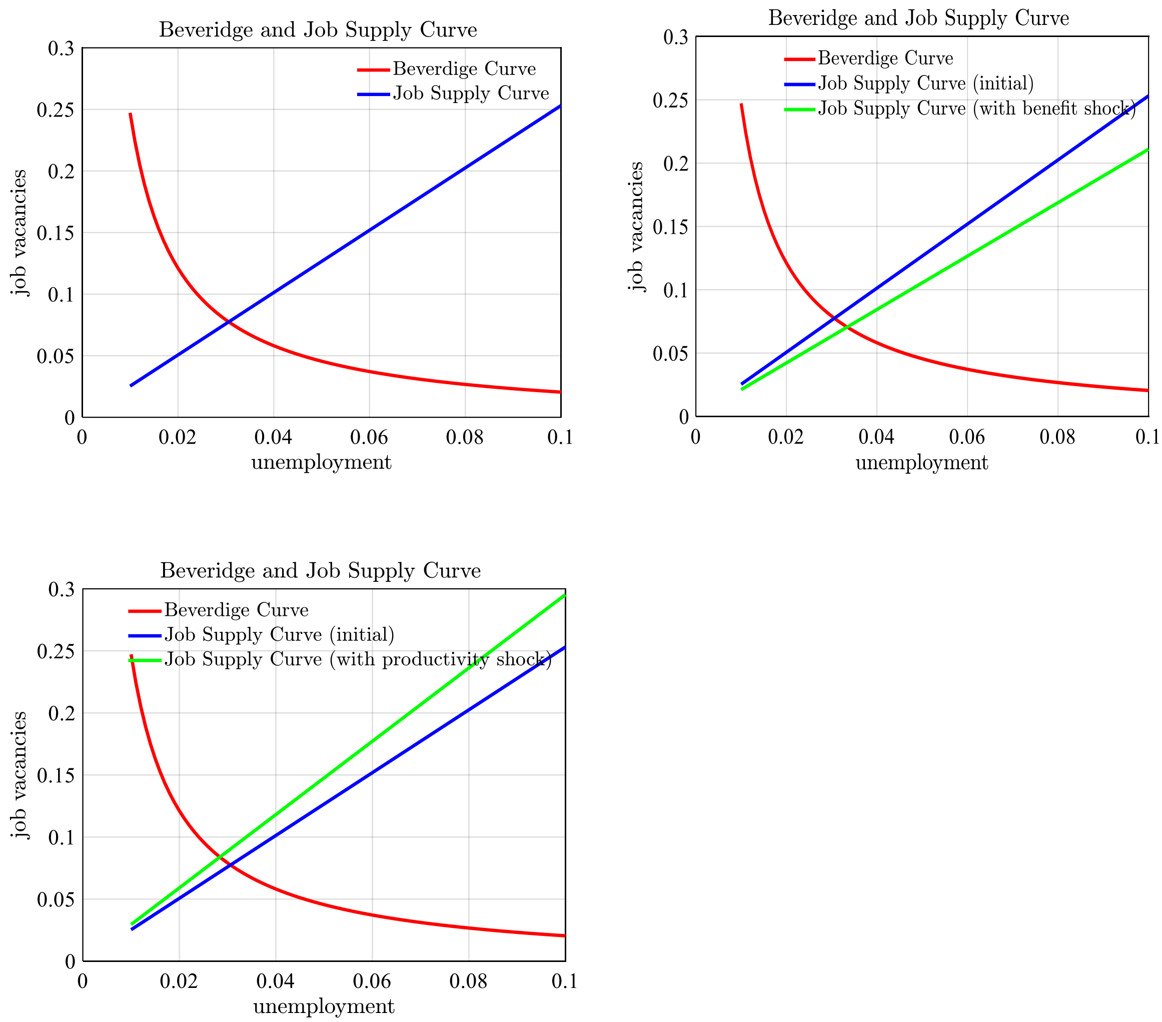
equation 72

$$u = (1-\beta)b + \beta(y + c\theta) - (\frac{c}{q(\theta)}\big[\delta + r + \beta\theta q(\theta)\big) \Rightarrow b - \beta b + \beta y + c\beta\theta - \left(\frac{c\delta}{q(\theta)} + \frac{cr}{q(\theta)} + c\beta\theta\right) = b - \beta b + \beta y - c\frac{(\delta+r)}{q(\theta)}$$

Since $b - \beta b > 0$ since we know that labor bargaining power ideally is around [18]$\beta = \frac{1}{2}$. Next, a simulation results of the Diamond-Mortensen-Pissarides model are presented on the following graph.From the graph it can be see that the Beveridge curve the association between unemployment rate and vacancy rate is not shifting as a result from technology or benefit shock. While the job supply curve is shifting towards right in a case of benefit shock and it is shifting to the left in a case of technology shock.

[18] These derivations are heavily borrowed from Josheski,Boshkov(2022).

Figure 10 Diamond-Mortensen-Pissarides canonical model with benefit shock and productivity shock



Source: Authors calculation based on a code available at: https://github.com/pdevlieger/MatLab-files

8.Gali (2010) Monetary policy and unemployment

In this model by Gali (2010) , households seek to maximize:

equation 73

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$$

Where $C_t = \left(\int_0^1 C_i(i)^{1-\frac{1}{\epsilon}} di\right)^{\frac{\epsilon}{1-\epsilon}}$ and $\beta \in (0,1)$. Total effort $L_t = N_t + \psi U_t$ where N_t, U_t are employed and unemployed respectively. Parameter $\psi \in (0,1)$ represents marginal disutility generated by an unemployed member relative to an employed one. Employment evolves according to: $N_t = (1-\delta)N_{t-1} + x_t U_t^0$. Household budget constraint is :

equation 74

$$\int_0^1 P_t(i) C_t(i) di + Q_t B_t \leq B_{t-1} + \int_0^1 W_t(j) N_t(j) dj + \Pi_t$$

$P_t(i)$ is the price of good i, B_t are bond purchases (one period) at a price Q_t, $W_t(j)$ is the wage paid by the firms (nominal terms), and Π_t is a lump-sum component of income (which may include, among other items, dividends from ownership of firms or lump-sum taxes).Optimal demand for all goods takes form:

equation 75

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t$$

Where $P_t \equiv \left(\int_0^1 P_t(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$ denotes the price index of final goods.The intertemporal optimality condition is given as:

equation 76

$$Q_t = \beta E_t \left\{\frac{C_t}{C_{t+1}} \frac{P_t}{P_{t+1}}\right\}$$

Technology available to all firms is $Y_t(i) = X_t(i)$, where $X_t(i)$ is the quantity of the (single) intermediate good used by firm i as an input. Profit maximizing condition is :$P_t(i) = \mathcal{M}^P(1-\tau)P_t^I$ where P_t^I is the price of intermediate good,$\mathcal{M} \equiv \frac{\epsilon}{\epsilon-1}$ is optimal markup,τ is subsidy on the purchases of intermediate goods. And $(1-\tau)P_t^I$ is the nominal marginal cost facing the final goods firm.Law of motion for aggregate price level is:

equation 77

$$\mathrm{p_t = \theta_p p_{t-1} + (1-\theta_p)p_t^* - law\ of\ motion\ of\ prices}$$
$$\mathrm{\pi_t^p = \beta E_t\{\pi_{t+1}^p\} - \lambda_p \hat{\mu}_t^p - inflation\ equation}$$

Where $\pi_t^p \equiv p_t - p_{t-1}$ which is price inflation, $\hat{\mu}_t^p \equiv \mu_t^p - \mu^p = p_t - (p_t^I - \tau) - \mu^p$ which is a deviation of log price markup from its desired steady state value and $\lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}$.In the intermediate goods sector, all firms have access to production function:$Y_t^I(j) = A_t N_t(j)^{1-\alpha}$,employment here evolves according to: $N_t(j) = (1-\delta)N_{t-1}(j) + H_t(j)$ where $\delta \in (0,1)$ is the exogenous separation rate, and $H_t(j)$ represents the measure of workers hired by the firm j in period t. Labor market frictions are introduced as cost per hire G_t. Job finding rate is $x_t \equiv H_t/U_t^0$ and the ratio of aggregate hires $H_t \equiv \int_0^1 H_t(j)dj$ and $G_t = G(x_t) = \Gamma x_t^\gamma$.Matching function is $M(V_t U_t^0)$where V_t represents the number of aggregate vacancies, and firms can post a vacancy at cost Γ.Under the assumptions of homogeneity of degree one in matching function, the fraction of filled vacancies posted per period is $\frac{M(V_t,U_t^0)}{V_t} \equiv q\left(\frac{V_t}{U_t^0}\right)$, and job finding rate is $x_t = \frac{M(V_t,U_t^0)}{V_t} \equiv p\left(\frac{V_t}{U_t^0}\right)$ where $p' > 0$.Fraction of vacancies $q(p^{-1}(x_t))$ is filled with resulting cost per hire $G_t = \Gamma/q(p^{-1}(x_t))$ .The optimal hiring policy is :$MRPN_t(j) = \frac{W_t(j)}{P_t} + G_t - (1-\delta)E_t\{\Lambda_{t,t+1}\, G_{t+1}\}$.Where $MRPN_t(j) = \left(\frac{P_t^I}{P_t}\right)(1-\alpha)A_t N_t(j)^{-\alpha}$., and $\Lambda_{t,t+k} \equiv \beta^k\left(\frac{C_t}{C_{t+k}}\right)$ is the stochastic discount factor for k period ahead of real payoffs. Or $MRPN_t(j) = \frac{W_t(j)}{P_t} + B_t$ where $B_t \equiv G_t - (1-\delta)E_t\{\Lambda_{t,t+1}G_{t+1}\}$. Average markup in final goods sector is: $\hat{\mu}_t^p = (a_t - \alpha\hat{n}_t) - [(1-\Phi)\widehat{\omega} + \Phi\hat{b}_t]$ where $\omega_t = w_t - p_t$ average log real wage and $\Phi \equiv \frac{B}{\left(\frac{W}{P}\right)+B}$ and $\hat{b}_t = \frac{1}{1-\beta(1-\delta)}\hat{g}_t - \frac{\beta(1-\delta)}{1-\beta(1-\delta)}(E_t\{\hat{g}_{t+1}\} - \hat{r}_t)$,where $\hat{g}_t = \gamma\hat{x}_t$ and $r_t$is real return on riskless return on one period bond. The average price markup in New-Keynesian model is given as: $\hat{\mu}_t^p = (a_t - \alpha\hat{n}_t) - \left[(1-\Phi)\hat{\omega}_t + \Phi\hat{b}_t\right] = -\hat{s}_t^N - \Phi\left(\hat{b}_t - \hat{\omega}_t\right)$,where $\hat{s}_t^n \equiv \hat{\omega}_T - (\hat{y}_t - \hat{n}_t)$is the log income share expressed as deviation of its mean.As for the monetary policy, monetary policy is described by simple Taylor rule:

equation 78

$$i_t = \rho + \phi_\pi \pi_t^p + \phi_y \hat{y}_t + v_t$$

Where $i_t \equiv -\log Q_t$ is a yield on one period riskless bond, and $\rho \equiv -\log \beta$ which is household discount rate, and v_t is an exogenous policy shifter, which follows AR(1) process. Next, we will pot this economy on two graphs first with technology shock after with monetary policy shock.

Figure 11 Orthogonalized technology shock

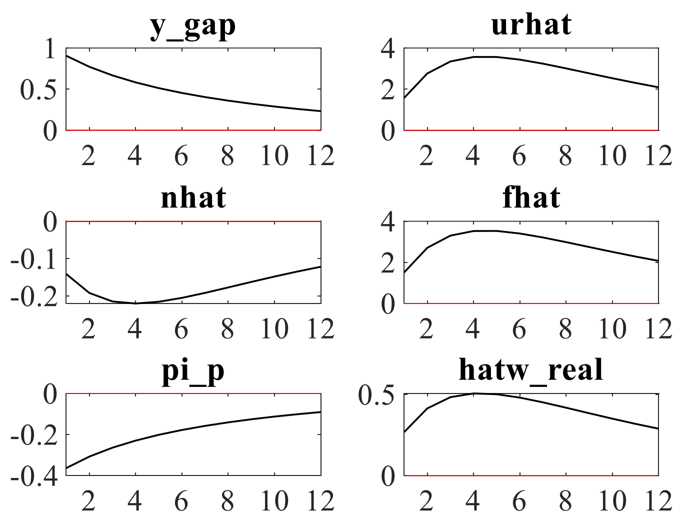
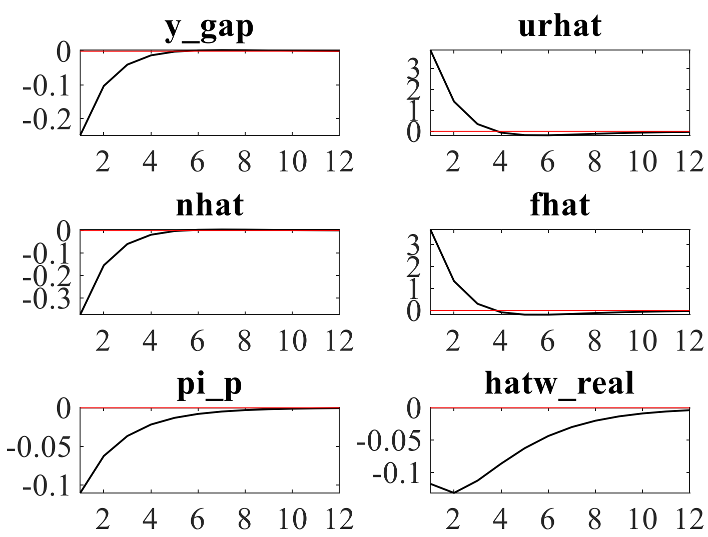


Figure 12 orthogonalized monetary policy shock



Source: Authors calculation based on a DYNARE code for Gali (2010)

Where in previous graphs : y_gap is the output gap; $urhat$ is the unemployment rate, $nhat$ is the employment rate,$fhat$ is the labor force; and pi_p price inflation, $hatw\_real$ is the real wage.

9.Rationing unemployment and Frictions unemployment in DSGE framework (Michaillat (2012))

In this model of Michaillat (2012) technology is a Markov process $\{a_t\}_{t=0}^{+\infty}$ .Workers have risk-neutral consumption and discount factor $\beta \in (0,1)$.This model does not include labor supply decision nor include working hours. Continuum of firms are indexed $i \in (0,1)$ are hiring workers, and at the end of period $t-1$ fraction s workers of the existing n_{t-1} existing worker-job matches are exogenously destroyed. Unemployed workers who are looking for a job and number of hires, also labor market tightness θ_t, and unemployment u_t are related through the job-finding probability$f(\theta_t)$ and probability f filling vacancy v_t is $q(\theta_t)$are modeled as:

equation 79

$$\theta_t = \frac{v_t}{u_t}$$
$$q(\theta_t) \equiv \frac{h(u_t, v_t)}{v_t} \equiv h(1, \theta_t)$$
$$u_t = 1 - (1-s) \cdot n_{t-1}$$
$$h_t = \int_0^1 h_t(i) di$$
$$f(\theta_t) = \frac{h_t}{u_t}$$

Beveridge curve relates labor market tightness θ and employment n:

$$n = \frac{1}{(1-s) + \frac{s}{f(\theta)}}$$
$$s \cdot n = [1 - (1-s)n] \cdot f(\theta)$$

This expression $s \cdot n = [1 - (1-s)n] \cdot f(\theta)$ is in steady-state inflows to unemployment $s \cdot n$ equal outflows $[1-(1-s)n] \cdot f(\theta)$.Wage schedule $w_t(i)$, real profit of firms $\pi_t(i)$,and the employment condition are given as:

equation 80

$$w_t(i) = w(n_t(i), \theta_t, n_t a_t) - \text{wage schedule}$$
$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \beta^t \cdot \pi_t(i) - \text{profit stream for firms}$$
$$\pi_t(i) = g(n_t(i), a_t) - w_t(i) \cdot n_t(i) - \frac{c \cdot a_t}{q(\theta_t)} \cdot h_t(i) - \text{real profit for the firm } i \text{ in period } t$$
$$\frac{\partial g}{\partial n(i)}(n_t(i), a_t) = w_t(i) + \frac{c \cdot a_t}{q(\theta_t)} + n_t(i) \cdot \frac{\partial w}{\partial n(i)}(n_t(i), \theta_t, n_t, a_t) - \beta(1-s)\mathbb{E}_t\left[\frac{c \cdot a_{t+1}}{q(\theta_{t+1})}\right] - \text{employment cond.}$$

In equilibrium : $0 < w_t < \frac{\partial g}{\partial n(i)}(n_t^*, a_t) - n_t^* \cdot \frac{\partial w}{\partial n(i)}(n_t^*, 0, n_t^*, a_t) + \beta(1-s)\mathbb{E}_t\left[\frac{c \cdot a_{t+1}}{q(\theta_{t+1})}\right]$.This condition implies that private efficiency is guaranteed if wages are low, and private efficiency guarantees that wage rigidity never causes the destruction of a match generating a positive bilateral surplus, a reasonable equilibrium requirement when rational workers and firms engage in long-term interactions see Barro (1977).The model with search-match with absence of job rationing is given as:

equation 81

$$w(n_t(i),\theta_t,n_t,a_t)=\frac{c\mathcal{B}}{1-\mathcal{B}}\left\{\frac{a_t}{q(\theta_t)}+\beta(1-s)\mathbb{E}_t\left[a_{t+1}\left(\theta_{t+1}-\frac{1}{q(\theta_{t+1})}\right)\right]\right\}-\text{wage schedule}$$
$$(1-\mathcal{B})=c\cdot\left[\frac{1-\beta\cdot(1-s)}{q(\theta)}+\beta\cdot(1-s)\cdot\beta\cdot\theta\right]-\text{eq. labor market tightness}$$
$$\lim_{c\to 0}\theta(c)=+\infty;\lim_{c\to 0}n(c)=1-\text{recruiting cost } c$$

In previous expression $\mathcal{B}\in(0,1)$ is the workers' bargaining power.In the model with wage rigidity :

equation 82

$$a\in[\underline{a},\overline{a}];\ w(n_t(i),\theta_t,n_t,a_t)=\omega\cdot a_t^{\gamma}-\text{technology } a \text{ and partial wage adjust } \gamma<1$$
$$1-\omega\cdot a^{\gamma-1}=c\cdot\frac{1-\beta\cdot(1-s)}{q(\theta)}-\text{firms optimallity condition}$$
$$a\geq\omega^{\frac{1}{1-\gamma}};\lim_{c\to 0}\theta(a,c)=+\infty\,;\lim_{c\to 0}n(a,c)=1\ -\text{technology and recruitment costs}$$

If technology is bounded $a\in[\underline{a},\overline{a}]$, rigid wages are privately efficient if $0\leq\omega\leq\underline{a}^{1-\gamma}$.In model with job rationing, rationing unemployment $u^R(a)$ and frictional unemployment$u^F(a,c)$ are defined as:

equation 83

$$u^R(a)=1-n^R(a)=1-\left(\frac{\alpha}{\omega}\right)^{\frac{1}{1-\alpha}}\cdot a^{\frac{1-\gamma}{1-\alpha}}$$
$$u^F(a,c)\equiv u(a,c)-u^R(a)$$

In previous $n^R(a)=\left(\frac{\alpha}{\omega}\right)^{\frac{1}{1-\alpha}}\cdot a^{\frac{1-\gamma}{1-\alpha}}$ and $n^R(a)\in(0,1)$; $a\in(0,a^R)$; $a^R=\left(\frac{\omega}{\alpha}\right)^{\frac{1}{1-\gamma}}$ and $\lim_{c\to 0}n(a,c)=n^R(a)$ and if $a\geq a^R$ then $\lim_{c\to 0}n(a,c)=1$ .Why does job rationing exist?, in static environment :

equation 84

$$a\cdot n^{\alpha-1}-\omega\cdot a^{\gamma-1}=[1-(1-s)\cdot\beta]\cdot\frac{c}{q(\theta)}$$

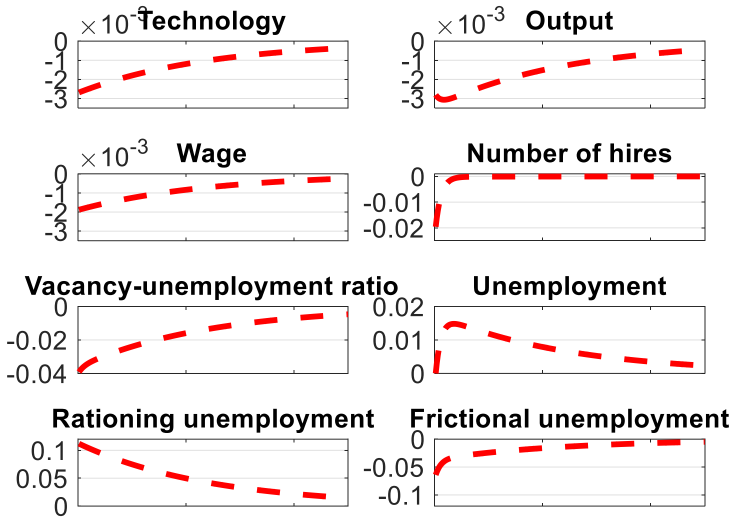
The presence of recruiting cost c creates wedge between marginal product of labor $\alpha\cdot a\cdot n^{\alpha-1}$ and wage $w=\omega\cdot a^{\gamma}$ so that in equilibrium $MPL>w$ or $\alpha\cdot a\cdot n^{\alpha-1}>\omega\cdot a^{\gamma}$. Cyclicality of the elasticity of tightness $\epsilon_c^{\theta}$and elasticity of unemployment with respect to recruiting cost $\epsilon_c^{u}$is given as:

equation 85

$$\epsilon_c^{\theta}=\frac{dln(\theta)}{dln(c)}=-\left[\eta+(1-\eta)\cdot(1-\alpha)\cdot u\cdot\frac{\alpha\cdot q(\theta)\cdot n^{\alpha-1}}{c\cdot[1-\beta\cdot(1-s)]}\right]^{-1}$$
$$\epsilon_c^{u}=\frac{dln(u)}{dln(c)}=-(1-u)\cdot(1-\eta)\cdot\epsilon_c^{\theta}\approx\epsilon_c^{u}=\left[\frac{\eta}{1-\eta}+\frac{u}{u^F}\right]^{-1}$$

In the following example as per Michaillat (2012) steady state unemployment is $\bar{u}=5.8\%$;$\bar{u}^R=1-\left(\frac{\alpha}{\omega}\right)^{\frac{1}{1-\alpha}}=2.1\%$ (steady-state rationing unemployment) and $\bar{u}^F=3.7\%$ (frictional steady-state unemployment).

Figure 13 IRFs as log-deviations of steady-state for US data for 1964:Q1--2009:Q2 with negative technology shock to the log linear model $-\sigma = -0.00269$ for period on x-axisf 250 weeks



Source: Authors calculations based on a code and data by Michaillat (2012)

From previous graph labor market tightness drops by 4% in response to a drop in technology by 0.27% ,implied elasticity of labor market tightness with respect to technology is $\frac{4}{0.27} = 14.8$.Here $s = 0.0095$ and $dln(u) = \frac{(1-\bar{u})}{\bar{u}} \cdot dln(n)$ since $n = \frac{1}{(1-s)+\frac{s}{f(\theta)}}$ employment could decrease at weekly rate of 1%, so unemployment could increase at weekly rate by 15%.Stochastic IRFs are :

equation 86

$$u_t^R \equiv \max\left\{0,1 - \left(\frac{\alpha}{\omega}\right)^{\frac{1}{1-\alpha}} \cdot (a_{t-1})^{\frac{1-\gamma}{1-\alpha}}\right\} - \text{stochasting rationing unemployment}$$

$$u^F \equiv u_t - u_t^R - \text{stochastic frictional unemployment}$$

After the technology shock wages are more rigid and rationing unemployment jumps up. As technology remains below steady state, the shortage of jobs remains (number of hires negative to zero) acute and rationing unemployment remains above steady state.

10.Conclusion

This paper as it turned out was more about unemployment than New Keynesian DSGE models. Despite the central role of unemployment in the policy debate, that variable has been—at least until recently—conspicuously absent from the new generation of models that have become the workhorse for the analysis of monetary policy, inflation and the business cycle, and which are generally referred to as New Keynesian,see Gali (2010).Discretionary monetary policy in a liquidity trap case pinpointed that:unemployment initially will be below its steady-state level, while productivity,vacancy rate,wages,labor market tightness will initialy be above steady-state evel and in 60-80 perod after the shock variables will restore to steady-state levels.Labor market with precautionary savings in a case of positive technology shock job finding rate will be almost 50% in first couple of months after the shock after 10 quarrters will fal down to 45%, unemployment initially wil, fall for 2-3 percentage points ,there will be no wage schedule impact,and average wage bill will move to its steady-state in more than 20 quarters.In a case of positive productivity shock in this model of precautionary savngs:capital will positively deviate from steady-state for more than 20 quarters,equity price will positively deviate from ss in almost 10 quarters,total assets in the conomy will positively deviate from ss for more than 20 quarters,interest rate annnualized will positively deviate for 5 quarters. Higher unemployment multiplier is associated with lower unemployment rate and lower optimal stimulus spending. Optimal stimulus spending is zero for a zero multiplier, increasing in the multiplier for small multipliers, largest for a moderate multiplier, and decreasing in the multiplier beyond that.In McCall job search model for lower levels of wage grid higher replacement rate is associated with higher levels of valuation function and at certain threshold of wage replacement rates valuation function are equaling themselves.Also in this model higher levels of unemployment compensation are associated with higher mean unemployment duration.In the canonical DMP model: Beveridge curve the association between unemployment rate and vacancy rate is not shifting as a result from technology or benefit shock. While the job supply curve is shifting towards right in a case of benefit shock and it is shifting to the left in a case of technology shock.In the monetary policy and unemployment model by Gali (2010) results and deviations of output gap and unemployment rate, employment rate, labor force, and price inflation and real wage are opossite in case of technology shock and monetary policy shock. In the rationing unemployment and frictional unemployment model in a case of negative technology shock,output fall fro about the same levels as the negative technology shock,followed by the fall in wages,and nub=mber fo hires,fall in vacancy/employment ratio,and a rise in unemployment in general of which at first there is rise in rationing unemployment and fall in frictional unemployment.

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