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FROM CHALLENGE TO OPPORTUNITY: MATHEMATICAL AND OPTIMIZATION TOOLS FOR CLOSED BALL-HYDROCYCLONE CIRCUIT PERFORMANCE

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Abstract: In this paper mathematical methods and optimization tools that make possibilities for appropriate, fast and sure presentation of some complex circuits in the mineral processing technologies will be shown. Some tools include the analytical methods of optimization, the constraints that appear in the form of equations. Such methods of optimization, in principle, determine or define the possible location of optimum, only stationary points, while discontinuous and border points must specifically to explore. Lagrange multipliers represent a simple procedure, which found great use in solving problems in the field of optimization.

Key Word: Lagrange, closed circuits, software

1. INTRODUCTION

After best fit flow rates have been calculated it is often necessary to adjust the experimental data to be consistent with the calculated flow rates.

All of the adjustment techniques are ways of distributing the mass balance errors Δ_i between the various measured values to give corrected or adjusted values (a_i, b_i, c_i) , $(\bar{a}_i, \bar{b}_i, \bar{c}_i)$, which are numerically consistent at the calculated flow rates. Then,

$$\Delta_i = a_i - \bar{\beta} \cdot b_i - (1 - \bar{\beta}) \cdot c_i$$

$$0 = \bar{a}_i - \bar{\beta} \cdot \bar{b}_i - (1 - \bar{\beta}) \cdot \bar{c}_i$$

The simplest adjustment is to assume measurement errors are proportional to component flow rates in each stream. Transposing the error equation yields:

$$a_i - \frac{\Delta_i}{2} = \bar{\beta} \cdot b_i + \bar{\beta} \cdot \frac{\Delta_i}{2} + (1 + \bar{\beta})c_i + (1 + \bar{\beta}) \cdot \frac{\Delta_i}{2}$$

This path does not ensure that the adjusted components add up to unity. If one of the components is measured by differences (as insoluble in chemical analysis) minor discrepancies can be absorbed into that component. If the components are completely determined (for example, screen size analysis) it is possible to correct the flow rates and normalize these flows. This method is quite arbitrary. The least squares method can also be used to distribute the errors to minimize the sum of squares of adjustments of the measured values at the best fit flows. Alternatively the experimental flows, that is,

measured assays by best fit flows, can be adjusted and the assays recalculated.

$$0 = (a_i - \Delta a_i) - \bar{\beta} \cdot (b_i - \Delta b_i) - (1 - \bar{\beta}) \cdot (c_i - \Delta c_i)$$

$$\Delta_i = +\Delta a_i - \bar{\beta} \cdot \Delta b_i - (1 - \bar{\beta}) \cdot \Delta c_i$$

Where: $\bar{a}_i = a_i - \Delta a_i$; $\bar{b}_i = b_i - \Delta b_i$; $\bar{c}_i = c_i - \Delta c_i$

Now the sum of squares to be minimized for each component is:

$$S_i = \Delta a_i^2 + \Delta b_i^2 + \Delta c_i^2$$

Equations can be combined, to eliminated Δa_i , and S_i minimized by taking the derivative with respect to each of the unknown (Δb_i and Δc_i) and setting the result to zero. It may be shown that:

$$\Delta a_i = +\Delta_i/k; \quad \Delta b_i = -\Delta_i \cdot \bar{\beta}/k$$

$$\Delta c_i = -\frac{\Delta_i(1 - \bar{\beta})}{k}$$

$$k = [1 + \bar{\beta}^2 + (1 - \bar{\beta})^2]$$

The equation shows that the calculation of all three residuals depends on only one number k once the best fit flow rate is known. This reduction in calculation was generalized by the French mathematician **Lagrange**.

2. THE METHOD OF LAGRANGE MULTIPLIERS

The method is used to simplify minimization (maximization) problems which are subject to conditions or constraints. The constraints are expressed in such a way that equal zero:

$$0 = +\Delta_i - \Delta a_i + \bar{\beta} \cdot \Delta b_i + (1 - \bar{\beta}) \cdot \Delta c_i$$

Regarding to minimized sum of squares it's modified (S_m) by adding each of these constraints equations multiplied by *Lagrange multipliers*, then:

$$S_m = S + \sum_j \lambda_j \cdot \text{constraint } j$$

$$S_m = \Delta a_i^2 + \Delta b_i^2 + \Delta c_i^2 + 2 \cdot \lambda_i \cdot [+ \Delta_i - \Delta a_i + \bar{\beta} \cdot \Delta b_i + (1 - \bar{\beta}) \cdot \Delta c_i]$$

The modified sum is then differentiated with respect to each of the unknowns (residuals and multipliers) and the *Lagrange multipliers* are used to substitute for the residuals thus reducing the required calculation:

$$\frac{\partial S_m}{\partial \Delta a_i} = 2\Delta a_i - 2\lambda_i = 0 \text{ или } \Delta a_i = \lambda_i$$

$$\frac{\partial S_m}{\partial \lambda_i} = 2[+\Delta_i - \Delta a_i + \bar{\beta} \cdot \Delta b_i + (1 - \bar{\beta}) \cdot \Delta c_i] = 0$$

$$\Delta_i = +\lambda_i[+1 + \beta^2 + (1 - \beta)^2]$$

$$S = \frac{\Delta a_i^2}{V_{ai}} + \frac{\Delta b_i^2}{V_{bi}} + \frac{\Delta c_i^2}{V_{ci}}$$

The application of the "curve-fitting" approach to general problems is mathematically complex, but it's essential for the applicants or skillers with desire and aim for programming in the computer programmes. The application of the *Lagrange multipliers* will be shown with closed circuit mill-hydrocyclone. Also, it is important to mention that estimation of the model parameters will be used the general methods of least squares, linear regression etc.

Table 1

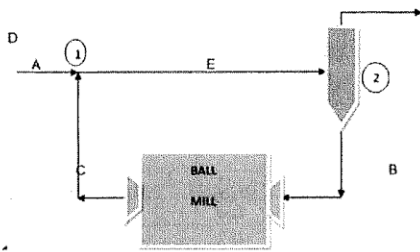
Tyler mesh	Circuit Feed	Hydrocyclone			Product
		Feed	Over-flow	Under-flow	
+8	0.1			nil	
+10	0.4			0.3	
+14	1.0	nil		0.2	nil
+20	1.2	0.4		0.2	0.1
+28	1.6	0.3		0.3	0.1
+35	2.2	0.3		0.6	0.2
+48	2.9	0.9	nil	1.2	0.7
+65	4.7	1.7	0.1	2.1	1.5
+100	8.1	4.7	0.3	5.7	4.9
+150	9.3	8.9	0.8	9.9	9.3
+200	12.8	21.6	2.6	25.4	24.6
+325	14.1	30.9	13.8	33.5	32.0
-325	41.6	30.3	82.4	20.6	26.6

Table 2

Tyler mesh	Flow residuals $\alpha = 6.0872$		Lagrange multipliers	
	Δ_1	Δ_2	λ_1	λ_2
+8	-0.10	0.0	0.0024	-0.0014
+10	-0.40	-1.53	-0.0114	0.0305
+14	-1.00	-1.02	0.0007	0.0103
+20	0.73	1.42	0.0023	-0.0235
+28	-0.28	0.30	-0.0108	-0.0109
+35	-1.39	-1.23	0.0160	0.0099
+48	-0.98	-0.63	0.0146	0.0013
+65	-1.98	-0.43	0.0408	-0.0168
+100	-4.42	-0.69	0.0947	0.0441
+150	-2.43	3.01	0.0985	-0.1042
+200	-6.46	-0.33	0.1477	-0.0804
+325	11.20	3.87	-0.2110	0.0617
-325	7.52	-2.75	-0.2148	0.1676

Table 3

Tyler mesh	Circuit feed	Hydrocyclone			Products
		Feed	Over-flow	Under-flow	
+8	0.1	0.0	0.0	0.0	0.0
+10	0.4	0.1	0.0	0.2	0.1
+14	1.0	0.1	0.0	0.2	0.1
+20	1.2	0.3	0.0	0.3	0.1
+28	1.6	0.3	0.0	0.4	0.1
+35	2.2	0.5	0.0	0.6	0.1
+48	2.9	1.0	0.0	1.2	0.6
+65	4.7	1.9	0.1	2.2	1.3
+100	8.0	5.0	0.4	5.9	4.4
+150	9.2	8.9	0.9	10.4	8.8
+200	12.7	22.0	2.7	25.8	23.9
+325	14.3	30.0	13.7	33.2	33.1
-325	41.8	30.0	82.2	19.8	27.7



$$A=1; A=D=1; E=\alpha; B=C=\alpha-1$$

Figure 1. Closed circuit mill-hydrocyclone

3. APPLICATION OF THE MINERAL PROCESSING SOFTWARES

Contemporary, some example of closed circuit of grinding – classifying applying computer programme for their solving will be shown (MINTEH-2 in Visual Studio 2008).

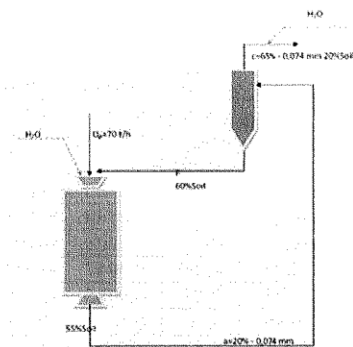


Figure 2. Closed circuit mill-hydrocyclone

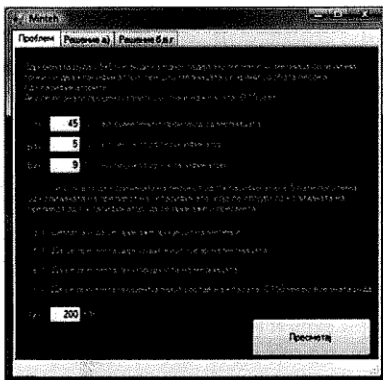


Figure 3. Closed circuit mill-hydrocyclone (Software decision)

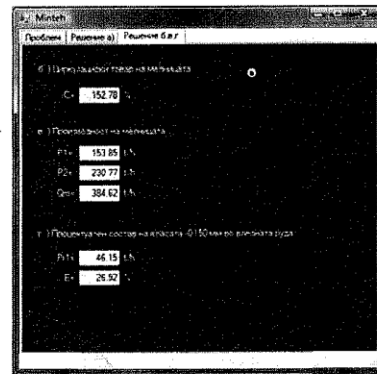


Figure 4. Closed circuit mill-hydrocyclone (Software decision)

4. CONCLUSION

It's clearly and simplify to conclude that the method of *Lagrange multipliers* is suitable way to represent the minimization or maximization of the known problems. The application of this method using the example of closed circuit Ball mill - Hydrocyclone is a good example for application of mathematical methods: *least squares, Lagrange multipliers, regression methods, Matrix notation* etc.

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