# Economy and Asymmetric Information: Mirrleesian Optimal Taxation as an Asymmetric Information Problem 

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#### Abstract

In this paper Mirrleesian optimal taxation will be reviewed. Models in optimal tax theory typically posit that the tax system should maximize a social welfare function subject to a government budget constraint, considering how individuals respond to taxes and transfers. James Mirrlees (1971) launched the second wave of optimal tax models by suggesting a way to formalize the planner's problem that deals explicitly with unobserved heterogeneity among taxpayers.There are static and dynamc versions of this model and we will review them or introduce them in this paper. Social welfare is larger when resources are more equally distributed, but redistributive taxes and transfers can negatively affect incentives to work and earn income in the first place. This creates the classical trade-off between equity and efficiency which is at the core of the optimal labor income tax problem. We will describe main theoretical findings in this literature as well as numerical examples with their policy implications.


Keywords: Optimal taxation, Mirrlees tax model, asymmetric information, non-linear tax rates, second-best analysis of taxes

JEL: H21

## 1. Introduction

This paper will review topic from optimal Mirrleesian taxation. In the classical framework initiated by Mirrlees (1971), the theory studies the maximization of a utilitarian social welfare function by a benevolent planner who only observes the pretax labor income of agents whose wages differ, but whose preferences are identical. The other studies have relaxed the assumptions in order to take heterogeneity among agents into account. These studies include: Mirrlees (1976), Saez (2001), Choné and Laroque (2010), see Fleurbaey , Maniquet (2018). Mainly approach is based on asymmetric information. Public policies apply to the individuals on the basis of what the government knows about them. Second welfare theorem ${ }^{1}$ states, that where a number of convexity and continuity assumptions are satisfied, an optimum is a competitive equilibrium once initial endowments have been suitably distributed. In general, complete information about the consumers for the transfers is required to make the distribution requires, so the question of feasible lump-sum transfers arises here. Usually the optimal tax systems combine flat marginal tax rate plus lump sum grants to all the individuals (so that the average tax rate rises with income even if the marginal does not), Mankiw NG, Weinzierl M, YaganD.(2009).Rigorous derivations of the optimal tax rates nclude: Atkinson,Stiglitz,(1980);Kaplow,(2008);Mirrlees(1976),Mirrlees(1986);Stiglitz,(1987);Tuomala,(1990).
The choice of the optimal redistributive tax involves tradeoffs between three kinds of effects: equity effect (it changes the distribution of income), the efficiency effect form reducing the incentives, the insurance effect from reducing the variance of individual income streams, Varian,H.R.(1980). Saez (2001) argued that "unbounded distributions are of much more interest than bounded distributions to address high income optimal tax rate problem". Saez (2001) investigated (four cases) ${ }^{2}$ and the optimal tax rates are clearly U-shaped, see Diamond

[^0](1998) too. Saez,S.Stantcheva (2016), define social marginal welfare weight as a function of agents consumption, earnings, and a set of characteristics that affect social marginal welfare weight and a set of characteristics that affect utility. Piketty, Saez,Stantcheva(2014),derived optimal top tax rate formulas in a model where top earners respond to taxes through three channels: labor supply, tax avoidance, and compensation bargaining. Dynamic taxation most famous examples in the literature are: Diamond-Mirrlees (1978);Albanesi-Sleet(2006),Shimer-Werning(2008),Ales-Maziero(2009),Golosov-TroshkinTsyvinsky(2011).Sizeable literature in NDPF studies optimal taxation in dynamic settings,(Golosov,Kocherlakota,Tsyvinski(2003),Golosov,Tsyvinski, and Werning (2006), Kocherlakota (2010).Here we will derive optimal linear, non-linear tax rates for top earners and we will derive results in heterogenous preferences environment for dynamic taxation. Optimal taxation is not to be confused with Pareto efficient taxes (see Werning (2007)).

## 2.Mirrlees framework optimal top tax rate : derivation

The effect of small tax reform in Mirrless (1971) model is examined in Brewer, M., E. Saez, and A. Shephard (2010), where indirect utility function is given as : $\boldsymbol{U}(\mathbf{1}-\boldsymbol{\tau}, \boldsymbol{R})=\boldsymbol{m a x}_{\boldsymbol{w}}((\mathbf{1}-\boldsymbol{\tau}) \boldsymbol{w}+\boldsymbol{R}, \mathbf{z})$, where $\boldsymbol{w}$ represents the taxable income $\boldsymbol{R}$ is a virtual income intercept, and $\boldsymbol{\tau}$ is an imposed income tax. Marshalian labor supply is $\mathrm{w}=\boldsymbol{w}(\mathbf{1}-\boldsymbol{\tau}, \boldsymbol{R})$, uncompensated elasticity of the supply is given as: $\boldsymbol{\varepsilon}^{\boldsymbol{u}}=\frac{(\mathbf{1}-\boldsymbol{\tau})}{\boldsymbol{w}} \frac{\boldsymbol{\partial} \boldsymbol{w}}{\boldsymbol{\partial}(\mathbf{1}-\boldsymbol{\tau})}$, income effect is $\boldsymbol{\eta}=$ $(\mathbf{1}-\boldsymbol{\tau}) \frac{\partial \boldsymbol{w}}{\partial \boldsymbol{R}} \leq \mathbf{0}$.Hicksian supply of labor is given as: $\boldsymbol{w}^{\boldsymbol{c}}((\mathbf{1}-\boldsymbol{\tau}, \boldsymbol{u}))$, this minimizes the cost in need to achieve slope $\mathbf{1}-\boldsymbol{\tau}$, compensated elasticity now is : $\varepsilon^{\boldsymbol{c}}=\frac{(\mathbf{1 - \tau )}}{\boldsymbol{w}} \frac{\partial \boldsymbol{w}^{c}}{\boldsymbol{\partial}(\mathbf{1}-\boldsymbol{\tau})}>0$, Slutsky equation now becomes: $\frac{\partial \boldsymbol{w}}{\boldsymbol{\partial}(\mathbf{1}-\boldsymbol{\tau})}=$ $\frac{\partial w^{c}}{\partial(1-\tau)}+\boldsymbol{z} \frac{\partial z}{\partial \boldsymbol{R}} \Rightarrow \varepsilon^{u}=\varepsilon^{c}+\boldsymbol{\eta}$, where $\boldsymbol{\eta}$ represents income effect : $\boldsymbol{\eta}=(\mathbf{1}-\boldsymbol{\tau}) \frac{\partial \boldsymbol{w}}{\partial \boldsymbol{R}} \leq \mathbf{0}$. With small tax reform taxes and revenue change i.e.: $\boldsymbol{d U}=\boldsymbol{u}_{\boldsymbol{c}} \cdot[-\boldsymbol{w} \boldsymbol{t}+\boldsymbol{d R}]+\boldsymbol{d w}\left[(\mathbf{1}-\boldsymbol{\tau}) \boldsymbol{u}_{\boldsymbol{c}}+\boldsymbol{u}_{\boldsymbol{z}}\right]=\boldsymbol{u}_{\boldsymbol{c}} \cdot[-\boldsymbol{z} \boldsymbol{t}+\boldsymbol{d R}]$.Change of taxes and its impact on the society is given as: $\boldsymbol{U}_{\boldsymbol{i}}=-\boldsymbol{u}_{\boldsymbol{c}} \boldsymbol{d} \boldsymbol{T}\left(\boldsymbol{w}_{\boldsymbol{i}}\right)$. Envelope theorem here says : $\quad \boldsymbol{U}(\boldsymbol{\theta})=\max _{\boldsymbol{x}} \boldsymbol{F}(\boldsymbol{x}, \boldsymbol{\theta}), \boldsymbol{s . t} \boldsymbol{c}>\boldsymbol{c}(\boldsymbol{x}, \boldsymbol{\theta}) \quad, \quad$ and the preliminary result is : $\boldsymbol{U}^{\prime}(\boldsymbol{\theta})=\frac{\partial F}{\partial \boldsymbol{\theta}}\left(x^{*}(\boldsymbol{\theta}), \boldsymbol{\theta}-\lambda^{*}(\boldsymbol{\theta}) \frac{\partial G}{\partial \boldsymbol{\theta}} \boldsymbol{x}^{*}(\boldsymbol{\theta}), \boldsymbol{\theta}\right)$. Government is maximizing :

$$
\begin{equation*}
0=\int G^{\prime}\left(u^{i}\right) u_{c}^{i} \cdot\left[\left(W-w^{i}\right)-\frac{\tau}{(1-\tau)} e W\right] \tag{1}
\end{equation*}
$$

1. mechanical effect is given as: $\boldsymbol{d} \boldsymbol{M}=\left[\boldsymbol{w}-\boldsymbol{w}^{*}\right] \boldsymbol{d} \boldsymbol{\tau}$,
2. welfare effect is : $\boldsymbol{d W}=-\overline{\boldsymbol{g}} \boldsymbol{d} \boldsymbol{M}=-\overline{\boldsymbol{g}}\left[\boldsymbol{w}-\boldsymbol{w}^{*}\right]$, and at last
3. the behavioral response is : $\boldsymbol{d B}=-\frac{\boldsymbol{\tau}}{\mathbf{1 - \tau}} \cdot \boldsymbol{e} \cdot \boldsymbol{w} \boldsymbol{d} \boldsymbol{\tau}$.

And let's denote that:

$$
\begin{equation*}
d M+d W+d B=d \tau\left[1-\bar{g}\left[w-w^{*}\right]-e \frac{\tau}{1-\tau} \cdot w\right] \tag{2}
\end{equation*}
$$

When the tax is optimal these three effects should equal zero i.e. $\boldsymbol{d} \boldsymbol{M}+\boldsymbol{d} \boldsymbol{W}+\boldsymbol{d} \boldsymbol{B}=\mathbf{0}$ given that: $\frac{\boldsymbol{\tau}}{1-\boldsymbol{\tau}}=$ $\frac{(\mathbf{1}-\bar{g})\left[\boldsymbol{w}-\boldsymbol{w}^{*}\right]}{\boldsymbol{e} \cdot \boldsymbol{z}}$, and we got $\boldsymbol{\tau}=\frac{1-\bar{g}}{1-\bar{g}+\boldsymbol{a} \cdot \boldsymbol{e}}, \boldsymbol{a}=\frac{\boldsymbol{w}}{\boldsymbol{w}-\boldsymbol{w}^{*}}$, and $\boldsymbol{d} \boldsymbol{M}=\boldsymbol{d} \tau\left[\boldsymbol{w}-\boldsymbol{w}^{*}\right] \ll d B=\boldsymbol{d} \boldsymbol{\tau} \cdot \boldsymbol{e} \frac{\boldsymbol{\tau}}{1-\boldsymbol{\tau}} \cdot \boldsymbol{w}, \boldsymbol{w h e n} \boldsymbol{w}^{*}>\boldsymbol{w}^{\boldsymbol{T}}$, where $\boldsymbol{w}^{\boldsymbol{T}}$ is a top earner income. Pareto distribution is given as:

$$
\begin{equation*}
1-F(w)=\left(\frac{k}{w}\right)^{a} ; f(w)=a \cdot \frac{k^{a}}{w^{1+a}} \tag{3}
\end{equation*}
$$

$\boldsymbol{a}$ is a thickness parameter and top income distribution is measured as:

$$
\begin{equation*}
w\left(w^{*}\right)=\frac{\int_{Z^{*}}^{\infty} s f(s) d s}{\int_{Z^{*}}^{\infty} f(s) d s}=\frac{\int_{Z^{*}}^{\infty} s^{-a} d s}{\int_{Z^{*}}^{\infty} s^{-a-1} d s}=\frac{a}{(a-1)} \cdot w^{*} \tag{4}
\end{equation*}
$$

Empirically $a \in[\mathbf{1 . 5}, \mathbf{3}], \boldsymbol{\tau}=\frac{\mathbf{1 - \overline { g }}}{\mathbf{1 - \overline { g } + \boldsymbol { a } \cdot \boldsymbol { e }}}$.General non-linear tax without income effects is given as:

$$
\begin{equation*}
\frac{T \prime\left(w_{n}\right)}{1-T \prime\left(w_{n}\right)}=\frac{1}{e}\left(\frac{\int_{n}^{\infty}\left(1-g_{m}\right) d F(m)}{w_{n} h(w)}\right)=\frac{1}{e}\left(\frac{1-H\left(w_{n}\right)}{w_{n} h\left(w_{n}\right)}\right) \cdot\left(1-G\left(\left(w_{n}\right)\right)\right. \tag{5}
\end{equation*}
$$

Where elasticity or efficiency $\boldsymbol{e}=\left[\frac{1-\boldsymbol{\tau}}{\boldsymbol{w}}\right] \times \frac{\boldsymbol{d} \boldsymbol{w}}{(\mathbf{1 - \tau})}$. Where $\boldsymbol{G}\left(\left(\boldsymbol{w}_{\boldsymbol{n}}\right)=\frac{\int_{\boldsymbol{n}}^{\infty} \boldsymbol{g}_{\boldsymbol{m}} \boldsymbol{d F}(\boldsymbol{m})}{\mathbf{1 - F}(\boldsymbol{n})}\right.$, and $\boldsymbol{g}_{\boldsymbol{m}}=\boldsymbol{G}^{\prime}\left(\boldsymbol{u}_{\boldsymbol{m}}\right) / \boldsymbol{\lambda}$ this is welfare weight of type $\boldsymbol{m}$.But non-linear tax witn income effect takes into account small tax reform where tax rates change from $\boldsymbol{d} \boldsymbol{\tau}$ to $\left[\boldsymbol{w}^{*}, \boldsymbol{w}^{*}+\boldsymbol{w}^{*}\right]$. Every tax payer with income $\boldsymbol{w}>\boldsymbol{w}^{*}$ pays additionaly $\boldsymbol{d} \boldsymbol{\tau} \boldsymbol{d} \boldsymbol{w}^{*}$ valued by ( $\mathbf{1}-\boldsymbol{g}(\boldsymbol{w})$ ) $\boldsymbol{\tau} \quad \boldsymbol{w}^{*}$.Mechanical effect is:

$$
\begin{equation*}
M=\tau d w^{*} \int_{z^{*}}^{\infty}(1-g(w)) d \tau d w^{*} \tag{6}
\end{equation*}
$$

Total income response is : I $=\boldsymbol{\tau} \boldsymbol{d} \boldsymbol{w}^{*} \int_{\boldsymbol{Z}^{*}}^{\infty}\left(-\boldsymbol{\eta}_{\boldsymbol{Z}} \frac{\boldsymbol{T}^{\prime}(\boldsymbol{w})}{1-\boldsymbol{T}^{\prime}(\boldsymbol{w})}(\boldsymbol{w})\right) \boldsymbol{h}(\boldsymbol{w}) \boldsymbol{d} \boldsymbol{w}$. Change at the taxpayers form the

We can rewrite FOC with respect to $\boldsymbol{l}_{\boldsymbol{n}}$ as:

$$
\begin{equation*}
\frac{\tau^{\prime}\left(w_{n}\right)}{1-\tau^{\prime}\left(w_{n}\right)}=\left(1+\frac{1}{e}\right) \cdot\left(\frac{\int_{n}^{\infty}\left(1-g_{m}\right) d F(m)}{n f(n)}\right) \tag{17}
\end{equation*}
$$

In previous expression $\boldsymbol{g}_{\boldsymbol{m}}=\frac{\boldsymbol{G}^{\prime}\left(\boldsymbol{u}_{\boldsymbol{m}}\right)}{\lambda}$ which is the social welfare on individual $\boldsymbol{m}$. The formula was derived in Diamond (1998). If we denote $\boldsymbol{h}\left(\boldsymbol{w}_{\boldsymbol{n}}\right)$ as density of earnings at $\boldsymbol{w}_{\boldsymbol{n}}$ if the nonlinear tax system were replaced by linearized tax with marginal tax rate $\boldsymbol{\tau}=\boldsymbol{\tau}^{\prime}\left(\boldsymbol{w}_{\boldsymbol{n}}\right)$ we would have that following equals $\boldsymbol{h}\left(\boldsymbol{w}_{\boldsymbol{n}}\right) \boldsymbol{d} \boldsymbol{w}_{\boldsymbol{n}}=\boldsymbol{f}(\boldsymbol{n}) \boldsymbol{d} \boldsymbol{n}$ and $\boldsymbol{f}(\boldsymbol{n})=\boldsymbol{h}\left(\boldsymbol{w}_{\boldsymbol{n}}\right) \boldsymbol{l}_{\boldsymbol{n}}(\mathbf{1}+\boldsymbol{e})$, henceforth $\boldsymbol{n} \boldsymbol{f}(\boldsymbol{n})=\boldsymbol{w}_{\boldsymbol{n}} \boldsymbol{h}\left(\boldsymbol{w}_{\boldsymbol{n}}\right)(\mathbf{1}+\boldsymbol{e})$ and we can write previous equation as:
$\frac{\tau^{\prime}\left(w_{n}\right)}{1-\tau^{\prime}\left(w_{n}\right)}=\frac{1}{e} \cdot\left(\frac{\int_{n}^{\infty}\left(1-g_{m}\right) \boldsymbol{d F}(m)}{w_{n} h\left(w_{n}\right)}\right)=\frac{1}{e} \cdot\left(\frac{1-H\left(w_{n}\right)}{w_{n} h\left(w_{n}\right)}\right) \cdot\left(1-G\left(w_{n}\right)\right)$
In the previous expression $\boldsymbol{G}\left(\boldsymbol{w}_{\boldsymbol{n}}\right)=\int_{\boldsymbol{n}}^{\infty} \frac{\boldsymbol{d} \boldsymbol{F}(\boldsymbol{m})}{\mathbf{1 - \boldsymbol { F } ( \boldsymbol { n } )}}$ is the average social welfare above $\boldsymbol{w}_{\boldsymbol{n}}$. If we change variables from $\boldsymbol{n} \rightarrow \boldsymbol{w}_{\boldsymbol{n}}$, we have $\boldsymbol{G}\left(\boldsymbol{w}_{\boldsymbol{n}}\right)=\int_{\boldsymbol{w}_{\boldsymbol{n}}}^{\infty} \frac{\boldsymbol{g}_{\boldsymbol{m}} \boldsymbol{d H}\left(\boldsymbol{w}_{\boldsymbol{m}}\right)}{\mathbf{1 - H ( \boldsymbol { w } _ { \boldsymbol { n } } )}}$. The transversality condition implies $\boldsymbol{G}\left(\boldsymbol{w}_{\mathbf{0}}=\mathbf{0}\right)=\mathbf{1}$.

### 2.2 Optimal linear tax formula

First modern treatment of optimal linear tax was provided by Sheshinski (1972).Optimal linear tax formulae is given as:
$\int_{0}^{\infty} \tau(w) f(n) d n=\int_{0}^{\infty}(w-\alpha-\beta w) f(n) d n=0$
$\boldsymbol{f}(\boldsymbol{n})$ is PDF of ability $\boldsymbol{n}, \boldsymbol{\alpha}$ is a tax parameter and is a lump-sum tax if $\boldsymbol{\alpha}<0$ and tax-subsidy if $\boldsymbol{\alpha}>0$ given to an individual with no income. $\mathbf{1}-\boldsymbol{\beta}$ is a marginal tax rate i.e. $\mathbf{0} \leq \boldsymbol{\beta} \leq \mathbf{1}$ so that marginal tax rate is non negative in the linear tax function which is $\boldsymbol{\tau}(\boldsymbol{w})=-\boldsymbol{\alpha}+(\mathbf{1}-\boldsymbol{\beta}) \boldsymbol{w}$, after tax consumption is $\boldsymbol{c}(\boldsymbol{w})=\boldsymbol{w}-\boldsymbol{\tau}(\boldsymbol{w})=\boldsymbol{\alpha}+$ $\boldsymbol{\beta} \boldsymbol{w}$.Optimal labor supply is given as: $\boldsymbol{\ell}=\hat{\boldsymbol{\ell}}(\boldsymbol{\beta} \boldsymbol{n}, \boldsymbol{\alpha})$.If $\boldsymbol{\lambda}$ is the lowest elasticity of labor supply function and it is equal to $\lambda=\lim _{n} \inf \left[\frac{\beta}{\hat{\ell}} \frac{\partial \hat{\ell}}{\partial \beta}\right]$ so that $\frac{\beta}{\hat{\boldsymbol{\ell}}} \frac{\partial \hat{\ell}}{\partial \beta} \geq \lambda$. Revenue maximizing linear tax rate is given as: $\frac{\tau^{*}}{1-\tau^{*}}=\frac{1}{e}$ or $\boldsymbol{\tau}^{*}=\frac{\mathbf{1}}{1+\boldsymbol{e}}$.Government FOC given $\left.\boldsymbol{S W F}=\int \boldsymbol{\omega}_{\boldsymbol{i}} G\left(\boldsymbol{u}^{\boldsymbol{i}}(\mathbf{1}-\boldsymbol{\tau}) \boldsymbol{w}^{i}+\boldsymbol{\tau} \boldsymbol{w}(\mathbf{1}-\boldsymbol{\tau})-\boldsymbol{E}, \boldsymbol{w}^{i}\right)\right) d \boldsymbol{f}(\boldsymbol{i})$ is :

$$
\begin{equation*}
0=\frac{s w F}{d \tau}=\int \omega_{i} G^{\prime}\left(u_{i}\right) u_{c}^{i} \cdot\left(\left(w-w^{*}\right)-\tau \frac{d w}{(1-\tau)}\right) d f(i) \tag{20}
\end{equation*}
$$

Social marginal welfare weight $\boldsymbol{g}_{\boldsymbol{i}}$ is given as: $\boldsymbol{g}_{\boldsymbol{i}}=\frac{\boldsymbol{\omega}_{i} \boldsymbol{G}^{\prime}\left(\boldsymbol{u}_{i}\right) \boldsymbol{u}_{\boldsymbol{c}}^{\boldsymbol{i}}}{\int \omega_{j} \boldsymbol{G}^{\prime}\left(\boldsymbol{u}_{j}\right) \boldsymbol{u}_{\boldsymbol{c}}^{\boldsymbol{j} \boldsymbol{d}(\boldsymbol{j})}}$. So that optimal linear tax formula is:

$$
\begin{equation*}
\tau=\frac{1-\bar{g}}{1-\bar{g}+e} \tag{21}
\end{equation*}
$$

where $\overline{\boldsymbol{g}}=\frac{\int g_{i} \cdot w_{i} d f(i)}{w}$.

### 2.3 Diamond ABC formula

Here in this paragraph a Diamond (1988) formula has been derived. Welfare weights are distributed with a CDF: $\boldsymbol{\Psi}(\boldsymbol{n})$ and PDF: $\boldsymbol{\psi}(\boldsymbol{n})$. The government maximization function is (objective function) is given as:

$$
\int_{\underline{n}}^{\bar{n}} u(n) \psi(n) d n
$$

Now by assumption $\int_{\underline{n}}^{\bar{n}} \boldsymbol{u}(\boldsymbol{n}) \boldsymbol{\psi}(\boldsymbol{n}) \boldsymbol{d} \boldsymbol{n}=\mathbf{1}$, which implies that $\lambda=\mathbf{1}, \lambda$ aggregates the social welfare weights across the entire economy.

$$
\begin{equation*}
\lambda=\int_{\underline{n}}^{\bar{n}} \Psi \boldsymbol{u}(n) \psi(n) d n \tag{23}
\end{equation*}
$$

FOC can be found as previously, form the Hamiltonian $\mathcal{H}=\left[\boldsymbol{\Psi}\left(\boldsymbol{u}_{\boldsymbol{n}}\right)+\boldsymbol{\lambda} \cdot\left(\boldsymbol{n} \boldsymbol{l}_{\boldsymbol{n}}-\boldsymbol{u}_{\boldsymbol{n}}-\boldsymbol{v}\left(\boldsymbol{l}_{\boldsymbol{n}}\right)\right)\right] \boldsymbol{\psi}(\boldsymbol{n})+$ $\boldsymbol{\phi}(\boldsymbol{n}) \cdot \frac{\boldsymbol{I} \boldsymbol{v}^{\prime}(\boldsymbol{l n})}{\boldsymbol{n}}$. In previous $\boldsymbol{\phi}(\boldsymbol{n})$ is the multiplier of the state variable. The FOC with respect to $\boldsymbol{l}$ is given as: $\boldsymbol{\lambda}$. $\left(\boldsymbol{n}-\boldsymbol{v}^{\prime}\left(\boldsymbol{l}_{\boldsymbol{n}}\right)\right)+\frac{\boldsymbol{\phi}(\boldsymbol{n})}{\boldsymbol{n}} \cdot\left[\boldsymbol{v}^{\prime}\left(\boldsymbol{l}_{\boldsymbol{n}}\right)+\boldsymbol{l}_{\boldsymbol{n}} \boldsymbol{v}^{\prime \prime}\left(\boldsymbol{l}_{\boldsymbol{n}}\right)\right]=\mathbf{0}$. FOC with respect to $\boldsymbol{u}$ is given as:

$$
\begin{equation*}
-\frac{d \phi(n)}{n}=\left[\Psi\left(u_{n}\right)-\lambda\right]=-\phi^{\prime}(n)-\lambda f(n) \tag{24}
\end{equation*}
$$

Or alternatively: $-\boldsymbol{\phi}(\boldsymbol{n})=\int_{n}^{\bar{n}}(\boldsymbol{f}(\boldsymbol{n})-\boldsymbol{\Psi}(\boldsymbol{n})) \boldsymbol{d} \boldsymbol{n}=\boldsymbol{\Psi}(\boldsymbol{n})-\boldsymbol{F}(\boldsymbol{n})$

$$
\begin{equation*}
\frac{\tau^{\prime}\left(w_{n}\right)}{1-\tau^{\prime}\left(w_{n}\right)}=\left(\frac{1+e}{e}\right) \cdot\left(\frac{\psi(n)-F(n)}{n f(n)}\right) \tag{25}
\end{equation*}
$$

To write ABC formula we divide and multiply by $\mathbf{1}-\boldsymbol{F}(\boldsymbol{n})$ :

$$
\begin{equation*}
\frac{\tau^{\prime}\left(w_{n}\right)}{1-\tau^{\prime}\left(w_{n}\right)}=\underbrace{\left(\frac{1+e}{e}\right)}_{A(n)} \cdot \underbrace{\left(\frac{\psi(n)-F(n)}{1-F(n)}\right)}_{B(n)} \cdot \underbrace{\left(\frac{1-F(n)}{n f(n)}\right)}_{C(n)} \tag{26}
\end{equation*}
$$

Where $\boldsymbol{A}(\boldsymbol{n})=\frac{\mathbf{1 + e}}{\boldsymbol{e}}$ is the elasticity and efficiency argument, $\boldsymbol{B}(\boldsymbol{n})=\frac{\boldsymbol{\psi}(\boldsymbol{n})-\boldsymbol{F}(\boldsymbol{n})}{\mathbf{1 - F ( n )}}$ measures the desire for redistribution, $\boldsymbol{C}(\boldsymbol{n})=\frac{\mathbf{1 - F ( n )}}{\boldsymbol{n f}(\boldsymbol{n})}$ measures the thickness on the right tail of distribution. In the Rawlsian case $\boldsymbol{\Psi}(\boldsymbol{n})=$ 1 previous formula will converge to:

$$
\begin{equation*}
\frac{\tau^{\prime}\left(w_{n}\right)}{1-\tau^{\prime}\left(w_{n}\right)}=\left(\frac{1+e}{e}\right) \cdot\left(\frac{1-F(n)}{n f(n)}\right) \tag{27}
\end{equation*}
$$

### 2.4 Formal derivation of optimal non-linear tax rates with income effects

Utility function takes form $\tilde{u}(c, l)=u(c)-v(l)$ where $u^{\prime}(c)>0 ; u^{\prime \prime}(c) \leq 0$. Elasticity of labor supply is :

$$
\begin{equation*}
\frac{v^{\prime}(l)}{u^{\prime}(c)}=\left(1-\tau^{\prime}(w)\right) n \tag{28}
\end{equation*}
$$

The uncompensated response of labor supply is given as:

$$
\begin{equation*}
\frac{\partial \partial^{u}}{\partial\left(1-\tau^{\prime}(w)\right) n}=\frac{u^{\prime}(c)+l\left(1-\tau^{\prime}(w)\right) n u^{\prime \prime}(c)}{v^{\prime \prime}(l)-\left(1-\tau^{\prime}(w)\right)^{2} n^{2} u^{\prime \prime}(c)} \tag{29}
\end{equation*}
$$

And uncompensated elasticity is implied:

$$
\begin{equation*}
\varepsilon^{u}=\frac{\frac{u^{\prime}(c)}{l}+\frac{v^{\prime}(l)^{2}}{\left.u^{\prime}(c)\right)^{\prime}} u^{\prime \prime}(c)}{v^{\prime \prime}(l)-\frac{v^{\prime}(l)^{2}}{u^{\prime}(c)^{2}} u^{\prime \prime}(c)} \tag{30}
\end{equation*}
$$

The response of labor to income changes is given as:

$$
\begin{equation*}
\frac{\partial l}{\partial y}=\frac{\left(1-\tau^{\prime}(w) n u u^{\prime \prime}(c)\right.}{v^{\prime \prime}(l)-\left(1-\tau^{\prime}(w)\right)^{2} n^{2} u^{\prime \prime}(c)} \tag{31}
\end{equation*}
$$

By using the Slutsky equation we have:
$\frac{\partial \boldsymbol{l}^{c}}{\partial\left(1-\boldsymbol{\tau}^{\prime}(w)\right) n}=\frac{\boldsymbol{u}^{\prime}(\boldsymbol{c})+l\left(1-\boldsymbol{\tau}^{\prime}(w)\right) n u{ }^{\prime \prime}(c)}{v^{\prime \prime}(l)-\left(1-\boldsymbol{\tau}^{\prime}(w)\right)^{2} n^{2} u^{\prime \prime}(c)}-\frac{l\left(1-\boldsymbol{\tau}^{\prime}(w)\right) n u^{\prime \prime}(c)}{v^{\prime \prime}(l)-\left(1-\tau^{\prime}(w)\right)^{2} n^{2} u^{\prime \prime}(c)}=\frac{u^{\prime}(c)}{v^{\prime \prime}(l)-\left(1-\boldsymbol{\tau}^{\prime}(w)\right)^{2} n^{2} u^{\prime \prime}(c)}$

Henceforth :

$$
\begin{equation*}
\varepsilon^{c}=\frac{\frac{v^{\prime}(l)}{l}}{v^{\prime \prime}(l)-\left(1-\tau^{\prime}(w)\right)^{2} n^{2} u^{\prime \prime}(c)} \tag{33}
\end{equation*}
$$

Here everything is as previous except now we cannot replace $\boldsymbol{c}(\boldsymbol{n})$ in the resource constraint by using def. of indirect utility here we will define consumption as expenditure function $\tilde{\boldsymbol{c}}(\widetilde{\boldsymbol{u}}(\boldsymbol{n}), \boldsymbol{w}(\boldsymbol{n}), \boldsymbol{n})$. Previous resource constraint for this economy with no income effects was:

$$
\begin{equation*}
\int_{\underline{n}}^{\bar{n}} c(n) f(n) d n \geq \int_{\underline{n}}^{\bar{n}} w(n) f(n) d n-E \tag{34}
\end{equation*}
$$

So this new function we will differentiate w.r.t. $\widetilde{\boldsymbol{u}}(\boldsymbol{n}), \boldsymbol{w}(\boldsymbol{n})$.Indirect utility is defined as :

$$
\begin{equation*}
\widetilde{\boldsymbol{u}}(n)=u(\tilde{c}(n))-v\left(\frac{w^{*}(n)}{n}\right) \tag{35}
\end{equation*}
$$

At optimum conditions that hold are:

$$
\begin{gather*}
\tilde{u}(n)=u^{\prime}(\tilde{c}(n)) \tilde{c}(n)  \tag{36}\\
0=u^{\prime}(\tilde{c}(n)) d \tilde{c}(n)-\frac{1}{n} v^{\prime}\left(\frac{w^{*}(n)}{n}\right) d w^{*}(n)
\end{gather*}
$$

If we rearrange we will get :

$$
\begin{align*}
\frac{\tilde{c}(n)}{\widetilde{u}(n)} & =\frac{1}{u^{\prime}(\tilde{c}(n))} \\
\frac{\tilde{c}(n)}{w^{*}(n)} & =\frac{v^{\prime}\left(\frac{w^{*}(n)}{n}\right)}{n u^{\prime}(\tilde{c}(n))} \tag{37}
\end{align*}
$$

Hamiltonian for this problem is given as:

Figure 2 Mirrleesian taxation: taxes and earnings schedule


In previous two figures we can see the schedules of taxes and earnings as well as skills and earning in the Mirrlees taxation model. What do studies tell? The compensated elasticity of labor supply with respect to real wage $\boldsymbol{\varepsilon}_{\boldsymbol{w}}^{*}$ has been estimate approximately to be 0.5 see Gruber, Saez (2002). Gruber, Saez (2002) estimate that for the US taxpayer with incomes above $100 \mathrm{~K} \$$ have elasticity around 0.57 . And those $<100 \mathrm{~K} \$$ have elasticity around 0.2 or even less.Next in table 4 FOC's for the Mirrlees model are presented.

Table 3 FOC's for the Mirrlees model
$\left.\begin{array}{ccccc}\text { iteration } & \begin{array}{c}\text { Func-c } \\ \text { ount }\end{array} & \begin{array}{c}\text { form o } \\ \text { f step }\end{array} & \begin{array}{c}\text { First-order optimal } \\ \text { ity }\end{array} \\ 0 & 3 & 1.37 \mathrm{E}-0\end{array}\right)$

In the next table 5 skills and consumption of agents that previously were depicted graphically are presented. Table 4 skills, consumption and earnings for the Mirrlees model
F(n)-skills

|  | x-cons. | $y$-income | $x(1-y)$ | z-earnings |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0.0424 | 0 | 0.0424 | 0 |
| 0.1 | 0.116 | 0.3894 | 0.0708 | 0.0869 |
| 0.5 | 0.18 | 0.4382 | 0.1011 | 0.1612 |
| 0.9 | 0.2888 | 0.4686 | 0.1535 | 0.2842 |
| 0.99 | 0.4315 | 0.4841 | 0.2226 | 0.4412 |

Table 6 depicts earning schedule, consumption, average tax rate and marginal tax rate correspondingly.

Table 5 average and marginal tax rates for Mirrlees model

| z-earnings | x-consumption | average tax <br> rate | marginal tax <br> rate |
| :---: | :---: | :---: | :---: |
| 0 | 0.0424 | -Inf | 0.2147 |
| 0.05 | 0.0847 | -0.54 | 0.2336 |
| 0.1 | 0.1271 | -0.1558 | 0.2223 |
| 0.2 | 0.214 | 0.0273 | 0.1993 |
| 0.3 | 0.3031 | 0.0817 | 0.1824 |
| 0.4 | 0.3937 | 0.1052 | 0.1698 |
| 0.5 | 0.4856 | 0.1171 | 0.1599 |

Optimal mirrleesian taxation is flat for a long range of top incomes $>1$.

## 6. Conclusion

Optimal tax rates as this paper shows depend on redistributive tastes of the supposedly benevolent social planers. The marginal social welfare weight on a given individual measures the value that society puts on providing an additional dollar of consumption to this individual. As the numerical solutions in the non-linear optimal tax rates showed that high tax rates are obtained when there unrealistically low uncompensated and compensated elasticities, also the shape parameter of Pareto distribution must be lower. For high tax countries e.g. countries with highest tax burden around $50 \%$ the area that provides such high tax rates is where compensated elasticity is between 0.2 and 0.5 and uncompensated elasticity and unrealistically high compensated elasticity between 0.5 and 0.8 but medium redistributive tastes $\overline{\boldsymbol{g}}=\mathbf{0 . 5}$. Or alternatively, if uncompensated elasticity is high $\boldsymbol{\varepsilon}_{\boldsymbol{u}}=\mathbf{0} .5$ than also the taste for redistribution must be high e.g. $\overline{\boldsymbol{g}} \in(\mathbf{0}, \mathbf{0} .25)$. For low tax countries the area where those taxes are provided is in high Pareto distribution parameter and very low taste for redistribution. These are very loose results and are conditioned by themselves and their combinations. In turn there is not straightforward solution to the optimal linear or non-linear labor income tax problem. Pareto efficient tax rates differ from those proposed by Mirrlees (1971).

In the dynamic Mirrlees approach, when it comes to the result for capital, capital is taxed to provide more efficient labor supply incentives when there is imperfect information (private distributions of ability unknown to other parties) and as a part of optimal insurance scheme against stochastic earning abilities. Intuition here is that savings affects incentive to work, so government needs to discourage savings to prevent the flowing deviation by highly skilled: 1) save more today; 2) work less tomorrow. That was the second model we reviewed and from there some optimal fiscal policy features are:1) On average wealth taxes across individuals are zero ex-ante ;2) However, they depend on future labor income-if labor income is below average, your capital tax is positive. If your labor income is above average, then your capital tax is negative. 3) So, this tax or this fiscal policy might be regressive for incentive reasons. So, in general about dynamic Mirrlees approach it can be concluded that: this approach assumes that agents' abilities to earn income are heterogeneous, stochastic, and private information. Tax instruments ex ante are unrestricted. The model solves for the optimal allocations using dynamic mechanism design (subject only to incentive compatibility constraints) and then considers how to implement these allocations using decentralized tax systems, see also Stantcheva (2020).This story also has normative element into it. Namely we must not forget principles of horizontal and vertical equity according to neo-classical economics defined by Feldstein (1976) when we define tax systems and marginal tax rates. Feldstein's Horizontal Equity Principle: Two people with the same utility before tax must have the same utility after tax and Feldstein's Vertical Equity Principle (No Reversals): If person $\boldsymbol{i}$ has greater utility than another person $\boldsymbol{j}$ before tax, then person $\boldsymbol{i}$ must have greater utility than person $\boldsymbol{j}$ after tax. Feldstein's no-reversals principle has important efficiency implications in a second-best world of imperfect information in which the government might not know how well-off certain people are, and they may have powerful incentive to hide private information about themselves, if the tax laws permitted reversals of utility

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[^0]:    ${ }^{1}$ Second fundamental theorem is giving conditions under which a Pareto optimal allocation can be supported as a price equilibrium with lump-sum transfers, i.e. Pareto optimal allocation as a market equilibrium can be achieved by using appropriate scheme of wealth distribution (wealth transfers) scheme (Mas-Colell, Whinston et al. 1995)
    ${ }^{2}$ Utilitarian criterion, utility type I and II and Rawlsian criterion, utility type I and II.

