

Union of Mathematicians of Macedonia - ARMAGANKA

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EQUATIONS AND ANALYSIS**

and

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Skopje, 2023

CONTENTS

1. **A GEOGRAPHICALLY WEIGHTED REGRESSION APPROACH IN REGIONAL MODEL FOR REAL ESTATE MASS VALUATION**
Natasha Malijanska, Sanja Atanasova, Gjorgji Gjorgjiev, Igor Peshevski, Daniel Velinov
2. **ПИСА ТЕСТИРАЊЕТО ВО СРЕДНИТЕ СТРУЧНИ УЧИЛИШТА**
Ана Димовска, Томи Димовски
3. **APPLICATION OF MARKOV CHAINS IN BIOLOGY**
Natasha Stojkovikj, Limonka Koceva Lazarova, Aleksandra Stojanova Ilievska
4. **CHAIN CONNECTED SET IN A SPACE**
Zoran Misajleski, Aneta Velkoska, and Emin Durmishi
5. **ABOUT THE T=GEOMETRIC INTRERPRATIONS ON THE BASIC INTERACTIONS AND SOME CONSEQUENCES**
Kostadin Trenchevski
6. **COSINE AND COTANGENT THEOREMS FOR A QUADRILATERAL, TWO NEW FORMULAS FOR ITS AREA AND THEIR APPLICATIONS**
V. Nenkov, St. Stefanov, H. Haimov, A. Velchev
7. **DEFORMED SPHERICAL CURVES**
Marija S. Najdanović, Miroslav D. Maksimović, Ljubica S. Velimirović, Svetozar R. Rančić
8. **ANALYSIS OF STUDENT ACHIEVEMENTS IN TEACHING COMPLEX NUMBERS USING GEOGEBRA SOFTWARE**
Dragana Nedić, Gordana Jotanović, Tijana Paunović, Aleksandar Kršić
9. **SEVERAL LOCI GENERATED BY A MOVING TRIANGLE BETWEEN TWO FIXED CIRCLES**
Prof. Sava Grozdev, Prof. Veselin Nenkov, Assoc. Prof. Tatiana Madjarova
10. **APPLICATION OF SEMIVARIOGRAMS AND KRIGING IN GEOTECHNICAL MODELLING**
Marija Maneva, Igor Peshevski, Milorad Jovanovski, Zoran Misajleski, Daniel Velinov
11. **INVERTIBILITY OF LINEAR COMBINATIONS OF K-POTENT MATRICES**
Marina Tošić, Jelena Vujaković
12. **STATISTICAL ANALYSIS OF REPAIR RATE FOR MAINTENANCE DECISION-MAKING**
Nataša Kontrec, Stefan Panić, Biljana Panić, Mejrima Ljajko
13. **GAMES IN MATHEMATICS INSTRUCTION**
Marina Manić¹, Eugen Ljajko², Marina Tošić
14. **Razvoj na elasticnosta na mislenjeto kaj uchenicite od pochetnoto obrazovanie**
Metodi Glavche, Malcheski Risto
15. **N-TUPLE WEAK ORBITS TENDING TO INFINITY FOR HILBERT SPACE OPERATORS**
Sonja Mančevska, Marija Orovčanec
16. **STEM APPROACH IN TEACHING MATHEMATICS**
Marija Miteva, Limonka Koceva Lazarova, Biljana Zlatanovska, Natasa Stojkovik
17. **АЛГЕБАРСКИ РАЦИОНАЛНИ ИЗРАЗИ ВО ОСНОВНО И СРЕДНО ОБРАЗОВАНИЕ - ПРОБЛЕМИ И НЕДОСТАТОЦИ**
Билјана Златановска¹, Марија Митева¹, Мирјана Коцалева Витанова
18. **CANTOR'S INTERSECTIONS THEOREM IN $(3,1,\nabla)$ -G METRIZABLE SPACES**
Tomi Dimovski, Pavel Dimovski
19. **WORKING WITH MATHEMATICALLY GIFTED STUDENTS AGED 18 TO 19**
Katerina Anevska¹, Valentina Gogovska², Risto Malcheski³,
20. **The Hanh-Banach Theorem for cyclic 2-subspaces generated by 5 and more elements**
Aleksa Malcheski
21. **2-topological vector space**
Aleksa Malcheski
22. **A PARTICULAR SOLUTION TO THE SPECIAL CASE OF A FOURTH-ORDER SHORTENED LORENZ SYSTEM**
Biljana Zlatanovska¹, Boro Piperevski², Mirjana Kocaleva Vitanova¹, Marija Miteva

23. SOLVING TASKS FROM LINEAR PROGRAMMING USING GEOGEBRA
Elena Karamazova Gelova ¹, Mirjana Kocaleva Vitanova ², Sonja Mančevska ³

STEM APPROACH IN TEACHING MATHEMATICS

Marija Miteva, Limonka Koceva Lazarova, Biljana Zlatanovska, Natasa Stojkovic

Abstract. Mathematics appears as essential subject in many academic studies. But students usually find math as difficult, and moreover consider its content as useless, thus avoid such studies. This is a reason for changing traditional approach in teaching mathematics and develop new one, which will emphasize problem-based learning. We will consider in this paper STEM approach in teaching mathematics and results of its implementation.

1. INTRODUCTION

The concept of STEM education is a concept which integrates Science, Technology, Engineering and Mathematics in the process of everyday teaching and learning. Education and teaching process should not only provide students with pure knowledge, but it should answer the question why do students need that knowledge and how to apply it in the future. During the process of learning, especially in the classes, the attention of the students at each age is usually at the highest level when they are considering a real life problem and are trying to solve it. This approach, i.e. including problem-based situations, characterizes STEM education. Thus, STEM approach means introducing certain concepts and their relations in the process of teaching, as something necessary for solving different problems that student will face up in the future, in their lives or carriers.

Students usually find mathematics as difficult subject in their education and are afraid of it, which is a reason for avoiding studying engineering, technology and anything else where mathematics appears as essential. This situation can be changed, as well as students' attitude toward mathematics, if STEM approach is implemented and real life problems are introduced in the classes. STEM approach can seriously contribute in improving the perception about mathematics among students. Moreover, project-based and problem-based learning and collaboration while solving certain problems can increase communication skills, creativity and critical thinking of students. It is important a STEM approach to be implemented in the earlier education, but is very important to become everyday practice in the higher education. About importance of STEM education, one can read [1-4].

In order to implement STEM approach in the process of education, teachers need an appropriate, well-developed curriculum for their lessons. Developing STEM curriculum progressively became research interest to many teachers nowadays. Some examples for real life problems can be found in [5]. In the

frame of the Erasmus+ project *Mathematics of the Future: Understanding and Application of Mathematics with the help of Technology, FutureMath* [6], teachers who participate have developed STEM curriculum for different math topics, together with variety of teaching materials and examples how to use different digital tools, in order to easier implement STEM approach on math classes. In this new approach teachers usually start lecture with real life problems. We will present some results of implementing such approach in the classes.

2. RESULTS OF APPLYING STEM APPROACH ON MATH LESSONS

Implementing STEM approach on math lessons, the students had an opportunity to face up with the new trends of teaching and almost all of the advantages of the STEM approach were achieved. By STEM as an educational approach, the students has the best introduction to each lecture via using a real problem. It is more interesting for students to consider and try to solve real life situations, than listening math lectures. The real problem motivates them to think about similar real problems, which are already known to them, without having in mind their connection with mathematics. Considering real life problems which need math knowledge to be solved, students realize that they have to achieve appropriate math knowledge first, in order to successfully solve the problem. Thus, learning math formulas and expressions become necessary, and students are not wondering anymore why they have to learn it. Presenting the new material and using computer applications and mathematical software, made the lesson more interesting and fun for the students then the previous methods of lecturing. Implementing all of this encourages the students to collaborate and discuss one with another, but also with the teacher via creative questions. These questions are related to requests for clarification of introduced new terms and curiosity to learn more, which are the basis for deeper knowledge. The lessons generally passed quickly, creatively, with fun, and with the mutual satisfaction of both the students and the teacher. The students passed the pilot lecture as the quality time spent because the new material was already introduced. The biggest result is satisfied students. The smiling and satisfied students, for the teacher mean successful organized lesson.

Gaining practical knowledge during the studies is also very important in the educational process. The students move through the educational process and they acquire knowledge from many different (but connected) areas. Later, this knowledge they should apply in real situations in life. The initial places where students can apply theoretical

knowledge in order to solve practical problems close to the real situations of their lives are the educational institutions. Educational institutions should teach the students how to do it. Only in this way, the students will be ready and successful for real life. Therefore, schools and universities must look for new ways and methodologies for practicing knowledge as a part of the process of teaching. Practical knowledge in teaching refers to students' knowledge of classroom situations and solving the practical problems they face in carrying out actions. The connection between Science, Technology, Engineering, and Mathematics guarantees the acquisition of practical knowledge in teaching. That is the STEM methodology. Having in mind that the new teaching methodology is based on solving practical problems, there is no doubt that students will gain practical knowledge simultaneously with the theoretical one. The new teaching methodology for practical knowledge per STEM means: connection of the real situations with the need for introducing new mathematical terms, which are provided in educational teaching programs; students should feel the need from introducing new mathematical knowledge; obtaining new knowledge with applications for solving a practical problem that is close to real problems of life; obtaining new knowledge which can be implemented in real situations, etc. The biggest benefit of gained practical knowledge is that the knowledge can be implemented in certain real-life situations. Applying different digital tools and software while solving certain problem-based situation students also gain practical knowledge.

We have organized several math lessons, with small groups of students (15-30), during which STEM approach was introduced. In order to receive feedback of the new methodology implemented, we have prepared short tests for checking the students' knowledge achieved on the lesson, and also a questionnaire as a survey about students' viewpoint for the new approach. It is interesting that almost all the students have answered the questions in the survey, although the questionnaire was not obligatory. In order to achieve objective answers, e-mail addresses of the students who answered the questions were not saved. Separate questionnaire was prepared for each piloting lesson, and the results are visible for each lesson separately. We have 107 answers to the 6 questionnaires with the same questions, but for the different lecture. There were 10 questions with the next offered answers: *I agree, I agree partially, I don't agree, I cannot say* and the students can choose only one of the offered answers. We will present in the next figures each question together with the results of the answers.

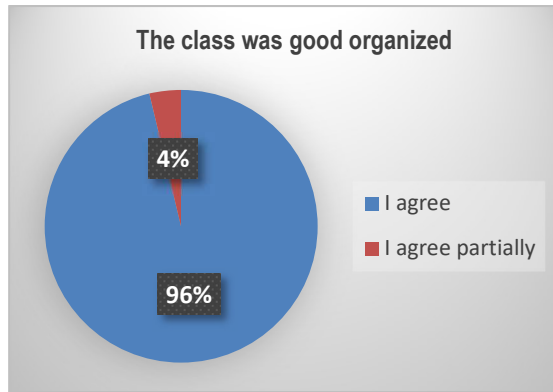


Figure 1: Answers to the first question

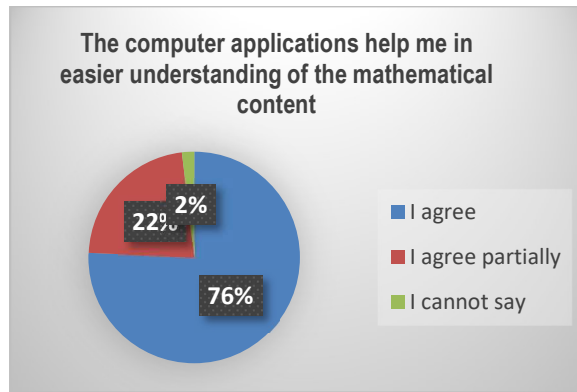


Figure 2: Answers to the second question

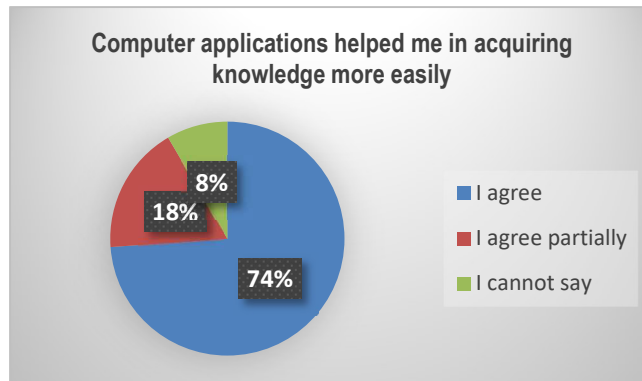


Figure 3: Answers to the third question

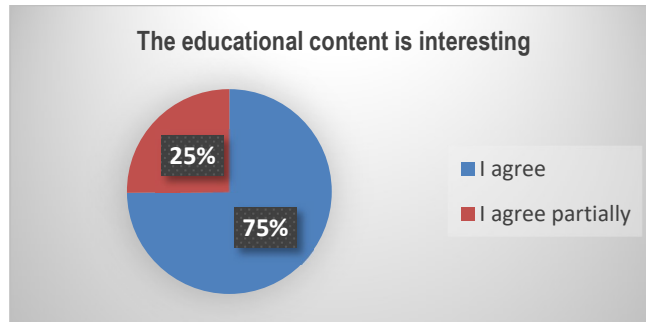


Figure 4: Answers to the fourth question

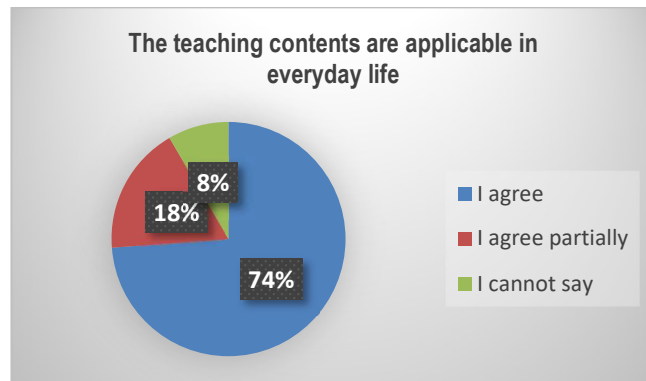


Figure 5: Answers to the fifth question

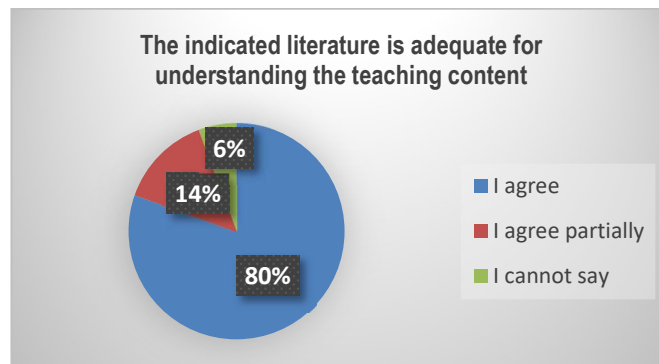


Figure 6: Answers to the sixth question

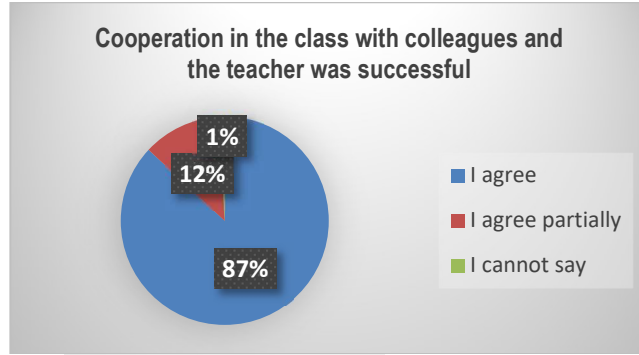


Figure 7: Answers to the seventh question

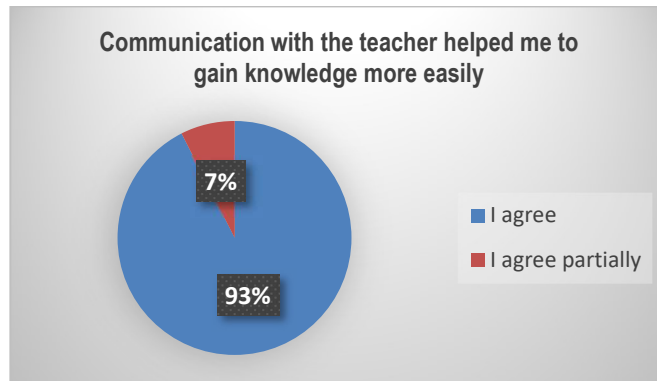


Figure 8: Answers to the eighth question

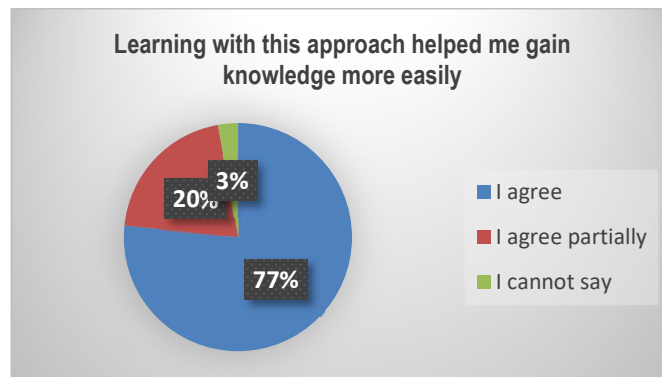


Figure 9: Answers to the ninth question

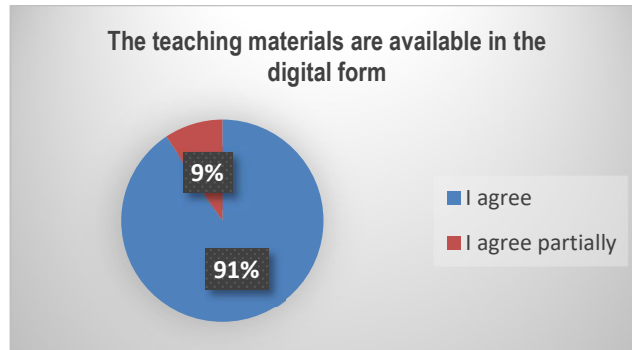


Figure 10: Answers to the tenth question

We believe that the answers are objective, because as we have said above, no email address was necessary to access the questionnaire, just the link of it was enough.

We have also prepared short test for checking students' knowledge immediately after the lesson. Unfortunately, we cannot boast with the results of the tests in this first phase of the implementation of the STEM approach, because we have not noticed big difference between these results and the results on the testing students' had before. But we are very pleased with the other results that we have notices with the implementation of the new method, because we managed to keep students' attention during the whole lesson and to burn the curiosity among them. They have very carefully joined the activities in the class; they were not boring and curiously have expected the solution of the problem set at the beginning of the lesson.

3. BENEFITS AND WEAK SIDES OF THE APPROACH

Both students and teachers have realized benefits from the implementation of the new methodology on the piloting lessons. Mathematics is usually understood by students as a science for itself, without any connection with reality. Using old methods during the math lessons, where math concepts were introduced with classical lectures, only with black board and chalk, full with mathematical theory, formulas and expressions, students are usually passive listeners. They are usually not involved in any activity on the classes, so their attention is decreasing continuously till the end of the lessons and they are getting boring. Thus, other method which will involve students in different activities during the lecture, as problem-solving situation, collaboration with others, etc. seems to be necessary to practice on everyday lessons. By connecting the 4 components of Science, Technology, Engineering and Mathematics, STEM has interesting access for presenting and introducing new material

for educational classes on each educational level. Implementing STEM approach on lessons and using different digital tools in order to easily achieve knowledge is big step towards to make lectures interesting, to increase students' attention on the lessons and contribute in reaching positive attitude among students toward mathematics. Students will become active problem solvers, they will develop creative thinking, and the most important of all they will change the perception for the process of math education, realizing that math knowledge is important for their carrier and everyday life. Therefore, STEM finds wide application in improving the educational process and general satisfaction of students and teachers.

Changing the students' perception about mathematics, as a result of the new approach in teaching, will on a long time make students to not be afraid anymore of mathematics and make students not to avoid study programs where mathematics appears as essential. Furthermore, implementing new teaching methodology in the education can attract students to study STEM fields, which will be big achievement for the universities.

There are many other findings emerging from the implementation of the new teaching methodology. By STEM methodology, the students think more broadly and more deeply than usual. The STEM methodology determines the way for the students to research new and creative ways to solve real-world problems and connect themselves to the fields that interest them. Using STEM methodology contributes in producing students who think critically with the integration of knowledge and skills from multiple areas. The students get creative ideas to apply the acquired knowledge in solving real problems. Later, these students will get up innovators, leaders, and educators of society. They will be creative people who will lead society forward in development and progress.

The newly developed teaching methodology based on the STEM approach can essentially change the way of teaching science, technology, engineering and mathematics, and more important of it, can essentially improve students' results of studying and their perception about education, generally. Thus, this new approach in teaching mathematics has to be preferred for the teachers to use.

According to all above described that characterizes the newly developed teaching methodology, based on the STEM approach, no doubly there are many strengths of it. This new teaching methodology differs a lot from the classical methods of teaching. One of its major strengths is that the students are in the centre of the attention. They are no

more passive listeners, but they are actively involved in the activities during the lessons. This new methodology and approach based on problem solving is strengthening students' creativity and problem-solving skills, as well as skills for collaboration and team work. The process of teaching with this new methodology has been changed from its roots. Students have new challenges on each new lesson, instead of being afraid from the material following. Teachers have also challenges to organize interesting lessons.

Although too many advantages, there are even some weaknesses of this new approach in teaching mathematics. Not each mathematical educational class can be organized via practical problems from real life's situations. In mathematics there exist terms with abstract nature which cannot be connected to practical problems. The good organization of a class with this methodology requires full dedication to the teacher and a lot of spent time.

Also, very often students do not have available appropriate digital tools and software which can help them in solving certain problems. Even the software and digital tools are available, very often they do not have an experience in working with it. This can be considered as a weakness, but as an advantage at the same time, because students simultaneously with the new lecture can practice digital tools and similar resources.

4. CONCLUSIONS

According to all above, the concept of STEM education and STEM principles can easily be implemented into STEM studies if teacher gives to the students a problem situation at the beginning of each lecture, in order students to realize that certain theoretical knowledge is necessary for solving such problem. In the continuation of the lesson, teacher can introduce theory of the subject, but students will realize it as something that they have to achieve in order to solve problem, not something useless and boring which is part of the subject's curriculum and that they have to memorize. In order to reach time on the lessons, teacher can give as homework to the students a problem situation related to the material which will be introduced on the next lesson. However, starting the lecture with a problem to be solved is the essential thing that makes this new approach, and changes the perception about mathematics and education at all.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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