



Mathematical Analysis and Simulation of Measles Infection Spread with SEIRV+D Model

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Agenda

Introduction

Model Description

- disease- free equilibrium (DEF) point

- Reproduction number

Case study

- data for N. Macedonia

- different vaccination rate

Summary

Introduction

Measles are most contagious infectious disease caused by measles virus .

transmitted by respiratory droplets of infected individual

Modelling measles is based on SIR (Susceptible-Infected-Recovered) model, since patients that have overcome the disease have gained permanent immunity

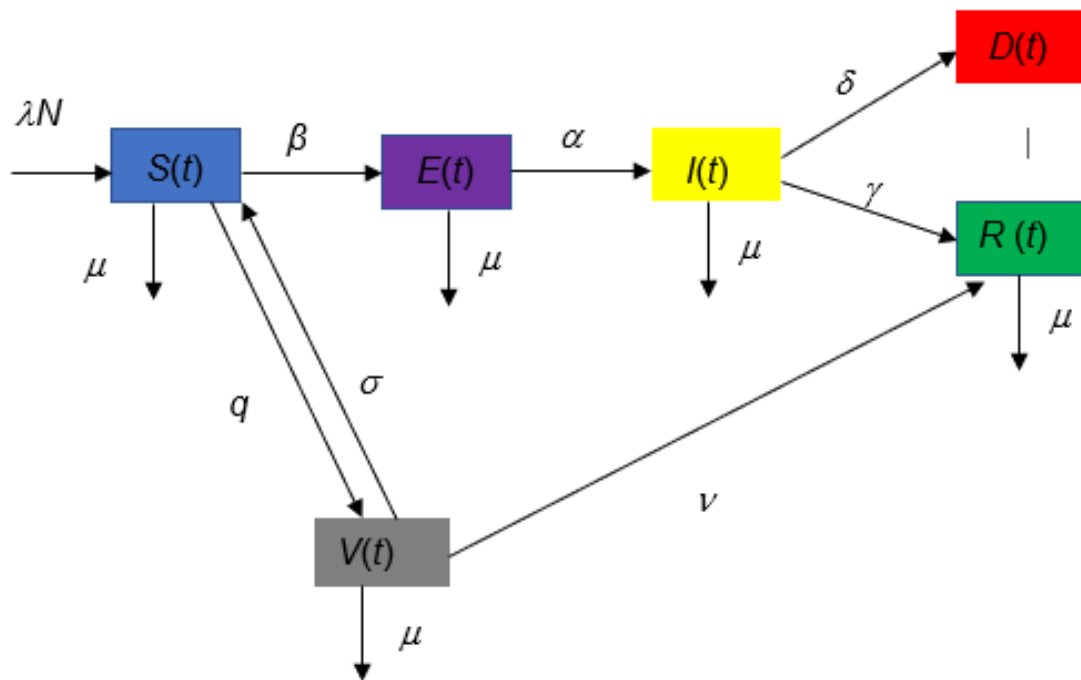
Model Description

The total population is divided into 6 compartments: susceptible, infected, exposed, recovered, vaccinated and death

$$N(t) = S(t) + E(t) + I(t) + R(t) + V(t) + D(t)$$

Susceptible- susceptible individuals that at risk of infection. Exposed individuals had contact with an infected individual but are not infectious. Infected individuals transmit the disease to others. Vaccinated have received vaccine and developed immunity. Deceased infected individuals are part of death compartment.

Model Description (2)



$$\frac{dS(t)}{dt} = \lambda N - \frac{\beta S(t)I(t)}{N} - qS(t) + \sigma V(t) - \mu S(t)$$

$$\frac{dE(t)}{dt} = \frac{\beta S(t)I(t)}{N} - \alpha E(t) - \mu E(t)$$

$$\frac{dI(t)}{dt} = \alpha E(t) - \gamma I(t) - \delta I(t) - \mu I(t)$$

$$\frac{dR(t)}{dt} = \gamma I(t) + vV(t) - \mu R(t)$$

$$\frac{dD(t)}{dt} = \delta I(t)$$

$$\frac{dV(t)}{dt} = qS(t) - \sigma V(t) - vV(t) - \mu V(t)$$

$$S(0) = S_0 \geq 0, E(0) = E_0 \geq 0, I(0) = I_0 \geq 0, R(0) = R_0 \geq 0, V(0) = V_0 \geq 0, D(0) = D_0 \geq 0$$



Model Description

The population is infection- free at disease free equilibrium point (DFEP)

Condition:
$$\frac{dS(t)}{dt} = \frac{dE(t)}{dt} = \frac{dI(t)}{dt} = \frac{dR(t)}{dt} = \frac{dV(t)}{dt} = \frac{dD(t)}{dt} = 0$$

$$(S^*, E^*, I^*, R^*, V^*, D^*) = \left(\frac{\lambda N(\sigma + \nu + \mu)}{(q + \mu)(\sigma + \nu + \mu) - \sigma q}, 0, 0, \frac{\nu q \lambda N}{\mu[(q + \mu)(\sigma + \nu + \mu) - \sigma q]}, \frac{q \lambda N}{(q + \mu)(\sigma + \nu + \mu) - \sigma q}, 0 \right)$$

Model Description

The basic reproduction number is a fundamental concept in epidemiology and is defined as expected number of new infections caused by single infected individual.

$$\mathcal{R}_0 = \frac{\beta\lambda(\sigma + \nu + \mu)}{(q + \mu)(\sigma + \nu + \mu) - \sigma q} \cdot \frac{\alpha}{(\alpha + \mu)(\gamma + \mu + \delta)}$$

$\mathcal{R}_0 > 1$ the disease-free equilibrium point is unstable, and the disease is spreading in the population.

$\mathcal{R}_0 < 1$ the disease-free equilibrium point is asymptotically stable, and the disease dies off

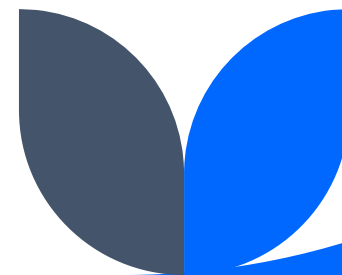
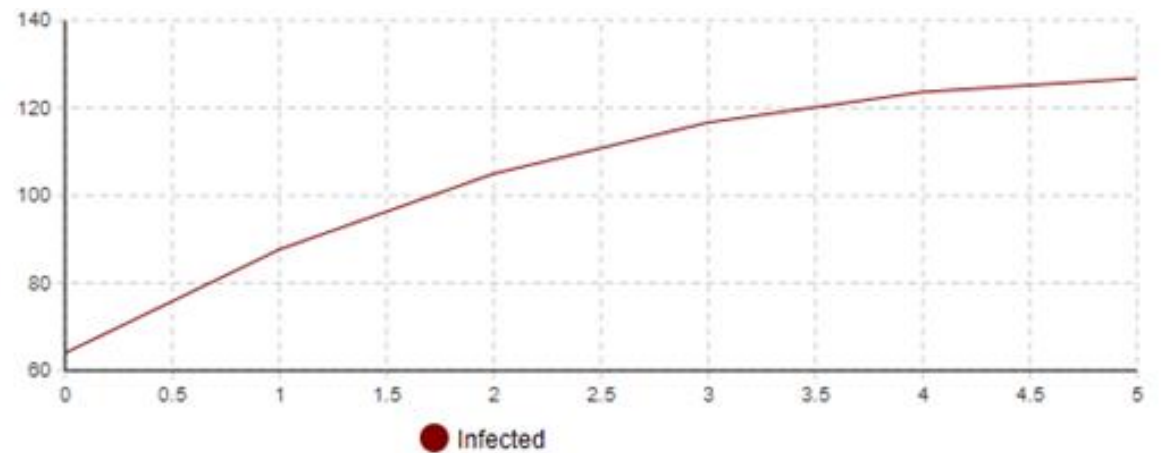
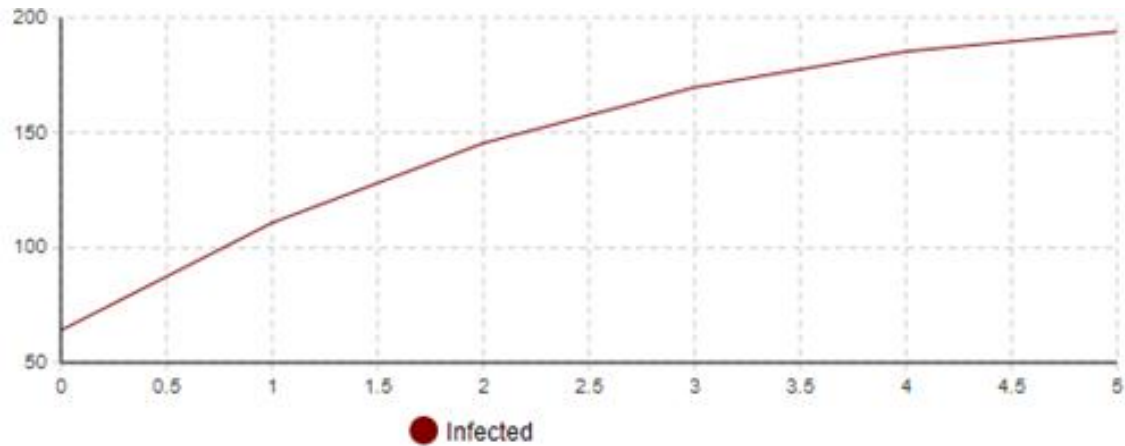
Case Study

Parameter estimation using data obtained by Institute of Public Health of N. Macedonia

Parameter	Description	Value
λ	Birth rate	0.014
β	Transmission rate	0.9
α	Incubation rate	0.125
γ	Recovery rate	0.142
δ	Mortality rate of measles	0.02
M	Natural mortality rate	0.0014
σ	Unsuccessful vaccination rate	0.007
ν	Successful vaccination rate	0.993
q	Vaccination rate	0.744

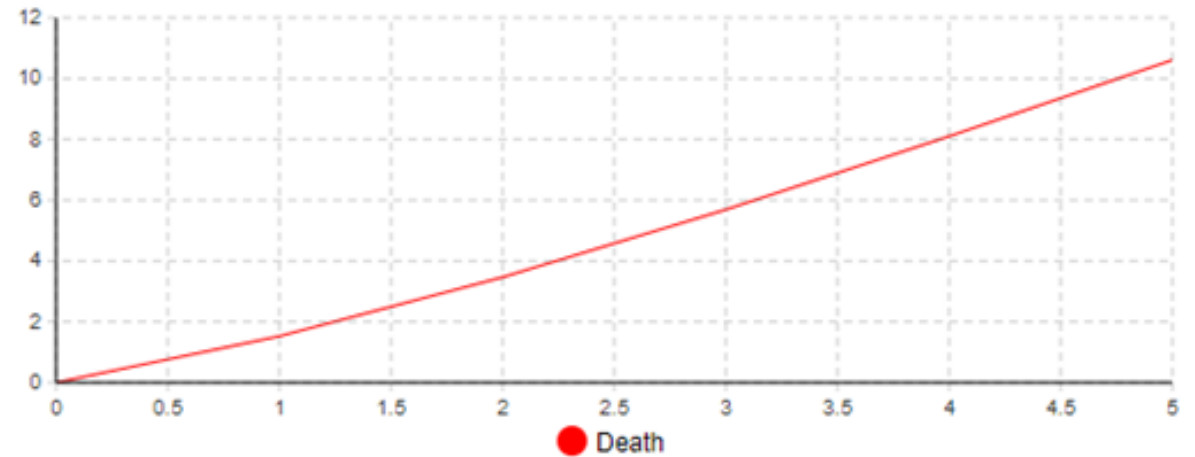
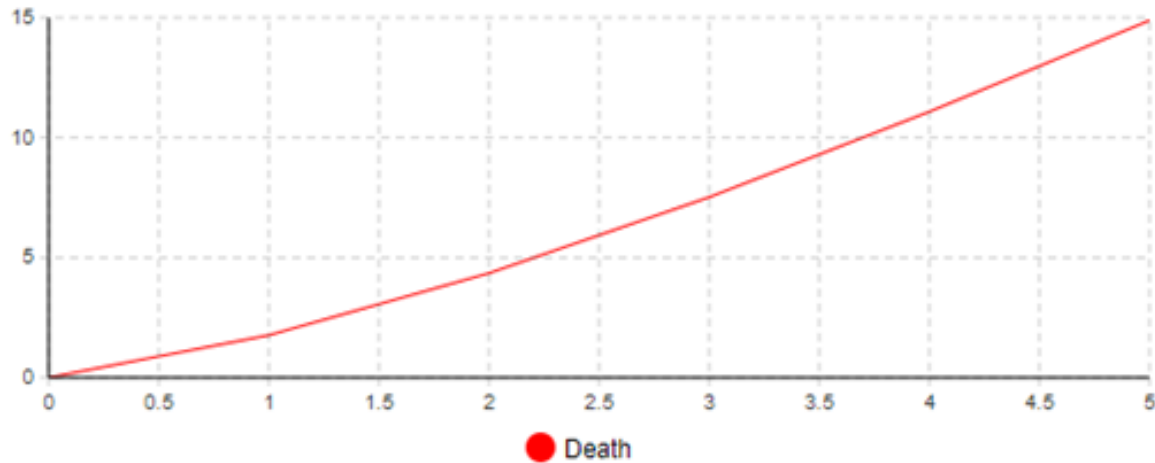
Case Study

Infected population for vaccination rate a) 0.744 and b) 0.95.



Case Study

Death population for vaccination rate a) 0.744 and b) 0.95.



Conclusion

The transmission dynamic of measles infection depends on various factors including mainly population density, vaccination rates and contact of individuals within the population. Mathematical models for infection disease had been proven to be a valuable tool for understanding the measles spread, predict future outcomes and outbreaks and evaluate the impact of vaccination.

The results from this simulation emphasize the significance of vaccination in controlling the measles disease and preventing the spread in the population. The vaccination has negative impact on prevalence thus the vaccination rate should be increased to prevent an outbreak of measles infection.



Thank you for your attention

