Brussels, 4 June 2019

COST 049/19

DECISION

Subject: **Memorandum of Understanding for the implementation of the COST Action "Mathematical models for interacting dynamics on networks" (MAT-DYN-NET) CA18232**

The COST Member Countries and/or the COST Cooperating State will find attached the Memorandum of Understanding for the COST Action Mathematical models for interacting dynamics on networks approved by the Committee of Senior Officials through written procedure on 4 June 2019.

Funded by the Horizon 2020 Framework Programme
of the European Union

MEMORANDUM OF UNDERSTANDING

For the implementation of a COST Action designated as

COST Action CA18232
MATHEMATICAL MODELS FOR INTERACTING DYNAMICS ON NETWORKS (MAT-DYN-NET)

The COST Member Countries and/or the COST Cooperating State, accepting the present Memorandum of Understanding (MoU) wish to undertake joint activities of mutual interest and declare their common intention to participate in the COST Action (the Action), referred to above and described in the Technical Annex of this MoU.

The Action will be carried out in accordance with the set of COST Implementation Rules approved by the Committee of Senior Officials (CSO), or any new document amending or replacing them:

a. “Rules for Participation in and Implementation of COST Activities” (COST 132/14 REV2);
b. “COST Action Proposal Submission, Evaluation, Selection and Approval” (COST 133/14 REV);
c. “COST Action Management, Monitoring and Final Assessment” (COST 134/14 REV2);
d. “COST International Cooperation and Specific Organisations Participation” (COST 135/14 REV).

The main aim and objective of the Action is to bring together leading groups in Europe working on analytical and numerical approaches to a range of issues connected with modelling and analysing mathematical models for dynamical systems on networks (DSN), in order to be able to address its research challenges at a European level. This will be achieved through the specific objectives detailed in the Technical Annex.

The economic dimension of the activities carried out under the Action has been estimated, on the basis of information available during the planning of the Action, at EUR 60 million in 2018.

The MoU will enter into force once at least seven (7) COST Member Countries and/or COST Cooperating State have accepted it, and the corresponding Management Committee Members have been appointed, as described in the CSO Decision COST 134/14 REV2.

The COST Action will start from the date of the first Management Committee meeting and shall be implemented for a period of four (4) years, unless an extension is approved by the CSO following the procedure described in the CSO Decision COST 134/14 REV2.

TECHNICAL ANNEX

OVERVIEW

Summary

Many physical, biological, chemical, financial or even social phenomena can be described by dynamical systems. It is quite common that the dynamics arises as a compound effect of the interaction between sub-systems in which case we speak about coupled systems. This Action shall study such interactions in particular cases from three points of view:

1. the abstract approach to the theory behind these systems,
2. applications of the abstract theory to coupled structures like networks, neighbouring domains divided by permeable membranes, possibly non-homogeneous simplicial complexes, etc.,
3. modelling real-life situations within this framework.

The purpose of this Action is to bring together leading groups in Europe working on a range of issues connected with modelling and analysing mathematical models for dynamical systems on networks. It aims to develop a semigroup approach to various (non-)linear dynamical systems on networks as well as numerical methods based on modern variational methods and applying them to road traffic, biological systems, and further real-life models. The Action also explores the possibility of estimating solutions and long time behaviour of these systems by collecting basic combinatorial information about underlying networks.

Areas of Expertise Relevant for the Action	Keywords
● Mathematics: Theoretical aspects of partial differential equations ● Mathematics: Numerical analysis ● Mathematics: Operator algebras and functional analysis	● dynamical systems on networks ● linear and nonlinear operator semigroups ● coupled systems of evolution equations ● sprectrum of quantum graphs ● numerical analysis of coupled PDEs

Specific Objectives

To achieve the main objective described in this MoU, the following specific objectives shall be accomplished:

Research Coordination

● Coordinate and direct research efforts by groups from different subdisciplines of mathematics.
● Study evolution equations on higher-dimensional multi-structures or ramified spaces, and develop the abstract theory needed for this study.
● Refine the previously known abstract techniques and develop new ones so that more general DSNs can be modelled and analysed using mathematical tools.
● Generalise available methods to handle nonlinear problems on networks, in particular discussing parametrisations of nonlinear operators with pure boundary interactions.
● Merge the research activities of groups that are currently separately working on spectral theory and existence and uniqueness issues for evolution equations, respectively.
● Enrich the theory of evolution equations on networks by new ideas and variational methods that have been developed in the context of optimal transport over the last two decades.
● Develop new numerical methods for discretisation and model reduction of DSNs which preserve important properties of the system and which can be used to analyse and simulate the system.
● Apply the developed theory to real-world engineering problems (e.g., vehicle traffic, water systems, data mining).

Capacity Building

● Establish an efficient and lasting network of researchers studying DSNs across Europe.

● Link several separate mathematical subfields (from functional analysis to graph theory and numerical analysis) in order to achieve breakthroughs.
● Foster the exchange between ITC and non-ITC researchers, with special focus on Early Career Investigators.
● Improve gender balance in mathematical research.
● Educate engineers and the professional public on relevant mathematical tools for tackling the investigation of DSNs.

TECHNICAL ANNEX

1 S&T EXCELLENCE

1.1 SOUNDNESS OF THE CHALLENGE

1.1.1 DESCRIPTION OF THE STATE-OF-THE-ART

As societies and technologies become more complex, there is an increasing need to develop highly sophisticated mathematical tools which enable the modelling, analysis, and control of these complicated systems. Successful mathematical modelling of physical phenomena is one of the greatest scientific achievements of the modern era, and there are well-known systems of equations which describe, for example, the flow of liquids, the probability distribution of quantum wave functions, vibrations of membranes and beams, and so on. More recently, scientists have started developing increasingly complex mathematical models for the challenging problems of our time, such as climate change, epidemics, and many economic phenomena. With the development of scientific computing, new frontiers have opened, giving us the possibility to calculate solutions for many previously inaccessible complicated systems and also to simulate their real-time behaviour.

Many physical, biological, chemical, financial and even social phenomena can be described by dynamical systems. It is quite common that the dynamics arise as a compound effect of the interaction between sub-systems, in which case we speak of coupled systems. The 21st century has seen the rise of ever more complex coupled systems consisting of thousands or even hundreds of millions of simpler sub-systems mutually linked in a network-like fashion. Nowadays one encounters such systems everywhere: from classical electricity grids, water supply, pipelines, road or railway networks to bloodstream or ensembles of bursting neurons, or even non-physical networks such as various social networks. One of the largest networks is, of course, the Internet. These systems are usually called dynamical systems on networks (DSNs). Their complexity is due to the heterogeneous dynamics of the systems comprising it, the diversity of interaction and communication media, and their huge scale in terms of the number of interacting systems and system interconnections.

The dynamics of many DSNs arise from the interaction of partial differential equations (PDEs) acting on its sub-systems. Partial differential equations of diffusive type on graph-like structures have been introduced by Lumer in the 1980s; many more dynamical processes living on the edges of a network have been studied ever since. These investigations have become very popular in operator theory and mathematical physics alike: differential operators on networks are nowadays most commonly referred to as “quantum graphs". Quantum graphs are known to be efficient low-dimension approximations of Hamiltonians: their properties have thus been studied intensively for over twenty years, at the crossroads between extension theory, spectral geometry, Dirichlet forms. Quantum graphs have already delivered most efficient toy models in control theory, quantum chaos, spectral theory, and inverse problems. At the same time, it is still an open question how beneficially they can be used in cryptography (using the damping term of a heat flow as a crypto key), analysis of nonlinear PDEs (as a 1-dimensional limit of nonlinear operators in tubular regions), or number theory (as an environment for the Berry-Keating-Hilbert-Pólya conjecture in connection with the Riemann Hypothesis), to name a few lines of active research. Another important class of DSNs arises if the state of each node in a network is governed by its own PDE, hence studying a dynamical process on a network entails examining a (typically large) system of coupled PDEs. The change in the state of a node depends not only on its own current state but also on those of its neighbours, and the network encodes the strength of mutual interaction of the nodes. Currently, available methods can only treat either DSNs with ordinary differential equations (ODEs) as the dynamical systems on the nodes or else DSNs with a quite simple topology and with a small number of nodes. The interaction of topology and dynamics of such systems is still poorly understood, especially in the case of highly decentralized systems. Moreover, not only can the network structure affect dynamical processes on that network, but also dynamical processes can affect the evolution of the network structure. In that case, we talk about evolving dynamical networks. This is an emerging topic of research with a large number of possible applications (modelling of epidemics in networks of small communities is a prominent example, since pioneering studies of Newman, Keeling, and Eames) but without a comprehensive mathematical theory behind it.

It is often the case that a DSN can be represented within the framework of an abstract evolution equation on a suitable state space complemented by an initial or final condition. As such, the model can be recast as a Cauchy problem on an abstract function space with some additional topological structure. Hence one can write the solution of the evolution equation in terms of the corresponding semigroup. The theory of semigroups of bounded linear operators, *C_0-semigroups* for short, is one of the most profitable approaches within the framework of abstract evolution equations. The abstract theory was initiated in the 1940s and 1950s by Hille, Yosida, Feller, Kato, and Phillips and it is now a major tool in most applications where time-dependent processes appear. Its extension to the nonlinear setting has delivered since the 1970s new tools for the study of important models in fluid-dynamics, gas kinetics, and population models, to name but a few applications. Not only in the analysis of PDEs, but also in control theory state-of-the-art results heavily rely on abstract techniques based on semigroups.

The theory of C_0-semigroups is not only a strong tool when proving analytical results about linear or nonlinear problems on networks and graphs, but also in numerical applications. Since the differential equations arising in applications of interest to us (such as fluid-dynamic models for traffic flow) have a rather complicated structure, their analytical solutions are usually not known. Thus, numerical methods are indispensable. In our context, it means that the C_0-semigroup needs to be approximated by such an object (not necessarily a C_0-semigroup in its own right) whose action can be implemented in computer code. This is usually done in two steps: first, the spatial differential operator is approximated by another generator which is easier to treat numerically, then the corresponding semigroup is approximated in time. The numerical methods should preserve the general structure as well as important properties of the system such as stability. The obtained discretized system is too large to be useful for analysing its corresponding dynamics, thus a subsequent model reduction is needed.

In the context of DSNs, the most promising framework seems to be based on port-Hamiltonian systems, since all important properties of a DSN are then preserved under an appropriate semi-discretization in space by mixed finite elements, thus resulting in a finite-dimensional differential-algebraic system. A further dimension reduction of these high dimensional models by a structure-preserving Galerkin projection can subsequently be implemented: this leads to reduced, lower-dimensional models that among further relevant properties preserve the port-Hamiltonian structure. They can thus be easily used for analysis, online simulation and control. This is an emerging field of research, with many possible applications but also with many open questions, both of theoretical and practical nature.

Variational methods are in many cases tightly related to PDEs: quadratic forms offer a powerful tool for investigating existence and uniqueness questions via semigroup theory; and their well-known refinement, Dirichlet forms, allow for additional bridges to the realm of stochastic analysis: accordingly, Markov processes are simply those gradient systems associated with Dirichlet forms. This correspondence has been thoroughly studied by Fukushima, among others; nonlinear versions of this theory are also well known within the community and rely upon Galerkin-type techniques developed by J.L. Lions. Not only do variational methods yield existence and uniqueness results, but they also provide essentially complete information about the long-time behaviour of the solution of the parabolic problem: they have been effectively used in image processing and denoising over the last 25 years. Since pioneering works by, among others, Buser, Cheeger, Gromov, Taylor, and Yau, such methods have been systematically exploited in spectral theory and differential geometry to study interplays between geometric properties of manifolds and features of parabolic equations thereon. In the context of networks, the typical question is: when dealing with a DSN, what can be said about convergence to the stationary solution or even the rate of such convergence if the fine structure of a huge network is not known, but rough information such as number of nodes and edges, connectivity, diameter/degrees of separation etc., is available? To this end, the above-mentioned spectral geometric theory on manifolds should be extended to graphs and networks. The underlying mathematical technology is based on novel but elementary ideas that connect discrete and continuous mathematics and have led to the development of isoperimetric inequalities and surgery methods: these have already proved to be powerful in the topological optimization of graphs with respect to the spectrum of Laplacian-type operators, and has paved the way for the development of several spectral clustering and segmentation algorithms in machine learning. In recent years, a new and unexpected twist has been given to variational methods by the introduction of gradient systems with respect to new families of metrics, such as Wasserstein distances in spaces of measures; this was a crucial idea in the solution of Monge's optimal transport problem widely popularized by, among others, Otto and Villani and has been successfully expanded to cover a plethora of nonlinear models, including traffic flows. Remarkably, evolutionary equations displaying this generalized variational structure may not be governed by a C_0-semigroup, but they are in fact associated with good discretization schemes and can be effectively studied by methods of convex optimization, like in the work of Brenier, Dal Maso and many more.

This shows that the study of DSNs involves a number of mathematical areas of research. To represent a DSN as a mathematical object and study its properties, one has to:

- analyse the corresponding system of coupled ODEs or PDEs using functional-theoretic and variational methods,
- analyse the topological structure of its underlying network,
- study the corresponding abstract evolution equation using the theory of semigroups,
- use existing and develop new numerical methods for PDEs and semigroups.

1.1.2 DESCRIPTION OF THE CHALLENGE (MAIN AIM)

The application of dynamical systems theory to areas outside mathematics is a vibrant and active field of research. The areas of application are diverse and multidisciplinary, ranging over all areas of applied science and engineering, including biology, chemistry, physics, finance, and industrial applied mathematics. The reliance on complex dynamical systems on networks is ubiquitous. For example, the synchronous grid of Continental Europe is the largest synchronous electrical grid (by connected power) in the world. At the beginning of 2018, the Kosovo electricity net production balance decreased during a period of a few weeks. This led to a small deviation of the European network's frequency (from 50Hz to 49.996Hz). In turn, this frequency deviation led to some electric clocks (like ones in ovens) being out of sync by up to 6 minutes. In this case, the problem could be promptly understood and solved using mathematical techniques for DSNs. This is just one of the myriads of examples of complex DSNs, the modelling and analysis of which is a major challenge of modern applied mathematics.

The topic of dynamical systems and differential equations on metric graphs has experienced a remarkable renaissance in the last decade. Broad projects devoted to both fundamental research and applications in biology and engineering have been funded by the ERC, NSF, and various national agencies and research institutes ever since. Although scientists and mathematicians have been studying such systems for some time, there is still much that is not well understood, especially in terms of a general mathematical framework for such systems.

The aim of this Action is to bring together leading groups in Europe working on a range of issues connected with modelling and analysing mathematical models for dynamical systems on networks, to be able to address its research challenges at a European level.

This COST Action will study DSNs from three points of view:

1. the abstract approach to the theory behind these systems,
2. applications of the abstract theory to coupled structures like networks, neighbouring domains divided by permeable membranes, possibly non-homogeneous simplicial complexes, etc.
3. modelling real-life situations within this framework.

If the phenomenon to be studied arises as a compound effect of (say two) sub-processes, the underlying equation can be often formally rewritten as a perturbed Cauchy problem, where one of the sub-processes is treated as a perturbation of the other. Hence, the concept of perturbed systems plays a large role in the modelling and analysis of DSNs. Consider a very large network, whose precise size is not actually known, but some of whose structural properties are well understood. One way to model this situation is to consider an infinite graph (about which combinatorially reasonable assumptions based on a priori knowledge about the structure of the network are made). On the edges of the network, some processes take place, and these are coupled via the vertices in which the edges meet. So, one is in the situation of coupled systems, and one can apply the semigroup methods if one chooses the appropriate state space. However, there are features of the whole problem which are not described by the classical theory (for example, continuous vs chaotic dependence on the initial data with respect to sup-norm over an infinite network). Therefore, it is important to introduce new concepts and techniques that allow for a broader class of state spaces (such as locally convex, Hadamard or merely metric spaces). Moreover, in a variety of real-world applications, nonlinear effects cannot be neglected. An important example is the macroscopic modelling of vehicular traffic on road networks, a research direction which is currently booming (think of self-driving vehicles, for example). In this area, “well-posedness” and stability of models are proved by investigating nonlinear systems of conservation laws. At present, these systems cannot be handled well within the abstract framework and the appropriate development of the nonlinear semigroup theory is currently an important ongoing direction of research. Note, that a system is called well-posed if there exists a unique solution which depends continuously on the initial data.

If one is interested not only in the topology of the network but mainly in the dynamical process taking place along the edges of the network, one needs to consider each edge as an interval and describe functions on it, thus generalizing the classical graph theoretical construction of line graphs to what are known as metric graphs. The most interesting and demanding part is that one has to interlink the functions on the edges that have a common node in the network via transmission conditions depending on the process under consideration. This provides a coupling between the various processes taking place on the edges. It turns out that perturbation techniques concerning coupled systems provide a good tool to study such equations on graphs. The two main challenges are, first, to characterize the well-posedness via appropriate transmission conditions and, second, to relate certain qualitative properties of the solutions to the chosen conditions.

Clearly, applications of theoretical results cannot proceed without a parallel development of appropriate numerical methods. It is essential that the discretization and model reduction of DSNs preserve important properties of solutions - such as their long-time behaviour and smoothness - and the rough topological structure of networks - like planarity. Numerical methods for PDEs usually assume the boundary conditions to be given, while more often than not the interaction of the distributed-parameter components with the other components in a network take place precisely via the boundary. On the other hand, finite-dimensional approximation methods for infinite-dimensional input-output systems (e.g., in a semigroup format) are not easily relatable to numerical techniques for solving PDEs and are mainly confined to linear PDEs. The challenge is to devise discretization/approximation methods which solve these issues. Additionally, new numerical frameworks should be robust with respect to the topological structure of the network: small changes in the topology of the network should be accommodated by a small update of the corresponding reduced order model.

It is clear that the coordinated investigation and development of DSNs is not only of considerable theoretical interest: it is extremely timely as the demand coming from numerous cutting-edge applications in machine learning, engineering, quantum technologies is high. The study of dynamical systems on networks thus has a high potential for innovations and breakthrough scientific results. Since it is also cross-disciplinary, requiring expertise in various fields of research, COST Actions are ideal for facilitating such an activity. This COST Action, with its networking tools, STMSs and Training Schools is an appropriate framework to foster progress and enable this subject to truly develop and achieve maximum effectiveness.

1.2 PROGRESS BEYOND THE STATE-OF-THE-ART

1.2.1 APPROACH TO THE CHALLENGE AND PROGRESS BEYOND THE STATE-OF-THE-ART

To tackle the challenges of the Action, firstly, the concept of C_0-semigroups will be widened in several ways. The analytical features of a semigroup will be generalized by relaxing its topological properties and allowing for a weaker dependence of the orbits on the initial data. This enables, for example, the study of semigroups on vector lattices. To make the theory applicable, a richer perturbation theory in these context needs to be developed. Furthermore, a novel approach to nonlinear semigroups on more general spaces will be developed.

Even though some progress in generalizing the C_0-semigroups as suggested above has been made, currently there is no unified, let alone mature, theory available. The further mentioned research directions open new research areas and are highly relevant for the purposes of this Action. By using coupled systems and the perturbation results mentioned above one can also study dynamical processes acting on hypergraphs or higher dimensional ramified spaces instead of one-dimensional networks. Since many application-driven problems are of nonlinear nature (i.e., traffic or gas flow) the development of the nonlinear part of the theory is crucial. The development of the theory of semigroups whose orbits depend Lipschitz-continuously on the initial data in the L^1-norm has been particularly dramatic. Often referred to as "Standard Riemann Semigroups", they were introduced by Bressan in the 1990s. They have proved particularly effective in the analysis of systems of 1D conservation laws. The general, unifying, case of systems in several space dimensions still seems formidably difficult.

Modern variational methods for nonlinear evolution equations have not yet achieved their potential in the context of ramified structures. This is due to intrinsic smoothness requirements within the theory that make the whole machinery of gradient systems in Wasserstein-type spaces of measures work, but fail to hold in discrete contexts; the Action will especially focus on the issue of implementing (possibly nonlinear) transition/interface conditions. Again, as in the linear context, these ideas lead to natural generalizations of the notion of solutions and to relaxations of their dependence on the initial data.

Variational methods will also be used to discuss stationary equations, especially concerning combinatorial estimates on the bottom of the spectrum and further spectral geometric features. This will ideally facilitate the investigation of long-time behaviour and also the search for optimal topologies with respect to the speed of convergence of solutions to parabolic PDEs towards steady state solutions. Related results will also have an impact in applications to the optimal partitioning of networks and manifold learning, two timely topics at the boundary between numerical analysis, data analysis, and calculus of variations.

Further challenges are posed by a variety of present-day applications. A key situation requiring the combined efforts of members of the Action is, for instance, the description of the spreading of pollutants caused by vehicles moving on a typical city road network. In heavily urbanized areas, pollutants concentrate above roads, forming a network of canyon- or tube-like structures with the roads at their bottom. The current engineering literature already offers a variety of *ad hoc* models, but a thorough analytic study still seems to be missing. While each of the basic phenomena, essentially the flow of traffic and the wind drawn diffusion of pollutant, are sufficiently well understood individually, the overall description still requires a well-established rigorous analytic framework. Coupled diffusive-hyperbolic systems on 1-dimensional structures are much more treatable both at an analytical and numerical level: It will be a major challenge to develop correct geometric approaches will enable efficient dimensional reduction techniques in this and further, similarly complicated models.

These application-driven results will have to encompass the use and, in most cases, the concomitant development of dedicated numerical methods. Here, the starting point will consist in a careful blending of the existing algorithms for the different types of equations that act as building blocks of the whole model. If a DSN has nice properties, the network will be modelled as a port-Hamiltonian descriptor system on a graph. In other cases, new theoretical frameworks will be needed. After a suitable discretization of the DSN, first by mixed finite element or finite difference methods, then by a time discretization, Krylov subspace methods will be employed for the construction of the reduced models which satisfy certain algebraic compatibility conditions which are required to ensure the well-posedness of the reduced models, the preservation of the key properties and the robustness with respect to the topology of DSN. Since it is not realistic to build a numerical framework applicable to a general DSN, methods for different classes of problems will have to be constructed. The topological robustness of the numerical approximation especially presents a novel challenge for which there are no satisfying state-of-the-art methods.

An interplay between model developers - including industrial stakeholders -, PDE experts and numerical analysts, as being proposed here, offers the best platform for working on the challenges and tasks raised by this Action.

1.2.2 OBJECTIVES

1.2.2.1 Research Coordination Objectives

- Coordinate and direct research efforts by groups from different subdisciplines of mathematics.
- Study evolution equations on higher-dimensional multi-structures or ramified spaces, and develop the abstract theory needed for this study.
- Refine the previously known abstract techniques and develop new ones so that more general DSNs can be modelled and analysed using mathematical tools.
- Generalise available methods to handle nonlinear problems on networks, in particular discussing parametrisations of nonlinear operators with pure boundary interactions.
- Merge the research activities of groups that are currently separately working on spectral theory and existence and uniqueness issues for evolution equations, respectively.
- Enrich the theory of evolution equations on networks by new ideas and variational methods that have been developed in the context of optimal transport over the last two decades.
- Develop new numerical methods for discretisation and model reduction of DSNs which preserve important properties of the system and which can be used to analyse and simulate the system.
- Apply the developed theory to real-world engineering problems (e.g., vehicle traffic, water systems, data mining).

1.2.2.2 Capacity-building Objectives

- Establish an efficient and lasting network of researchers studying DSNs across Europe.
- Link several separate mathematical subfields (from functional analysis to graph theory and numerical analysis) in order to achieve breakthroughs.
- Foster the exchange between ITC and non-ITC researchers, with special focus on Early Career Investigators.
- Improve gender balance in mathematical research.
- Educate engineers and the professional public on relevant mathematical tools for tackling the investigation of DSNs.

2 NETWORKING EXCELLENCE

2.1 ADDED VALUE OF NETWORKING IN S&T EXCELLENCE

2.1.1 ADDED VALUE IN RELATION TO EXISTING EFFORTS AT EUROPEAN AND/OR INTERNATIONAL LEVEL

Understanding networks, such as the Internet or the interconnections in the human brain, is one of the most important, current challenges of humankind. "The Human Brain Project", one of the EU H2020 Flagship projects, has been set up to achieve one of these goals by methods of neuroscience, scientific computation, and modelling. There are also some existing COST Actions that study aspects of networks from different viewpoints:

- IC1104 - Random Network Coding and Designs over GF(q)
- TU1305 - Social networks and travel behaviour
- CA15104 - Inclusive Radio Communication Networks for 5G and beyond
- CA16207 - European Network for Problematic Usage of the Internet
- CA16228 - European network for game theory
- CA16222 - Wider Impacts and Scenario Evaluation of Autonomous and Connected Transport

However, none of these aims at the *mathematical study of the dynamics* that take place on the network. In general, *mathematics* seems to be under-represented in the existing COST actions.

- TD1409: Mathematics for industry network

is the most related action in terms of technological impacts of mathematical sciences to industrial stakeholders. However, its scope again relies mostly on discrete mathematics while this Action fits into continuous mathematics.

- CA15225 - Fractional-order systems: analysis, synthesis and their importance for future design is the only COST Action that addresses mathematical problems concerning systems, but not on networks or multi-structures. It thus seems that the topics presented here have not yet addressed by any previous COST Actions.

Networks of actors, be they endowed with dynamic processes or not, have been an important topic in mathematics, computer science, traffic engineering, and theoretical physics in recent decades; in particular, topology optimization with respect to properties of dynamical systems has been studied by means of graph theory, operations research, control theory, variational methods, and game theory. Several ERC consolidator and advanced grants, as well as NSF grants and DFG Collaborative Research Centres (SFB) on related topics, have already been approved; these include, among others,

- ERC-2008-AdG - From discrete to continuous: understanding discrete structures through continuous approximation
- ERC-2008-AdG - Expander Graphs in Pure and Applied Mathematics
- ERC-2013-CoG - Limits of discrete structures
- ERC-2015-CoG - Networks in Time and Space
- ERC-2015-AdG - High Dimensional Expanders, Ramanujan Complexes and Codes
- NSF 1536397 - Graph-Based Control Design for Network Dynamics with Time Delays
- NSF 1652492 - Principled Structure Discovery for Network Analysis
- DFG-SFB Transregio 154: Mathematical Modelling, Simulation and Optimization using the Example of Gas Networks

This Action has its roots in this existing stream of research. It is however unique among major European actions in that its focus lies upon networks which most interesting features are *interactions*, rather than actors themselves: in other words, the focus is moved from the analysis of the structure of a network to the dynamic processes taking place on its edges. Here methods from analysis, operator theory, and dynamical systems are relevant. Through the experts involved, the Action combines knowledge from all these mathematical areas in an optimal way to carry out fundamental research concerning DSNs, but also to provide potential applicability for real-life problems.

2.2 ADDED VALUE OF NETWORKING IN IMPACT

2.2.1 SECURING THE CRITICAL MASS AND EXPERTISE

The network of proposers includes leading European experts in the fields of operator semigroups, evolution equations, conservation laws, networks, and numerical analysis. The participation is balanced among the main objectives of the Action. A critical mass of researchers is present which assures the possibility of a proper interdisciplinary approach that will bring synergetic effects.

Apart from mathematicians, the network has members from physical sciences and different branches of engineering as well. Representatives of industry are present among the proposers. This will ensure the dissemination of mathematical technology from the Working Groups to businesses and applied research labs. Industrial stakeholders will profit from the theoretical results, which will be conversely motivated and directed by real-world investigations. The proposed network is thus capable of addressing all the presented challenges and objectives.

The network is geographically extended across the whole of Europe, ensuring the diversity of approaches and the spread of knowledge. It initially included 31 proposers from 14 COST member countries, 8 of which are ITCs, and one Near Neighbour country. Gender (around 40% female) and age balance (around 40% ECIs) of the group are an important feature of the proposing group. Each partner will participate in at least one WG activity, according to their expertise and scientific background. The appropriate combination of senior researchers who are already well-established and younger early career researchers will result in a knowledge rich and vivid atmosphere.

Furthermore, a key priority of the Action will be gender and outreach activities, in order to build a truly diverse and inclusive community and allow for the largest possible pool of outstanding researchers to participate in the network.

2.2.2 INVOLVEMENT OF STAKEHOLDERS

Among the Action proposers, there are also 5 industrial stakeholders from different branches related to the challenges of the Action. These include engineering companies working in traffic modelling and in gas and water distribution as well as IT companies specialized in machine learning, optimization of networks, numerical simulation, and data mining. During the Action, special events, to be called *Problem-Solving Sessions*, will be organized in order to attract even more industrial collaborators.

Special attention will be given to the younger generation. Concretely, this will be realized by organizing Training Schools and Short-Term Scientific Missions with leading experts. Moreover, the Action facilitates the mobility needed for joint supervisions of Ph.D. students and possible secondments at industrial partners.

2.2.3 MUTUAL BENEFITS OF THE INVOLVEMENT OF SECONDARY PROPOSERS FROM NEAR NEIGHBOUR OR INTERNATIONAL PARTNER COUNTRIES OR INTERNATIONAL ORGANISATIONS

Among the secondary proposers oh the Action there is a partner from a Near Neighbour Country (Morocco) in order to strengthen and widen the existing scientific contacts. This cooperation complies with the EC Policies; indeed, the latest AU-EU Summit of November 2017 stressed the need for deepening R&I cooperation with a view to delivering on the first joint strategic priority for the following years: “Investing in people - education, science, technology and skills development”.

Partners from NNCs usually have limited possibilities for research travels (due to both legal and financial issues), thus the Action is a great chance for them to stay involved in the contemporary research in their area. The Action will benefit from the highly motivated participants from the NNC. Female and younger researchers will be especially encouraged to join, which is already the case for the chosen secondary proposer.

3 IMPACT

3.1 IMPACT TO SCIENCE, SOCIETY AND COMPETITIVENESS, AND POTENTIAL FOR INNOVATION/BREAK-THROUGHS

3.1.1 SCIENTIFIC, TECHNOLOGICAL, AND/OR SOCIOECONOMIC IMPACTS (INCLUDING POTENTIAL INNOVATIONS AND/OR BREAKTHROUGHS)

Much of today’s science and engineering builds on advances in the mathematical sciences. The European Science Foundation in the document "Mathematics and Industry" proclaimed: "It is evident that, in view of the ever-increasing complexity of real-life applications, the ability to effectively use mathematical modelling, simulation, control and optimization will be the foundation for the technological and economic development of Europe and the world." This Action will directly contribute to the advancement of three of the four highlighted mathematical topics (namely mathematical modelling, simulation and control) and indirectly to all of them. The Action's contribution will have an immediate and significant impact on these fields of research. For the first time, the European scientific community will have the capability to join forces in the field of mathematical modelling of interacting dynamics on networks.

Modern research within the mathematical sciences is characterized by an ever-increasing degree of collaboration, and collaborative efforts often feature two or more fields within the discipline. The clichéd image of the solitary researcher from mathematics folklore has never been further from reality than it is now. This reality also suggests what has long been the experience of leading mathematicians: that the various subfields in mathematics depend on one another in ways that can be highly unpredictable; and so more individuals need to collaborate in order to bring all the necessary expertise to bear on contemporary problems. This Action will greatly expedite this process in the fields of dynamical systems, semigroup theory, PDEs, and functional and numerical analysis, to name just a few of the areas of research featured within the scope of the proposal. The increased collaboration will lead to greater cross-fertilization of these fields.

The expected scientific and innovation breakthroughs of the Action will be achieved by:

- Widening understanding and developing new a mathematical toolkit for modelling and analysis of coupled PDEs, including those with a very high number of interactions.
- Developing a semigroup approach to nonlinear DSNs as well as numerical methods based on modern variational methods and applying them to road traffic and biological systems, and further real-life models which to date have not yet properly addressed.
- Exploring the possibility of estimating solutions and long-time behaviour of DSNs by collecting basic combinatorial information about underlying networks.
- Extending the theory of C_0-semigroups to a broader class of spaces which enables considering more general DSNs, as required in applications.

Regarding socio-economic development, the Action will:

- Help female researchers fulfil their potential in research and academia through the creation of the WISE network and other Action activities designed to promote gender inclusion.
- Help ECIs to obtain leading roles in research and to develop their capacity to work in a competitive international environment.
- Create additional opportunities for Action members, and other researchers working in the areas covered by the Action, as well as their students, and Postdocs, to further boost their career.
- Support and promote the development of mathematical expertise and profile in multiple ITCs as well as Morocco.
- Increase the visibility of the institutions involved by generating synergies and multidisciplinary collaborations.

The technological impact of the Action in the short-term will consist of:

- finding solutions and bringing new insights to existing problems proposed by partners from industry,
- proposing new lines of research based on the business challenges of the partners from industry,
- introducing Ph.D. students and ECIs to mathematical challenges coming from real-world problems and thus shortening the innovation cycle.

The long-term impact on technology will consist of:

- stimulating cooperation and establishing a lasting and productive environment at a European level between players involved in mathematical research of network dynamics and industrial partners,
- increasing the profile of mathematics in industry,
- stimulating greater awareness in industry of the power of mathematics to solve real-world problems.

3.2 MEASURES TO MAXIMISE IMPACT

3.2.1 KNOWLEDGE CREATION, TRANSFER OF KNOWLEDGE AND CAREER DEVELOPMENT

There has been a striking expansion in the impact of the mathematical sciences on other fields, as well as an expansion in the number of mathematical sciences subfields that are being applied to challenges outside of the discipline. This expansion has been ongoing for decades, but it has accelerated greatly over the past 10-20 years. The role of mathematics as a natural driver of industrial innovation has increased accordingly. This is especially true in the context of communications technology, but also in other areas where heterogeneous networks appear: the technology of expander graphs is a paradigmatic success story at the boundary of discrete mathematics, theoretical computer science, and electrical engineering. This Action aims to achieve similar feats, in particular, a breakthrough in space-continuous network models.

The Action will create a suitable framework to start a long-term collaboration of researchers of various fields of research connected with the modelling and analysis of interacting dynamical systems on networks. The industrial sector will benefit either directly through the delivery of numerical methods for particular applications or indirectly through a deeper understanding of the dynamical properties of networks.

Workshops, Training Schools, and Short-Term Research Missions will create a lively and strong interaction and training network within Europe. The attendance at these events may represent cornerstones in building careers. Fruitful contacts with industrial stakeholders are very important for young people entering the labour market as well as for firms seeking to recruit a skilled workforce.

3.2.2 PLAN FOR DISSEMINATION AND/OR EXPLOITATION AND DIALOGUE WITH THE GENERAL PUBLIC OR POLICY

In order to reach out to the scientific community outside the Action, and to the general public and policy stakeholders beyond them, the following steps are planned.

1. The Action webpage will be set up and updated in a timely fashion to reflect and report on all its activities.
2. A Public Relations Officer (PRO) will be chosen from among the ECIs. The tasks of the PRO include updating the Action webpage and posting short updates of the Action activities to various social media accounts (such as Instagram, Twitter, Facebook) in order to reach interested students and younger researchers. She or he will also be responsible for organizing different outreach activities of the Action.
3. Each year at least one thematic workshop will take place in one of the participating countries. These workshops will be open to non-Action members as well.
4. The dissemination of Action challenges will be expedited by the publication of basic review articles in workshop proceedings and on the Action webpage, during the initial phase of the Action.
5. During the main conferences, public lectures aimed at the broad public will be organized with the aim of conveying fundamental results in modern mathematical research.
6. Four Training Schools for students and a special ECI meeting will be organized to attract new young researchers and to build up new scientific relations.
7. Problem-Solving Sessions will be organized in collaboration with partners from industry.
8. A founding meeting of WISE (Women In Semigroup theory and Evolutionary problems) will take place. An article about it will be published in the European Women in Mathematics Newsletter.
9. A satellite conference of the 8th European Congress of Mathematics devoted to the topics of the Action will be organized.
10. In the last year, a final Action conference will take place, where all the achievements and new questions will be presented, and the possibilities for future cooperation will be planned. The conference will be open to non-Action members as well.
11. The results of the joint work will be published in relevant journals and every paper will be uploaded to the open access platform arXiv.
12. Action members will present their work at major conferences in their fields.
13. The main achievements and meetings will be promptly communicated by the PRO to local and European media.

4 IMPLEMENTATION

4.1 COHERENCE AND EFFECTIVENESS OF THE WORK PLAN

4.1.1 DESCRIPTION OF WORKING GROUPS, TASKS AND ACTIVITIES

The Action will be governed by a Management Committee (MC), consisting of nationally nominated experts chosen during the first weeks of the Action. Under the supervision of the MC, five Working Groups will be established. Each Working Group (WG) will be coordinated by a WG Leader and a Vice-Leader, elected during the kick-off meeting. WG Leaders are responsible for the coherence of the scientific work and the completion of specific deliverables and milestones.

WG1: C_0-semigroups and beyond

The theory of C_0-semigroups, time-continuous linear dynamical systems, is a useful tool to study evolution equations. There is a one-to-one correspondence between these objects and well-posed abstract Cauchy problems on Banach spaces. In many situations, however, the notion of C_0-semigroups is not satisfactory, and depending on the framework there are a couple of competing approaches. This Working Group will generalize the theory of semigroups by relaxing the classical theory and studying some novel continuity concepts. These are, for example, important to obtain the well-posedness on L- or continuous function spaces on infinite networks.

Especially important for applications will be the study of systems driven by two competing processes: (Additive) perturbation theory asks for conditions on a linear operator B such that the sum A + B (defined in an appropriate sense) generates a C_0-semigroup, whenever A does. In this case, a formula (or an approximation) relating the perturbed and the unperturbed semigroups is also desired.

Tasks:

1. Generalize the theory of C_0-semigroups by modifying analytic or algebraic features.
2. Study examples which are appropriate for the theory (in collaboration with W2 and W3).
3. Obtain stability results for the semigroup in terms of spectral properties of the generating operator.
4. Develop perturbation results for the semigroups discussed above.
5. In collaboration with WG5 develop appropriate numerical methods for applications.
6. Use perturbation results for questions regarding control of the system.

WG2: Nonlinear problems

Often, linear models are first-order approximations in the descriptions of intricate phenomena. Their study is a mandatory first step whose results open the way to more accurate portraits of the problems being studied. On the other hand, general nonlinear equations typically generate a variety of behaviours that can hardly be described and classified along the lines of their linear counterparts. In this context, numerical integrations, or *experiments*, play a crucial role. They are often the only tool available to have a first insight into the qualitative behaviour of the solutions to the various nonlinear models.

This project aims at a maximal exploitation of the known linear theories to obtain relevant information on specific nonlinear models. A prime example is offered by a class of predator-prey models that leads to mixed hyperbolic-parabolic systems consisting of equations that, when seen separated, are linear.

Tasks:

1. Fully develop a semigroup approach for nonlinear DSNs in a general metric space setting (in collaboration with W1 and W4).
2. Develop appropriate nonlinear perturbation results.
3. Study stability questions for the nonlinear case.
4. In collaboration with WG5 develop suitable numerical methods for applications.

WG3: Networks and similar structures

The cell cycle, as well as the exploitation of a biological resource or demographic models, lead to renewal equations (i.e., balance laws with possibly nonlocal boundary conditions) on graphs. Their analytic treatment has some features that resemble that of models for vehicular traffic on road networks. Coupled systems of PDEs of different types can then be stated on various geometric structures, in particular graphs or networks. While the development of a very general abstract theory comprising all these situations might look ineffective in this particular case, the use of subtle analytic tools in specific applications is likely to be highly productive.

Tasks:

1. Apply the theory developed in WG1 and WG2 to DSNs.
2. Develop the appropriate approach to study dynamical processes on multi-structures, especially hypergraphs and ramified spaces.
3. Use the semigroup approach to study nonlinear dynamical systems on networks.
4. In collaboration with WG5 test *ad hoc* numerical algorithms dedicated to specific models.

WG4: Variational methods on graphs and networks

Proceeding from classical variational methods for the analysis of Laplace-type operators, the aim of this Working Group is twofold: on the one hand, classic spectral geometry applies to elliptic operators generating semigroups that are Markovian (i.e., they describe the time evolution of expectation with respect to a Markov process); the plan is to investigate possible generalizations to generators of semigroups that have such a probabilistic interpretation only after a suitable transient; nonlinear contexts will also be studied. On the other hand, insights that have recently allowed the extension of gradient systems with respect to Wasserstein metric in spaces of measures will be extended to nonlinear dynamical systems on further discrete structures. A particular challenge is that the implementation of boundary conditions, which is a central issue in the theory of networks, is not yet well understood in the context of flows on metric spaces.

Tasks:

1. Develop variational methods for nonlinear dynamical systems on graphs (in collaboration with W3).
2. Prove suitable isoperimetric inequalities for contexts that lack maximum principles.
3. Exploit spectral geometric information to settle questions related to the long-time behaviour of the dynamical system on graphs.
4. In collaboration with WG5 develop appropriate numerical methods for applications.

WG5: Numerical methods and applications

Since the differential equations which arise in applications like cell cycle, biological resource and demographic models, or vehicular traffic on road networks have a rather complicated structure, the analytical approach to the analysis of the dynamics of the corresponding system is not viable. Hence numerical methods are highly important for the success of the project.

The present project aims at finding most suitable numerical methods for applications appearing in the working plans of WG1-4. To this end, the existing approximation results will be analysed, and new techniques will be developed by taking into account the special requirements of modelling on networks.

The list of applications which will be tackled includes wave propagation in pipeline and road networks, dynamics of chemical networks, geodetic networks and multi-physics system simulations. These applications have important ramifications in real life and the development of successful numerical methods for them would be highly beneficial to the industrial partners.

Tasks:

1. Determine the spatial differential operator in problems proposed by WG2 and WG3, and provide appropriate space discretization techniques.
2. Develop numerical methods specially designed for problems proposed by WG2 and WG3.
3. Use the semigroup approach, and especially the results of WG1 and WG4, to prove the convergence of the new numerical methods.
4. Generalize the results to broader classes of problems in network modelling.
5. Apply the developed numerical methods to the applications proposed by the industrial partners.

Tasks of all WGs:

1. Report WG activities to the MC.
2. Regular weekly email correspondence and monthly remote web-conferences.
3. Short-term Scientific Missions (STSMs) relating to the objectives of the WG.
4. Communication with other WGs.

Activities and deliverables of all WGs:

1. Basic scientific review article in the first 2 years of the Action.
2. Each among WG 1-4 will organize one Training School for students.

3. Joint and separate workshops with other WGs.
4. Problem-Solving Sessions with industrial partners.
5. One-page summary reports for STSMs prepared by the visiting researcher.
6. Joint scientific publications.
7. A final report in the form of a white paper.

4.1.2 DESCRIPTION OF DELIVERABLES AND TIMEFRAME

A Short-term Scientific Mission (STSM) Coordinator will be elected to manage calls and applications for research stays, with special attention given to Ph.D. students, Postdocs and other ECIs. Calls for STSMs will be launched two times per year, in January and July. STSM applications will be assessed within one month. The assessment will be based on excellence and impact, prioritizing applications from countries that have less capacity in the field of the Action. The MC will be responsible for approving STSMs and Training School applications.

An Equal Opportunity Agent (EOA) will be nominated to ensure gender, geographical and age balance in all Action activities. The EOA will also strive to organize family-friendly research stays and meetings, with emphasis placed on ensuring the availability of organized child-care.

The MC will be supported by the Core Group (CG), consisting of the Chair, Vice-chair, STSM Coordinator, EOA, and the WG Leaders. The CG will manage the day-to-day operations of the Action, monitor milestones and prepare documents and decisions for the MC, including a financial plan to share the resources within the Action budget in an appropriate fashion.

Activities and Milestones:

1. MC/CG Meetings take place at the beginning of each year to monitor the Action.
2. Training Schools for students are organized each year: each of WG1-4 organizes in cooperation with WG5 one during the Action lifetime.
3. Workshops are organized each year, separate and/or jointly with more WGs.
4. Every year a Problem-Solving Session is prepared in collaboration with industrial stakeholders.
5. Two larger Action Conferences merging the skills are organized.
6. A Mid-term Review after 2 years and a Final Review at the end of the Action.

Deliverables:

1. Action webpage with regular updates of the Action activities
2. Review articles for each Training School
3. Report of each Training School, including supporting material
4. Public STSM reports
5. Book of abstracts for each workshop organized within the Action
6. List of peer-reviewed publications
7. WG1 "C0-semigroups and beyond" final report published as white paper
8. WG2 “Nonlinear problems” final report published as white paper
9. WG3 “Networks and similar structures” final report published as white paper
10. WG4 “Variational methods on graphs and networks” final report published as white paper
11. WG5 “Numerical methods and applications” final report published as white paper
12. Proceedings, containing a collection of survey articles on mathematical methods for DSNs and applications to different problems published after the Final Action Conference

4.1.3 RISK ANALYSIS AND CONTINGENCY PLANS

The general structure and proposed working methods have been assessed not to entail any highly likely risk factors that could hinder the successful development of the Action. However, the following low or medium risks have been identified (L=Likelihood, I=Impact) and according to contingency plans have been developed.

- Significant difficulties in obtaining theoretical mathematical results (L: Medium, I: High): It is extremely difficult to foresee the precise form and scope of theoretical mathematical results. With this in mind, rather general objectives have been formulated, in order to provide space for fine-tuning of concrete research goals on the way. The WG Leaders will report to the MC and the CG during the regular web conferences on progress towards the research goals. In the event of major difficulties or delays in a WG, the CG will assist the WG Leader in seeking necessary specialists and invite them to join a certain Action activity in order to overcome the difficulties. In extreme cases, the CG may divert some additional financial resources to invite additional experts or organize a special session.

- Lack of involvement of Action members (L: Low, I: High): The partners are chosen carefully via personal contacts, and are all highly motivated for cooperation. Meetings and research visits are planned regularly, to sustain this motivation during the Action lifetime and beyond. The CG will regularly assess, with the support of the WG Leaders, the progress of each group and, in the case of non-engagement by member(s), seek clarification from the corresponding member(s) and actively encourage them to resume participation. In extreme cases, upon agreement of the CG and the affected member(s), the latter may be invited to withdraw from the Action. In this event, at the following web conference the CG will draw up a list of candidate replacements and, upon majority decision, issue an invitation to the preferred candidate(s) to join the Action.

- Partner leaves the Action (L: Low, I: Low): The core network is large enough to ensure robustness also in this case. In case of a withdrawal, the CG will choose a replacement in accordance with the strategy sketched in the previous point.

- New individuals joining the Action have inconsistent goals (L: Medium, I: Low): New members will have to choose an appropriate WG and will be invited to adjust their goals according to the objectives of the WG. The WG Leader will constantly monitor the progress of the work and is entitled to make some adjustments of the research goals to the special interests and abilities of the participants. The WG Leader will report to the CG in the case of perceived difficulties and, an individual may be encouraged to switch to another WG.

- Different WGs do not interact (L: Medium, I: Medium): Organization of joint workshops and research visits among members of different WGs will boost interactions. The CG will regularly assess the state of interaction of the WGs and inform the relevant WG Leaders when it identifies problems. In the event of a persistent lack of communication, the CG may upon majority decision organize an extraordinary joint workshop or suggest vice-versa STSMs of ECIs from both WGs to boost cooperation.

- Difficulties in involving younger researchers or in maintaining gender balance (L: Low, I: Low): By planning every major event within the Action gender and age representation will be monitored constantly in order to ensure continued inclusiveness. STSM applications, new invitations, and after the first year requests, to join the Action will be assessed on the basis of gender, age and geographic balance, and in the event of approximately equal qualification of candidates, wherever possible preference will be given to under-represented groups (younger researchers, females, and researchers from ITCs and NNCs).

- Industrial partners are not interested in the research (L: Medium, I: Medium): The industry partners already involved in the Action have been selected carefully, on the basis of existing collaboration and interests for R&D. Moreover, the Action will continuously make efforts to foster links with new industry partners. Any new members, upon joining the Action, will be invited to consult with any relevant industrial partners they may have, and their contacts with industry will be assessed as a relevant factor when determining whether to issue an invitation.

4.1.4 GANTT DIAGRAM

Activity		MC/CG Meetings	WG Workshops	Problem Sessions	Training Schools	Special Meetings	Action Conferences	Mentoring if ECIs	STSMs	Reporting
Year 1	3		WG 1+2+3+4+5							
	6									
	9		WG 1							
	12		WG 2+3			WISE	ECM			
Year 2	15		WG 4+5							
	18									
	21		WG 1+3							
	24		WG 2+5							MTR
Year 3	27		WG 4							
	30		WG 2							
	33		WG 3+5			ECI				
	36		WG 1+4							
Year 4	39		WG 3							
	42		WG 2+4							
	45		WG 1+5							
	48						Final			FR