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CONTENT

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PRACTICAL APPLICATION OF THE REFRACTION METHOD

BLAGICA DONEVA, MARJAN DELIPETREV, GJORGI DIMOV

Abstract. Seismic refraction is a surface geophysics method that utilizes the refraction of seismic waves on geology layers and rock/soil units to characterize the subsurface geologic conditions. The method involves a geophysical principle governed by Snell's Law, which is a formula used to describe the relationship between seismic wave angles of refraction when passing through a boundary between two different isotropic media.
The Seismic Refraction method depends on the principal that seismic waves possess varying compression and shear wave velocities within differing types of soil and rock material. Analysis of refracted wavelet arrival times, velocities, and geophone geometries can be used to estimate general soil types and approximate depths to strata boundaries, water tables and/or the upper bedrock surface to be determined.
Seismic refraction is exploited in engineering geology, geotechnical engineering, and exploration geophysics.

1. Introduction

Since in natural conditions the boundaries of the media are not horizontal, in field research along the profile, it is necessary to perform refractive measurements in two directions (wave sources are inverse in relation to the receiver device). When performing explorations, it is necessary to make a plan of the position of the wave source and the hodochrone, which should be examined from these points. Depending on the number of boundaries and the thickness of the layers, hodochrone systems can be complex, i.e., each boundary can correspond to one hodochrone system. The length of individual hodochrone s depends on the depth of the boundary that is determined and the thickness of the bottom layer that separates that boundary. If the layer is very thick, the length of the hodochrone, obtained by excitation of waves at the same point of the source, is determined by a rational amount of explosives, which ensures that refractive waves of satisfactory quality are registered from that limit.

In order to ensure continuous monitoring of the boundary, a new flash point is inserted in such a way that it is positioned, in relation to the geophone device, so that a connection can be established with the hodochrone obtained from the previous point.

2. Determination of limit velocities

If the limit is flat and there is an incline, then the limit velocity is given by equation

$$V_g = \frac{2V_d \cdot V_u}{V_d + V_u}$$

where Vg - limit velocity
Vd - apparent velocity obtained via direct hodochrone,
Vu - apparent velocity obtained via inverse hodochrone.

If the boundary is curvilinear, the mean value of the velocity is obtained by using a series of intervals on the direct and inverse hodochrone at which the apparent velocities are determined.

The calculation is done in such a way that the average velocities V are determined for each pair of bound parts of the hodochrone. Then the mean value of the velocity limit is obtained according to the equation

$$\frac{1}{V_g} = \frac{1}{n}\sum_{i}^{n}\frac{1}{V_{gi}}$$

3. Determination of depth with t_0 method

With an irregular shape of the boundary surface that separates the two media through which the elastic waves propagate at different velocities, the hodochrone will not be a straight but a curved line. [1]

Figure 1 shows the vertical cross-section of the land with two hodochrone s obtained during ignition in the direct and opposite directions. It is necessary for at least one hodochrone to go to the intersection with the ordinate of the other ignition point.

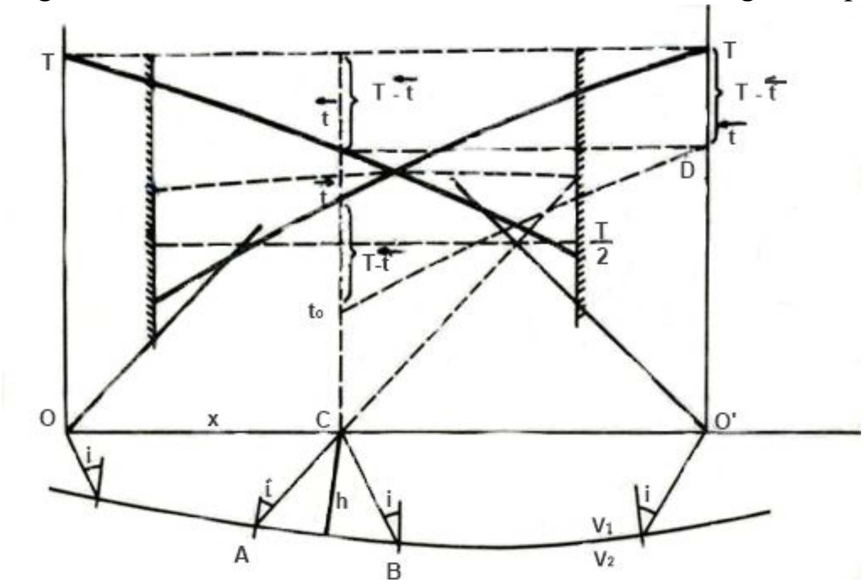

Fig. 1. Determination of depth with t_0 method

The first hodochrone is the one obtained by ignition at point O and the second is the one obtained by ignition at point O'.

The time required for a refracted wave to reach an arbitrarily chosen point C from the source point O, with abscissa x, will be marked with $\vec{t}$, and the time required for a refracted wave to reach the same point C from the flash point O' will be marked with $\overleftarrow{t}$. If we conduct an auxiliary hodochrone through point C, which is oriented as the first hodochrone, it will intersect the ordinate that passes through the beginning 0 of the second hodochrone at point D. The time required for the refracted wave to travel from C to O ' is $\vec{t}$ (Figure 1). Since the branches of the first and auxiliary hodochrone s are parallel to each other, the segment of the second branch of the auxiliary hodochrone will intersect the ordinate passing through the point C at the point corresponding to the time $t_{0(x)}$. That time amounts

$$t_{0(x)} = \vec{t} - \left(T - \overleftarrow{t}\right) = \vec{t} + \overleftarrow{t} - T \tag{1}$$

T - the time of arrival between O and O'.

From Fig. 1 the following is obtained

$$\vec{t} = t_{(OA)} + t_{(AC)}$$
$$\overleftarrow{t} = t_{(O'B)} + t_{(BC)}$$
$$T = t_{(OA)} + t_{(AB)} + t_{(BO')}$$

If the third formula is subtracted from the sum of the first two equations, we obtain

$$\vec{t} + \overleftarrow{t} - T = t_{(AC)} + t_{(BC)} - t_{(AB)} \tag{2}$$

From (1) and (2) follows

$$t_{0(x)} = t_{(AC)} + t_{(BC)} - t_{(AB)} \tag{3}$$

Provided that the part of the boundary surface between points A and B does not deviate much from the plane, the time delay $t_{0(x)}$ is given by the equation

$$t_{0(x)} = \frac{2h}{V_1} \cos i \tag{3a}$$

i - critical angle at which the beam falls on the boundary surface.

Replacing (3a) in (1)

$$\frac{2h}{V_1} \cos i = \vec{t} + \overleftarrow{t} - T$$

V_1 - velocity of the beam before refraction

Solving this equation by h, the expression for the depth to the boundary surface is obtained

$$h = \frac{V_1}{2\cos i}\left(\vec{t} + \overleftarrow{t} - T\right) \tag{3b}$$

In addition to the requirement that the boundary surface does not deviate much from the plane, "t_0" method can be applied if it also satisfies the following conditions:

- that the wave propagates along the boundary surface, i.e., there is no influence of the sinking of the wave,
- that the radius of curvature of the boundary surface is much larger than the depth at which it is located,
- that the speed limit V_2 does not change abruptly along the profile.

3.1. Determination of velocities by the method of differences

In order to be able to use equation 3b, we need to calculate the value beforehand $\frac{V}{2\cos i}$. Velocity V_1 is determined from the right branches of the hodochrones, and we obtain the critical angle by knowing the velocities V_1 and V_2.

Velocity V_2 can be determined from the so-called hodochrone of the time difference of the occurrence of refracted waves to individual points on the profile, during ignition at points O and O' (Figure 2). It is given with the equation:

$$Q_{(x)} = \vec{t} - \overleftarrow{t} + T \tag{4}$$

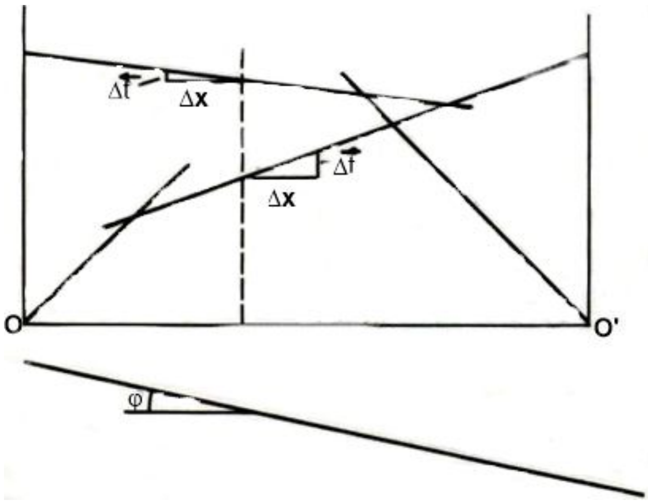

Fig. 2. Determination of velocities by the method of differences

The coefficient of tangent directions on the hodochrone Q is given by the expression:

$$\frac{dQ_{(x)}}{dx} = \frac{\overrightarrow{dt}}{dx} - \frac{\overleftarrow{dt}}{dx} \tag{5}$$

It can be shown that with a continuous boundary surface (without sudden bends or faults) the hodochrone Q is a curve that practically does not deviate from the line. Therefore, it can be assumed that the hodochrone Q (constructed based on the time t taken from the first and second hodochrone es) is a straight line whose direction coefficient is based on the equation (5)

$$\frac{\Delta Q}{\Delta x} = \frac{\overrightarrow{\Delta t}}{\Delta x} - \frac{\overleftarrow{\Delta t}}{\Delta x} \tag{6}$$

From Figure 2 it is seen that

$$\frac{\overrightarrow{\Delta t}}{\Delta x} = \frac{1}{V_d} \qquad and \qquad \frac{\overleftarrow{\Delta t}}{\Delta x} = -\frac{1}{V_u} \tag{7}$$

Replacing equation (7) in the expression (6), it is obtained that

$$\frac{\Delta Q}{\Delta x} = \frac{1}{V_d} + \frac{1}{V_u}$$

If in the equation $\frac{1}{V_g} = \frac{\frac{1}{V_d}+\frac{1}{V_u}}{2\,cos\varphi}$ is replaced $\frac{1}{V_d} + \frac{1}{V_u}$ with $\frac{\Delta Q}{\Delta x}$, it is obtained that

$$\frac{1}{V_g} = \frac{\frac{\Delta Q}{\Delta x}}{2\,cos\varphi} \qquad \text{or} \qquad V_g = 2\,\frac{\Delta x}{\Delta Q}\,cos\varphi \tag{8}$$

If the dip of the area boundary is less than 10° - 15°, it can be assumed that cosφ = 1, and formula (8) can be written as

$$V_g = 2\,\frac{\Delta x}{\Delta Q}$$

4. Determination of depth by the wavefront method

Over time, the wave front, which propagates from a source, occupies different positions in the environment in which it spreads. To each position of the wave front can be assigned the corresponding moment t_1, t_2, t_3… etc. and vice versa, to each moment a certain position of the wave front in a given environment corresponds. In order to enable the observation of the position of the wave front at different times, the term isochron was introduced, i.e., surfaces which at certain different moments coincide with the wavefront. Each isochron is characterized by a certain value of time. [1]

In Figure 3 several isochrons of refracted waves from the boundary surface MN with two sources O and O' are observed: Isochrones of the first time field are marked with times t_{A1}, t_{A2}, t_{A3} …, and the other with t_{B1}, t_{B2}, t_{B3},… Isochrones of these two fields intersect. It is seen that two isochrones (t_A and t_B, $t_{A'}$, and t_B', etc.) intersect at points C, C', C" on the boundary surface. These isochrones have the property that the sum of their times is equal to constant time T. So,

$$t_{An} + t_{Bn} = T \tag{9}$$

It follows that the position of the boundary surface can be determined as the geometric point of intersection of isochrones of two fields of time of refractive waves with two sources, and whose sum of times is equal to T.

Figure 3 also shows the construction of such isochrons that meet the condition given by relation (9), hence the method of construction of the boundary surface MN.

The propagation velocity in the lower environment will be

$$V_2 = \frac{\overline{CC'}}{\Delta t} \qquad \text{or} \qquad V_2 = \frac{\overline{CC'} + \overline{C'C"}}{2\Delta t}$$

The wavefront method is used to construct graphic surfaces that are not flat. Usually, in the construction of isochrones, the times differ by one constant Δt (fig. 3).

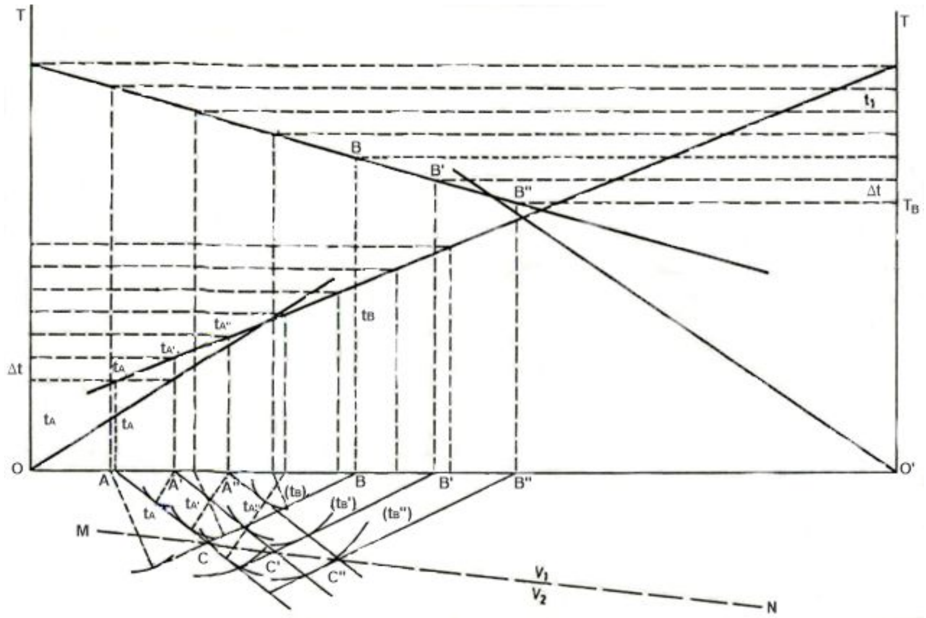

Fig. 3. Determination of depths by the wavefront method

5. Generalized refraction reciprocal method

The generalized refractive method is based on the calculation of the function time - depth and values of velocities from direct and inverse time of wave encounters at different distances XY between receivers arranged symmetrically along the refractive profile.

This method makes it possible to determine the position of the wavy refractor at any depth, based on the data of direct and inverse times of wave encounters obtained along the refractive profile.

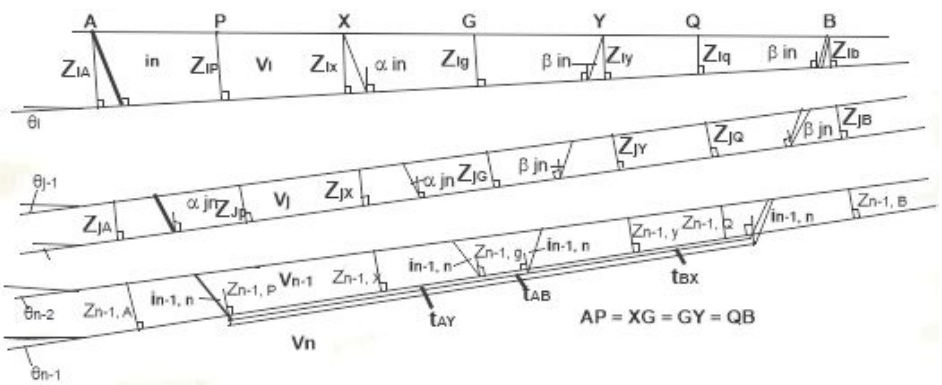

Fig. 4. Schematic representation of generalized reciprocal refractive method

(Derecke Palmer 1980)

The time of occurrence of waves in two positions of receivers separated by a variable distance XY is used for speed analysis and calculation of time - depth curves. At optimal distances XY rays to each receiver are reflected from almost the same point of the refractor and the analysis gives more accurate speeds and depths (Figure 4).

5.1. Velocity analysis

When the boundaries are flat within the refractive model, the velocity is determined in the previously exposed ways based on the direct and inverse distance-time function. When this is not the case or when the velocities of all layers above the refractor are not known, using the symbols in Figure 4 (Derecke Palmer, 1980), [5] the refractor velocity can be expressed, with satisfactory accuracy, by the following relation, which is the function for velocity analysis V

$$t_V = \frac{1}{2}(t_{AY}) - t_{BX} + t_{AB} \tag{10}$$

In the routine interpretation, the values of t_V, calculated using equation (10), are plotted on the diagram as a function of distance for different values of XY.

The procedure is shown in Figure 4, which represents the analysis of the velocity function for a given model of the environment. For optimal XY distance the speed function is a straight line. For other XY distances, the analyzed velocity curve deviates from the straight line in a short interval which is a function of the optimal value of XY, on either side of the given model.

5.2. Function time - depth

After determining the velocity, the next procedure in defining the shape of the wavy refractor is to determine the function time - depth. [3]

Using the symbols from figure 4, generalized function depth - time is defined with the following equation

$$t_G = \left[\frac{1}{2}t_{AY} + t_{BX} - \left(t_{AB} + \frac{XY}{V'_n}\right)\right] \tag{11}$$

where the apparent refractive velocity is determined from the velocity function analysis.

Based on Figure 4, it could be seen that

$$\begin{aligned} Z_{jx} &= Z_{jG} + GX\,sin\theta \\ Z_{jy} &= Z_{jG} - GX\,sin\theta \end{aligned} \tag{12}$$

for flat boundaries, equation (11) reduces to the following expression:

$$t_G = \sum_{j=1}^{n-1} Z_{jG}\left(cos\alpha_{jn} + cos\beta_{jn}\right)\Big/ 2V_j \tag{13}$$

Equation (13) associates the time-depth function with layer thicknesses and contains an expression

$$V_{jn} = \frac{2V_j}{\left(cos\alpha_{jn} + cos\beta_{jn}\right)} \tag{13a}$$

where V_{jn} is called conversion factor.

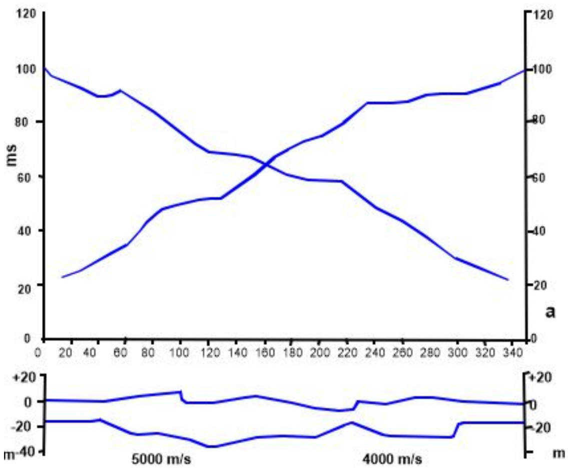

Fig. 5a. Model of terrain with hodochrone es

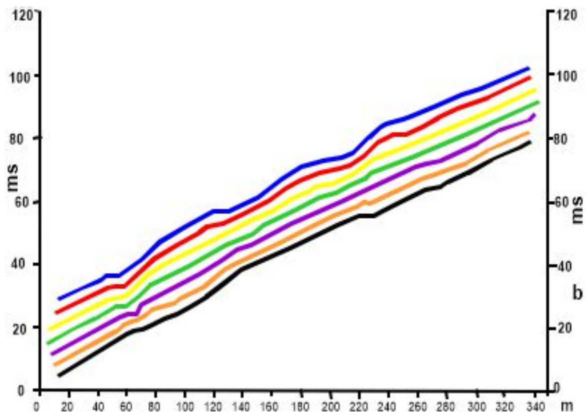

Fig. 5b. Velocity function for XY values of 0 - 30 m (Derecke Palmer, 1980)

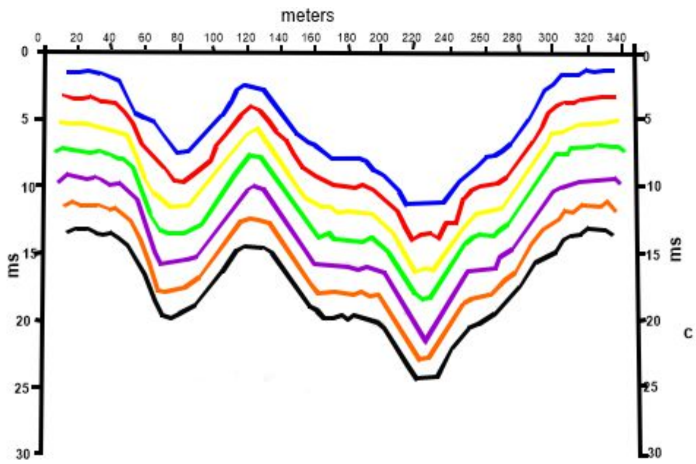

Fig. 5c. Time - depth curve for different XY values (Derecke Palmer, 1980)

Figures 5a, 5b, 5c show the model of the environments, the corresponding hodochrone es, the functions of velocity analysis for XY values from 0 - 30 m, and time curves t_0 for XY values from 0 - 30 m (Figure 5c).

In general, αj_n and β_{jn} depend on the variation of the boundary drop, which can be random and then it is impossible to determine. Usually, the effects of the dip are neglected and Equation 13a is approximated by the following relation

$$V_{jn} = \frac{V'_n V'_j}{\left(V'^2_n + V'^2_j\right)^{1/2}} \tag{13b}$$

6. Wave distribution on a curved line

When the velocity of the medium changes continuously with depth, the path of the ray will have a curved shape. If the speed increases or decreases with depth, the ray path will become curved - concave or convex. The ray starting from the wave source and falling on the boundary surface at an angle of 90° will return after reflection along the same curved path to the source point (Figure 8a). [2]

If receivers S_1 and S_2 are placed on both sides of the wave source, symmetrically with respect to the flash point 0, as seen in Figure 8a, then the difference in the time of occurrence of the reflected wave to the two receivers will be dT. Since the wavefront is bent at an angle θ, it crosses the path dx sinθ during the time dT.

If the velocity of the seismic wave in the upper part of the layer is denoted by V_1, then the time difference will be

$$dT = \frac{dx\, sin\theta}{V_1} \tag{14}$$

Geometrically speaking, it follows that $\theta = \alpha_1$ so it can be written that

$$\sin \alpha_1 = V_1 \frac{dT}{dx} \tag{15}$$

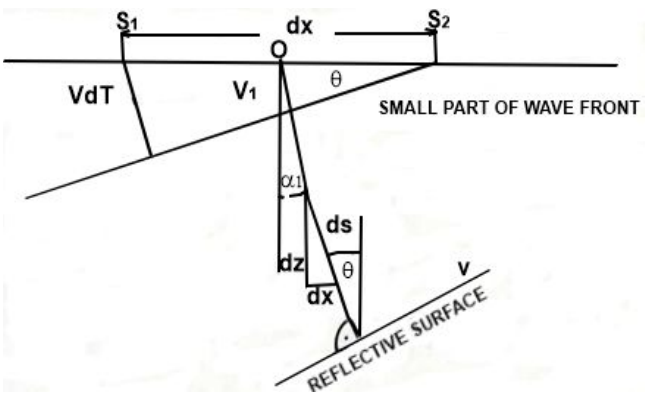

Fig. 6. Ray trajectory with continuous velocity change with depth

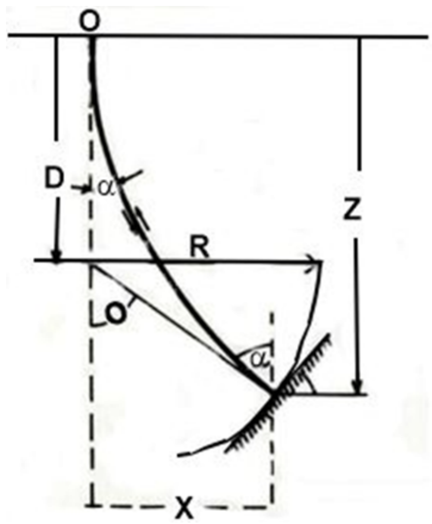

Fig. 7. Ray trajectory with linear velocity change with depth

From Figure 6 it is seen that the angle of incidence θ of the reflecting surface is equal to the angle that the reflected ray creates with the vertical at the point of reflection. Using the basic relation for this case, it can be written

$$sin\,\theta = \frac{V}{V_1}\sin\alpha_1 \tag{16}$$

replacing sin α_1 in equation (16) with the expression from equation (15) the following relation is obtained

$$sin\,\theta = V\frac{dT}{dx}$$ where V is the velocity in the reflection point.

For simplification, let's mark the parameter $\frac{\sin\alpha_1}{V_1} = p$.

By replacing this parameter in the relation that determines the angle which the beam forms with the vertical at a certain depth (Figure 8a) $\sin\alpha = \frac{V}{V_1}\sin\alpha_1$, it can be written $\sin\alpha = pV$.

If the elementary distance along the arc of the ray is denoted with ds, horizontally with dx and vertically with dz (Figure 6), then the following relation is established

$$\frac{dx}{dz} = tg\,\alpha$$

and since $\sin\alpha = pV$ and $\cos\alpha = \sqrt{1-(pV)^2}$

then

$$\frac{dx}{dz} = \frac{pV}{\sqrt{1-\ (pV)^2}}$$

If the ray is reflected from the boundary at a point that is horizontally distant from the point of the wave source, then

$$x = \int_0^x dx \tag{17}$$

Based on the relation (17) it is obtained that

$$x = \int_0^z \frac{pV\,dz}{\sqrt{1-\ (pV)^2}}$$

From figure 6 it is seen that

$$\frac{dz}{ds} = \cos\alpha \tag{18}$$

The distance ds the ray will pass for the time dt, or

$$ds = Vdt \tag{19}$$

Based on the relations (18) and (19), bearing in mind that $cos\ \alpha\ = \sqrt{1\ -\ (pV)^2}$, we obtain

$$\frac{dt}{dz} = \frac{1_1}{\mathrm{V}\cos\alpha} = \frac{1}{V\sqrt{1-\ (pV)^2}}$$

The total propagation time of the T wave from the source to the reflection border and back to the receiver will be

$$T = \int_0^t dt \qquad \text{or} \qquad T = \int_0^t \frac{pV\,d(pV)}{V\sqrt{1-(pV)^2}}$$

6.1. Velocities of a linear depth function

Assuming that the velocity changes linearly with depth or that V = V_1 + az, then $dz = \frac{dV}{a}$

Replacing this expression in the equation (17), we obtain

$$X = \frac{1}{ap}\int_{pV_1}^{pV} \frac{pVd(pV)}{\sqrt{1-\ (pV)^2}}$$

Solving this integral gives the expression for the horizontal distance of the point of ray reflection from the wave source

$$X = \frac{1}{ap}\left[\sqrt{1-(pV_1)^2} - \sqrt{1-\ (pV)^2}\right]$$

By a similar procedure, the expression for T is found

$$T = \frac{2}{a}\left[arc\ a\cos hyp\left(\frac{1}{pV_1}\right) - arc\ a\cos hyp\left(\frac{1}{pV}\right)\right]$$

where is

$$arc\cos hyp\left(\frac{1}{X}\right) = \ln\left(\frac{1}{X} + \frac{\sqrt{1-x^2}}{X}\right)$$

If we simplify the obtained results for X and T, we get the equation from them

$$X^2 + \left[Z - \frac{V_1}{a}\left(\cos hyp\,\frac{aT}{2} - 1\right)\right]^2 = \left(\frac{V_1}{a}\right)^2 \sin hyp^2\,\frac{aT}{2}$$

from which it can be seen that the reflected wave registered at the source point, at time T, comes from a reflection plane tangent to a radius sphere $\left[\frac{V_1}{a}\sin hyp\ \frac{aT}{2}\right]$, and whose center is on the depth $\left[\frac{V_1}{a}\cos hyp\ \frac{aT}{2}-1\right]$.

7. Conclusion

The seismic refraction survey is a very important geophysical technique used in the investigation of subsurface characteristics.

Applied seismic methods comprise sending impulses underground and registering the resulting refracted arrivals from subsurface interfaces on a number of receivers positioned on or near the surface. Times elapsed from sending to receiving seismic waves depend on the depths of the studied structures and the propagation velocities of seismic waves along the paths of their propagation from the source to the refractor (or reflector) to the receiver.

This paper presents methods for the determination of limit velocities and the depth with different methods. Such methods are determination of depth with t_0 method, method of wave front, and reciprocal refraction method.

The results obtained are generalized expressions that relate travel time, distance (depth) and velocity of refracted waves.

The velocity of refracted waves is the most fundamental parameter in these methods. It depends on the elastic properties as well as on bulk densities of the media and it varies with mineral content, lithology, porosity, pore fluid saturation, and degree of compaction.

During their propagation within the subsurface, seismic waves are refracted when elastic contrasts occur at boundaries between layers and rock masses of different rock properties. [4]

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