

## SIMULATING THE DIAMOND-PISSARIDES-MORTENSEN MODEL: SEARCH MODEL THAT GIVES REALISTIC ACCOUNT OF UNEMPLOYMENT

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### Abstract

This paper is about the conventional search models of unemployment. An as considerable number of authors point out that negatively - sloped Beveridge curve is the result of an aggregate demand shock. The shock that creates a positive movement between vacancies and unemployment “loops” around the Beveridge curve is due to matching efficiency and job destruction. This positive co-movement of vacancies and unemployment occurs after recessions. It is why we include RBC model to see the co-movement of IRF function of 6 macro variables including: Labor and wages. New- Keynesian DSGE model was included out of fancy. Unemployment dynamics due to: output, matching efficiency, vacancy advertising cost, unemployment benefits, and exogenous separation rate is studied at the end.

**Key words:** DPM model, labor market search, unemployment

**JEL Classification:** J01

### INTRODUCTION

John Maynard Keynes in his *The General Theory of Employment, Interest and Money* (1936), argued that capitalist economy could poses equilibria that are characterized by the persistent involuntary unemployment, see also, Akerlof, Yellen (1987). Keynes defines involuntary unemployment : ....“Men are involuntarily unemployed If, in the event of a small rise in the price of wage-goods relatively to the money-wage, both the aggregate supply of labour willing to work for the current money-wage and the aggregate demand for it at that wage would be greater than the existing volume of employment”. There is a distinction between voluntary and involuntary unemployment<sup>1</sup>, or “for instance in the latter case it is not true that real wages must be lower if employment is to be higher<sup>2</sup>...moreover involuntary unemployment arises from avoidable co-ordination failures and externalities”, Hahn,(1987). As the Beveridge curve movements (relationship between market tightness (vacancies over unemployment)  $\theta = \frac{v}{u}$  and unemployment rate  $u$ ) is to be interpreted as a decrease in the efficiency of matching process between workers and jobs, see Diamond (2011). So if  $\lambda$  exogenous separation rate increases, real wage will decline and involuntary unemployment will rise (vacancy setting curve will move to the left). Keynes analyzed that the key departure from the self-interested maximizing behavior is the assumed stickiness of money wages. Workers typically resists money wage reduction but...“not to resist real money wage reductions”. Keynesian theory of involuntary unemployment is compatible with search theory, since the worker in question may have reservation wage below those of his type who are being hired. In that view Dasgupta, Raj(1986) regard involuntary unemployment as a manifestation of

<sup>1</sup> Unemployment to an average person is an involuntary idleness (Andolfatto, 2006 in The New Palgrave Dictionary of Economics, 2nd edition, 2008). This is inconsistent in the way in which unemployment is in fact defined and measured. Because according to International Labor Organization (ILO) conventions, which are followed by most of the nation's labor force surveys, unemployment relates to those individuals that are unemployed but are actively searching job. Those unemployed who are not actively included in search are classified as non-participants.

<sup>2</sup> For Keynes worker is involuntary unemployed if the market wage for his labor exceeds his shadow wage, which is a wage at which a worker would be indifferent between not accepting and accepting job offer, see Hahn (1987).

horizontal inequity. This paper will treat unemployment not as demand problem but as matching or structural problem and not so as an inadequate aggregate demand problem. For instance, this was described in simple words in Kocherlakota (2010) statement: "Firms have jobs but can't find appropriate workers. The workers want to work but can't find appropriate jobs". This inference was simply taken to imply that one should not be concerned with stimulating the aggregate demand through monetary and fiscal policy. Like this the unemployment in 1950's-1960's in US was also been described as structural rather than a result of inadequate aggregate demand, see Solow (1964). On the other hand Samuelson, Solow (1960) paper has been widely known to be the first paper to have drawn US Philips curve. Both authors though report that Philips curve had disappeared during the Great Depression and suggested that persistent high unemployment by 1933 could well reduce mobility and increase structural unemployment<sup>3</sup>. Samuelson, Solow (1960) continue to argue about the relation between wages and unemployment .." They argued that imperfect competition was the right setting for studying cost push inflation. They emphasized that many factors were at work in the labor market including labor reallocation, labor mobility, collective bargaining; thus no simple or single explanation was likely to account for the relation between wages and unemployment"... see Blanchard, Diamond (1991). Solow later papers were also explorations of those themes, in his 1969 Manchester lectures Solow emphasized that with multiple equilibria (incomplete models)<sup>4</sup>.. "the economy might be jolted out of an underemployment equilibrium". Wage setting curve (WS) and Vacancy setting curve (VS) equilibrium defines market tightness and real wage. After a change in the: efficiency of matching process, exogenous separation rate, bargaining power, unemployment benefits, advertising costs of vacancy, real wage and market tightness  $\theta$  changes, also Beveridge curve (association between market tightness  $\theta$  and unemployment rate  $u$ ) moves left or right, see Bhattacharya et al. (2017). In general equilibrium setting of the neoclassical model and labor market as a centralized marketplace with perfect information devoting time to search for a job is non worthy, and individuals either become employed or unemployed and the solution is Pareto efficient. In the search model of unemployment labour market is decentralized place where the search model postulates wage distribution and distribution of offers per unit time. Latter is Poisson distribution with  $\lambda$  arrival rate, and some separation rate  $s$ , and job finding rate  $f$  from where unemployment rate  $u$  could be determined by notion of the reservation wage  $w_r$  with CDF  $F(w_r)$  and  $F(w)$  wage offer cumulative distribution function, with benefits  $b$  see Mortensen (2011)<sup>5</sup>. Summers(1986) finds a strong negative relation between changes in unemployment and the growth of high-wage jobs ,see Burda (1988). This is in line with notion that voluntary unemployment exists near full employment, otherwise there exists involuntary unemployment which is dependent on the reservation wage (minimum wage) and efficiency wage. Papers on efficiency wage theories and explanations of involuntary unemployment such as: Yellen (1984), and Shapiro, Stiglitz (1984) are worth mentions. Though this paper is about search theory of unemployment with a special emphasis on the Diamond-Mortensen-Pissarides as a central model around the research. First literature review will be followed by

<sup>3</sup> "...one could argue that by 1933 much of the unemployment had become structural, insulated from the functioning labor market, so that in effect the vertical axis ought to be moved over to the right. This would leave something more like the normal pattern.", Samuelson, Solow (1960) of the US Philips curve for US

<sup>4</sup> Other than that his later work included themes on collective bargaining and unemployment, like in his Wicksell lecture, Solow developed a view of labor market .." as a market with constant reallocation of labor and addressed the question of how much of unemployment was due to low demand and how much to structural factors.", see, Blanchard, Diamond (1991).

<sup>5</sup> Worth the mentioning...  $\frac{u}{1-u} = \frac{s}{f}$  ;  $f = \lambda(1 - F(w_r))$  ; and  $u = \frac{s}{s + \lambda(1 - F(w_r))}$  ;  $w_r = b + \lambda \int_{w_r}^{\infty} (J(w) - V) dF(w)$  ;  $J(w)$  denotes the future earning associated with a job that offers wage  $w$  , and  $V$  is the value of vacancy when unemployed

numerical examples and certain conclusions about the search and matching theory of unemployment.

## 2. Literature review on search theory of unemployment

The idea that a theory of unemployment can be built on the assumption that trade in the labor market is an economic activity was first explored by a number of authors in the late 1960's, in what is now known as search theory, see for example Stigler (1962) or economics of information and job search see McCall, J. (1970). The most influential papers in this tradition were Alchian (1969), Phelps (1968), and Mortensen (1970); they were collected with other contributions in the Phelps volume (Phelps et al. 1970). The driving thought to this research came from Phelps's (1967) and Friedman's (1968) reappraisal of the Phillips curve and the natural rate approach to which this led. Early search theory assumed the existence of a distribution of wage offers for identical jobs; unemployment arose in equilibrium because workers rejected low-wage jobs. This aspect of the theory was criticized both on logical grounds (these models take into account the supply side of the market) (Rothschild 1973) and on empirical grounds (Tobin 1972<sup>6</sup>; Barron 1975). An equilibrium model that met Rothschild's criticisms, but with a trivial role for workers looking for alternative jobs, was first presented in Lucas; Prescott (1974). Early applications of the concept of the matching function that downplay the role of reservation wages include Hall (1979), Pissarides (1979), and Bowden (1980). Diamond and Maskin (1979) used the similar concept of "search technology" in a related context. The application of zero-profit conditions for new jobs, leading to a closed model with endogenous demand for labor, was first discussed in Pissarides (1979, 1984b). The Nash solution was first applied in this context with fixed numbers of traders by Diamond (1982b), though earlier papers by Mortensen (1978) and by Diamond and Maskin (1979) discussed similar sharing rules for the division of the surplus from a job match. Despite its importance there are very few attempts to derive the matching function from primitive assumptions about trade. Hall (1979), Pissarides (1979), and Blanchard and Diamond (1994) have borrowed Butters's (1976) urn-ball game to derive an exponential function. The out-of-steady-state analysis of unemployment and vacancies was first discussed in Pissarides (1985a, 1987). In Pissarides (1985a), imputed unemployment income is assumed fixed, but the model contains more other features than the models in this review. In Pissarides (1987) unemployment income was allowed to depend on wealth. Large literature testifies on the importance on matching frictions and job rationing<sup>7</sup> as a source of unemployment, Michaillat (2012)<sup>8</sup>. Labor markets see constant job creation, job destruction, and a very large flows of workers see for example Blanchard and Diamond (1989). Next, the fact that when there is search on the job the optimal policy can be described by two reservation wages was first noted in a partial context by Burdett (1978). The early literature is surveyed by Mortensen (1986). Jovanovic's (1979) model of turnover uses the latter mechanism and is built into an equilibrium search model in Jovanovic (1984)<sup>9</sup>. Vacancy chains caused by quitting are studied by Contini and Revelli (1997) and

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<sup>6</sup> Tobin (1972) explained that the job seeking theory of Phelps et.al. (1970), is useful in explaining the voluntary frictional unemployment. But in the Beveridge curve reality –"vacancies should not be less than unemployment. But because of limited capital stocks and interdependence among skills, jobs cannot be indefinitely multiplied without lowering their marginal productivity". .."Our wise and benevolent planner would not place people in jobs yielding less than the marginal value of leisure. Given this constraint on the number of jobs, he would always have to keep some workers waiting, and some jobs vacant"..wrote Tobin (1972) acknowledging that there must be involuntary unemployed workers.

<sup>7</sup> Models of job ration include efficiency wage models, Solow 1979, Akerlof gift-exchange model (1982), insider-outsider models such as Lindbeck ; Snower 1988, and social norm models Akerlof (1980).

<sup>8</sup> This survey in modeling the matching frictions used the literature and it imposed a vacancy posting costs, see: Pissarides 1985; Mortensen and Pissarides 1994; Pissarides 2000; Shimer 2005; Hall 2005a. About the wage schedule in such an environment: The marginal product of labor always exceeds the flow value of unemployment, normalized to zero, so there are always mutual gains from matching. But there is no compelling theory of wage determination there. In other models such as those as: Hall 2005b; Shimer 2012, the labor market rapidly converges to an equilibrium in which inflows to and outflows from employment are large.

<sup>9</sup> In these models Jovanovic assumes that workers productivity is unknown to the firm at the beginning. Over time firm and the worker gain information about the value of the job.

Akerlof, Rose, and Yellen (1988). Some papers have focused on costly search and the cost of advertising vacant jobs, see Howitt, McAfee (1987).

## Materials and methods

### 3.Diamond-Mortensen-Pissarides (DMP) model

Economists had been using search models for more than 50 years to describe labor market more closely. And the seminal work of Diamond(1982b); Pissarides (1985); and Mortensen and Pissarides (1994), had become a framework for macroeconomists to study unemployment, Bhattacharya et al. (2017).Some important standard textbook in macroeconomics that use DMP framework include: Carlin and Soskice (2006); Williamson (2013); Chugh (2015).DMP model has been accepted throughout macroeconomics in the economics of business cycles, Merz (1995) ;Andolfatto (1996), in the New Keynesian model, see Gertler, Trigari (2009),in the area of monetary policy, see Blanchard and Gali (2010);and in the field of endogenous disasters, Petrosky-Nadeau, Zhang, and Kuehn (2015) ,see Petrosky-Nadeau, Zhang(2017).As per Hall (2012), DMP model is a central component of modern macroeconomics.

### 4.DMP framework

Matching function is given as: $mL = m(uL, vL)$ , it is concave and homogenous of degree 1. Homogeneity or constant returns to scale. Where  $u$  is unemployment rate,  $v$  -vacancy rate,  $uL$  unemployed worker  $L$ -total labor force, and  $vL$  job vacancies. Vacancy to filled jobs equals  $\frac{v}{u}$  is denoted to  $\theta$ <sup>10</sup> and equals to:  $\theta = m\left(\frac{u}{v}, 1\right)$ . Also,  $\delta t$  is a small time interval during some vacant job is matched to an unemployed person, with a probability  $q(\theta)\delta t$ . To a related Poisson process  $\lambda = \frac{m(uL, vL)}{uL}$  where  $\lambda = \theta q(\theta)$  and has elasticity  $1 - \eta(\theta) \geq 0$ . The mean duration of unemployment is  $1/\theta q(\theta)$ . Worker goes from employment to unemployment with probability  $\lambda \delta t$ , the mean number of workers who enter unemployment during a small time interval is  $\lambda(1 - u)L\delta t$ , and the mean number who leave unemployment is  $mL\delta t$ , or we can rewrite the latter as:  $u\theta q(\theta)L\delta t$ , where  $\theta q(\theta)\delta t$  is the transitional probability of unemployed. The evolution of mean unemployment is given as:

equation 1

$$\dot{u} = \lambda(1 - u) - \theta q(\theta)u$$

In the steady-state the mean rate of unemployment is given as:  $\lambda(1 - u) = \theta q(\theta)u$ . he equation that determines unemployment in terms of two transition states is  $u = \frac{\lambda}{\lambda + \theta q(\theta)}$ . Job creation rate is defined as the ratio of the number of jobs created to employment  $\frac{m(v, u)}{1 - u}$ , and job destruction rate is similarly defined as the ratio of the total number of jobs destroyed to employment  $\frac{\lambda(1 - u)}{1 - u}$ . Let  $J$  be the present-discounted value of expected profit from an occupied job and  $V$  the present-discounted value of expected profit from a vacant job. With a perfect capital market, an infinite horizon and when no dynamic changes in parameters are expected,  $V$  satisfies the Bellman equation:

equation 2

$$rV = -pc + q(\theta)(J - V).$$

<sup>10</sup>  $\theta = \frac{v}{u}$  is a market tightness, and for the firms probability of filling a vacancy is given as:  $\frac{m(u, v)}{v} = m\left(\frac{1}{\theta}, 1\right) \equiv q(\theta)$ , and  $q'(\theta) < 0$ ; and for the workers probability of finding a job is:  $\frac{m(u, v)}{u} = m(1, \theta) \equiv \theta q(\theta)$ . There flowing applies :  $\lim_{\theta \rightarrow 0} [\theta q(\theta)] = \lim_{\theta \rightarrow 0} q(\theta) = 0$  and  $\lim_{\theta \rightarrow \infty} [\theta q(\theta)] = \lim_{\theta \rightarrow \infty} q(\theta) = +\infty$



A job is an asset owned by the firm. In a perfect capital market the valuation of the asset is such that the capital cost,  $rV$ , is exactly equal to the rate of return on the asset: The vacant job costs  $pc$  per unit time and changes state according to a Poisson process with rate  $q(\theta)$ . The equilibrium condition for the supply of vacant jobs is  $V = 0$ , implying that  $J = \frac{pc}{q(\theta)}$ . This is the second key equation of the equilibrium model. For an individual firm,  $1/q(\theta)$  is the expected duration of a vacancy. The flow capital cost of the job is  $rJ$ . In the labor market, the job yields net return  $p - w$ , where  $p$  is real output and  $w$  is the cost of labor. The job also runs a risk  $\lambda$  of an adverse shock, which leads to the loss of  $J$ . Hence  $J$  satisfies the condition,  $rJ = p - w - \lambda J$ . The firm takes the interest rate and product value as given, but the wage rate is determined by a bargain between the meeting firm and worker as  $p - w - \frac{(r+\lambda)pc}{q(\theta)} = 0$ . Let  $U$  and  $W$  denote the present-discounted value of the expected income stream of, respectively, an unemployed and an employed worker, including the imputed return from nonmarket activities. The unemployed worker enjoys (expected) real return  $z$  while unemployed, and in unit time he expects to move into employment with probability  $\theta q(\theta)$ . Hence  $U$  satisfies:  $rU = z + \theta q(\theta)(W - U)$ . Employed workers earn a wage  $w$ ; they lose their jobs and become unemployed at the exogenous rate  $\lambda$ . Hence the valuation placed on them by the market,  $W$ , satisfies:  $rW = w + \lambda(U - W)$ . The permanent incomes of unemployed and employed workers, in terms of the returns  $z$  and  $w$  and the discount and transition rates:

equation 3

$$rU = \frac{(r+\lambda)z + \theta q(\theta)w}{r+\lambda + \theta q(\theta)}; rW = \frac{\lambda z + [r + \theta q(\theta)]w}{r+\lambda + \theta q(\theta)}$$

The job is worth to the worker:  $rW_i = w_i - \lambda(W_i - U)$  the job rate for this job satisfies:

equation 4

$$w_i = \operatorname{argmax}(W_i - U)^\beta (J_i - V)^{1-\beta}$$

$\beta$  is labor's share of the total surplus that an occupied job creates,  $0 \leq \beta \leq 1$ ,  $\beta = \frac{1}{2}$  is the most plausible value. Now,  $rU$  -reservation wage,  $\beta(p - r)$  fraction of net surplus they create by accepting the job, product value net of what they give up<sup>11</sup>,  $rU \Rightarrow rU = z + \frac{\beta}{1-\beta} pc\theta$ . Aggregate wage equation that holds in equilibrium, is given as:  $w = (1 - \beta)z + \beta p(1 + c\theta)$ .

#### 4.1 Steady-state equilibrium

Now, recall that the number of jobs is equal to employment,  $(1 - u)L$ , plus job vacancies,  $\theta uL$ ; therefore, if we know  $\theta$  and  $u$ , we also know the number of jobs. Henceforth,  $\theta$  is the labour market tightness or  $\theta = \frac{v}{u}$  and  $u = \frac{\lambda}{\lambda + \theta q(\theta)}$  unemployment rate and  $p - w - \frac{(r+\lambda)pc}{q(\theta)} = 0$  wage rate is determined by a bargain between the meeting firm and worker. Also,  $w = (1 - \beta)z + \beta p(1 + c\theta)$  aggregate wage equation that holds in equilibrium and  $(1 - \beta)(p - z) - \frac{r+\lambda + \beta \theta q(\theta)}{q(\theta)} pc = 0$  equilibrium condition for  $\theta$ . If we let  $z = \rho w$ , where  $\rho$  is the replacement rate (a policy parameter), then the wage equation becomes:  $w = \frac{\beta(1+c\theta)}{1-(1-\beta)\rho} p$ . The job creation condition now becomes  $1 - \frac{\beta(1+c\theta)}{1-(1-\beta)\rho} - \frac{(r+\lambda)c}{q(\theta)} = 0$ .

#### 4.2 Capital

Now we define  $k = \frac{K}{pN}$  capital stock per efficiency unit labor.  $f(k) = F\left(\frac{K}{pN}, 1\right)$  output per efficiency unit of labor,  $f'(k) > 0$ ;  $f''(k) < 0$   $J$  is determined by the asset-valuation condition

<sup>11</sup> It is intuitive for a market equilibrium if we note that  $pc\theta$  is the average hiring cost for each unemployed worker (since  $pc\theta = pcv/u$  and  $pcv$  is total hiring cost in the economy).

equation 5

$$r(J + pk) = pf(k) - \delta pk - w - \lambda J$$

equilibrium condition for the firm's capital stock, is given as:  $f'(k) = r + \delta$ . We restate here the equilibrium conditions with this generalization:  $f'(k) = r + \delta$  equilibrium condition for the firm's capital stock, and now we have:  $p[f(k) - (r + \delta)k] - w - \frac{(r+\lambda)pc}{q(\theta)} = 0$ , wage rate is determined by a bargain between the meeting firm and worker:  $w = (1 - \beta)z + \beta p(f(k) - (r + \delta)k + c\theta)$  aggregate wage equation that holds in equilibrium  $u = \frac{\lambda}{\lambda + \theta q(\theta)}$  unemployment rate.

#### 4.3 Out of steady-state dynamics

Let again  $V$  denote the asset value of a vacant job. With a perfect capital market and perfect foresight it satisfies the arbitrage equation:  $rV = -pc + \dot{V} + q(\theta)(J - V)$ . Expected capital gains from changes in the valuation of the asset  $\dot{V}$ , yield  $-pc$  and expected capital gains from the chance of finding a worker to take the vacancy  $q(\theta)(J - V)$ . The value of a filled job,  $J$ , satisfies a similar arbitrage condition. In the absence of capital we get:  $rJ = p - w + \dot{J} - \lambda J$  - is the expected capital gain from changes in job value during adjustment. Our assumption that firms exploit all profit opportunities from new jobs, regardless of whether they are in the steady state or out of it, implies that  $V = \dot{V} = 0 \Rightarrow J = \frac{pc}{q(\theta)} \Rightarrow \dot{J} = (r + \lambda)J - (p - w)$ .

#### 4.4 Endogenous job destruction in DMP search model

Reservation productivity  $R$ , defined by  $J(R) = 0$ . By the reservation property, firms destroy all jobs with idiosyncratic productivity  $x < R$  and continue producing in all jobs with productivity  $x \geq R$ . Therefore the flow into unemployment (job destruction) is given by  $\lambda(R)(1 - u)$ . As before, the flow out of unemployment is equal to job creation,  $m(v, u) = \theta q(\theta)u$ . The evolution of unemployment is therefore given by

equation 6

$$\dot{u} = \lambda G(R)(1 - u) - \theta q(\theta)u$$

And its steady-state value is given by:  $u = \frac{\lambda G(R)}{\lambda G(R) + \theta q(\theta)}$ . The asset value of a job with productivity in the range  $1 \geq x \geq R$  satisfies:  $rJ(x) = px - w(x) + \lambda \int_R^1 J(s) dG(s) - \lambda J(x)$ . For the worker the returns from working at a job with idiosyncratic productivity  $x$  satisfy:  $rW(x) = w(x) + \lambda \int_R^1 W(s) dG(s) + \lambda G(R)U - \lambda W(x)$ . As before, we assume that the wage rate divides the job surplus in fixed proportions at all  $x$ , so the sharing rule that generalizes is:

equation 7

$$(x) - U = B[J(x) + W(x) - V - U]; \forall: 1 \leq x \leq R$$

Noting that all jobs are created at maximum idiosyncratic productivity,  $x = 1$ , the expected profit from a new job vacancy satisfies:  $rV = -pc + q(\theta)[J(1) - V]$ . Here  $q(\theta)$  is the rate at which workers arrive to job vacancies.  $J(1) = \frac{pc}{q(\theta)}$ . The unemployed worker's expected returns as:  $rU = z + \theta q(\theta)[W(1) - U] = z + \frac{\beta}{1-\beta} pc\theta$ .

##### 4.4.1 Wage equation in the endogenous job destruction in DMP search model

This is the wage equation here:

equation 8

$$w(x) = (1 - \beta)z + \beta p(x + c\theta); (r + \lambda)J(x) = (1 - \beta)(px - z) - \beta pc\theta + \lambda \int_R^1 J(s) dG(s).$$

$$(r + \lambda)J(x) = (1 - \beta)p(x - R); (r + \lambda)J(x) \\ = (1 - \beta)(px - z) - \beta pc\theta + \frac{\lambda(1 - \beta)p}{r + \lambda} \int_r^1 (s - R)dG(s).$$

the expected gain from a new job to the firm must be equal to the expected hiring cost that the firm has to pay :  $(1 - \beta) \frac{1-R}{r+\lambda} = \frac{c}{q(\theta)}$ . The job destruction condition is derived by evaluating it at  $x = R$  and substituting the result into the zero-profit condition for the reservation job:  $R - \frac{z}{p} - \frac{\beta c}{1-\beta} \theta + \frac{\lambda}{r+\lambda} \int_R^1 (s - R)dG(s) = 0$ . Suppose that  $z$  is a fixed proportion of the mean wage rate observed in the market, write  $z$  as a proportion of the mean wage, the effect of  $p$  on the reservation productivity disappears i.e.  $z = \rho E[w(x)|x \geq R]$ . Here  $0 \leq \rho \leq 1$  is the replacement rate. Expected value of wage is given as:  $E[w(x)|x \geq R] = (1 - \beta)z + \beta p[E(x|x \geq R) + c\theta]$  and the fixed proportion of mean wage rate is  $z = \frac{\rho\beta}{1-\rho(1-\beta)} p[E(x|x \geq R) + c\theta]$ . The new job destruction condition here is:

equation 9

$$R - \frac{\rho\beta}{1 - \rho(1 - \beta)} (E(x|x \geq R)) - \frac{\beta}{1 - \beta} \frac{c\theta}{1 - \rho(1 - \beta)} + \frac{\lambda}{r + \lambda} \int_R^1 (s - R)dG(s) = 0$$

For the analysis of additive shifts, we suppose that all idiosyncratic productivities  $x$  depend on an additive shift parameter  $h$ , such that  $x(h) = x + h$ . Thus, in examining the effects of a change in the variability of the productivity distribution, we write  $x(h) = x + h(x - \bar{x})$  and  $(1 - \beta)(1 + h) \frac{1-R}{r+\lambda} = \frac{c}{q(\theta)}$  from which follows that  $(1 + h)R - h \bar{x} + \frac{(1+h)\lambda}{r+\lambda} \int_R^1 (s - r)dG(s) = \frac{z}{p} + \frac{\beta}{1-\beta} c\theta$ . By differentiation it can be shown that at  $h = 0$  both market tightness and the reservation productivity rise. Differentiation of previous equation gives:

equation 10

$$\left[ 1 - \frac{\lambda}{r + \lambda} [1 - G(R)] \right] \frac{\partial R}{\partial h} = \bar{x} - R - \frac{\lambda}{r + \lambda} \int_R^1 (s - R)dG(s) + \frac{\beta}{1 - \beta} c (\partial \theta / \partial h)$$

Differentiation with respect to  $h$  gives:  $\frac{c\eta(\theta)}{\theta q(\theta)} \frac{\partial \theta}{\partial h} = \frac{1-\beta}{r+\lambda} \left[ 1 - R - \frac{\partial R}{\partial h} \right]$  and the elasticity notation is given as:  $\eta(\theta) = - \frac{\partial q(\theta)}{\partial \theta} \frac{\theta}{q(\theta)}$  <sup>12</sup>.

#### 4.5 Wage bargain implications

We consider finally the implications of a higher labor share in the wage bargain,  $\beta$ .

equation 11

$$\left[ 1 - \frac{\lambda}{r + \lambda} [1 - G(R)] \right] \frac{\partial R}{\partial \beta} = \frac{1}{1-\beta} \left[ \frac{c\theta}{1-\beta} + \beta c \frac{\partial \theta}{\partial \beta} \right]; \frac{c\eta(\theta)}{\theta q(\theta)} \frac{\partial \theta}{\partial h} = - \frac{1-R}{r+\lambda} - \frac{1-\beta}{r+\lambda} \frac{\partial R}{\partial \beta}$$

the reservation productivity is independent of labor's share, and the net effect of labor's share on market tightness becomes:  $\frac{\partial \theta}{\partial \beta} = - \frac{\theta}{(1-\beta)\eta}$ .

#### 4.6 Capital in the endogenous job destruction model

<sup>12</sup> Furthermore for the average productivity  $\bar{x}$ :  $\left[ 1 - \frac{\lambda}{r+\lambda} [1 - G(R)] \right] (1 - R) - \bar{x} + R + \frac{\lambda}{r+\lambda} \int_R^1 (s - R)dG(s); 1 - \bar{x} - \frac{\lambda}{r+\lambda} \int_R^1 (1 - s)dG(s); 1 - \bar{x} = \int_0^1 (1 - s)dG(s); \bar{x} - R - \frac{\lambda}{r+\lambda} \int_R^1 (s - R)dG(s) + \frac{\beta\theta q(\theta)}{\eta(\theta)} \frac{1-R}{r+\lambda}; \bar{x} - R - \frac{\lambda}{r+\lambda} \int_R^1 (s - R)dG(s) + \frac{\beta}{(1-\beta)\eta(\theta)} c\theta$

Aggregate capital in this economy is : $K = L(1 - u)pk \int_R^1 x dG(x)$  and aggregate output  $F(L(1 - u), K)$ , or in per unit terms : $Y = L(1 - u)pf(k) \int_R^1 x dG(x)$ . The job creation condition, which as before satisfies it is:  $(1 - \beta) \frac{1-R}{r+\lambda} [f(k) - (r + \delta)k] = \frac{c}{q(\theta)}$ . The job destruction condition is derived from :  $[f(k) - (r + \delta)k] \left[ R + \frac{\lambda}{r+\lambda} \int_R^1 (s - R) dG(s) \right] = \frac{z}{p} + \frac{\beta}{1-\beta} c\theta$ . Some empirical studies of job flows to support this theory include: Leonard (1987) and Dunne, Roberts, and Samuelson (1989) for the United States; Konings (1995) and Blanchflower and Burgess (1993) for the United Kingdom; Boeri and Cramer (1992) for Germany; Broersma and den Butter (1994) and Gautier (1997) for the Netherlands; Lagarde, Maurin, and Torelli (1994) for France; Albaek and Sorensen (1995) for Denmark; and Contini et al. (1995) for countries of the European Union. This model is based on Mortensen, Pissarides (1994)

#### 4.6 Search intensity and job advertising

Now, the job matching technology as:  $m = m(su, av)$  also the efficiency units of job vacancies as  $av$ , thus, let  $s$  be a variable measuring the intensity of search by workers, and let  $a$  be a variable measuring job advertising. Key equations here are:  $q_i^w = \frac{s_i}{s_u} m(su, av)$ ;  $\theta = \frac{v}{u}$ ;  $q_i^w = q^w(s_i, s, a\theta)$ . Index  $i$  denotes worker,  $s_i$  are efficiency search units. The process that transfers a job from a vacant state to a filled one for each efficiency unit of advertising supplied is Poisson with rate  $m(su, av)/av$ . Transition probability in unit time is :  $q_j = \frac{a_j}{a_v} m(su, av)$ . The equilibrium condition for unemployment, the Beveridge curve, is given, as before, by :  $u = \lambda/(\lambda + q\theta(s, a, \theta))$ . In general, we assume that the cost of  $s_i$  units of search is  $\sigma_i$ , where:  $\sigma_i = \sigma(s_i, z)$ ,  $\sigma_s(s_i, z) > 0$ ,  $\sigma_{ss}(s_i, z) > 0$ ,  $\sigma_z(s_i, z) \geq 0$ <sup>13</sup>. We assumed that the cost of a vacancy,  $pc$ , is out of the control of the firm. Here we assume that the cost depends on the level of advertising that the firm chooses for the job. We write:  $c = c(a_j)$ ,  $c'(a_j) > 0$ ,  $c''(a_j) \geq 0$ . The firms expected profit from one more job vacancy is:  $rV_j = -pc(a_j) + q(a_j, \cdot)(J - V_j)$  also,  $-pc(a_j) + \frac{\partial q_i}{\partial a_j}(J - V_i) = 0$ , and  $\frac{\partial q_i}{\partial a_j} = \frac{q(s, a, \theta)}{a}(J - V_j) = 0$ . The final result about the choice of advertisement is :

equation 12

$$J = \frac{pc(a_j)}{q(a_j, \cdot)}; \frac{c'(a) a}{c(a)} = 1$$

wages are given by :  $w = (1 - \beta)[z - \sigma(s, z)] + \beta p(1 + c\theta)$ . And the steady state search effort and unemployment are :  $s\sigma_s(s, z) = \frac{\beta}{1-\beta} pc\theta$ ;  $u = \frac{\lambda}{\lambda + q(s, a, \theta)}$ . If  $z$  represents entirely the imputed value of leisure, then net income during unemployment,  $z - \sigma(s, z)$ , has to be recalculated as the imputed value of total hours net of the hours of search,  $h(s)$ ,  $z$  is the unemployment income. In the linear case the marginal cost of search is  $\sigma_s(s, Z) = zh'(s)$ , and so the condition for equilibrium intensity, becomes  $zsh'(s) = \frac{\beta}{1-\beta} pc\theta$ ,  $z = \rho w$   $cs = \rho wh'(s)$  from

where  $w = \frac{\beta(1+c\theta)}{1-(1-\beta)[b+\rho(1-h(s))]} p$ . The equilibrium condition for search intensity then becomes:

equation 13

$$\frac{\rho s(1 + c\theta)h'(s)}{1 - (1 - \beta)[b + \rho(1 - h(s))]} = \frac{c\theta}{1 - \beta}$$

<sup>13</sup> The optimal  $s_i$  satisfies :  $-\sigma_s(s_i, z) + \frac{\partial q_i^w}{\partial s_i}(W - U_i) = 0$ ;  $W - U = \frac{w+z+\sigma(z,s)}{r+\lambda+q^w(s,a,\theta)}$ ;  $-\sigma_s(s_i, z) + \frac{w+z+\sigma(z,s)}{r+\lambda+q^w(s,a,\theta)} \frac{q^w(s,a,\theta)}{s} = 0$



#### 4.7 Stochastic job matchings

When the jobs and workers are brought together, one pair may be more productive than the other. The new feature now introduced is the ex post match specific heterogeneity. We refer to this extension of the model as stochastic job matchings. Then the rate of job contacts is given by  $m = m(u, v)$ . Because all firms and workers are ex ante identical, the reservation productivity  $\alpha_r$  is common to all job-worker pairs. So if all productivities  $\alpha \geq \alpha_r$  are accepted, the fraction of acceptable job contacts is:  $\int_{\alpha_r}^1 dG(\alpha) = 1 - G(\alpha_r)$ . Process of arriving at the job  $q = [1 - G(\alpha_r)] \frac{m(u, v)}{v} = [1 - G(\alpha_r)]m$ ,  $[1 - G(\alpha_r)]m$  is the rate of job matching. And workers move from unemployment to employment at the rate *equation 14*

$$q^w = [1 - G(\alpha_r)] \frac{m(u, v)}{v} = [1 - G(\alpha_r)]\theta q(\theta)$$

##### 4.7.1 The choice of reservation wage

In general, the wage rate offered will depend on the productivity of the job match<sup>14</sup>,  $w_j = w(a_j)$ ;  $w_r = w(\alpha_r)$ ;  $q_i^w = \theta q(\theta)[1 - G(\alpha_{ri})]$ . The reservation wage then becomes: *equation 15*

$$w_{ri} = \frac{(r+\lambda)z + q_i^w w_i^e}{r+\lambda+q_i^w}; w_r = \frac{(r+\lambda)z + q^w w^e}{r+\lambda+q^w}$$

##### 4.7.1 The choice of hiring standard

Hiring standard  $a_f$  satisfies:  $a_f^e = E(a|a \geq a_f)$ . Where:  $rJ_f^e = pa_f^e - w_f^e - \lambda J_f^e$ ;  $rJ_j = pa_j - w(a_j) - \lambda J_j$  and  $rV = -pc + q_f(J_f^e - V)$ .  $q_f$  is the rate at which vacant job becomes filled:  $q_f = q(\theta)[1 - G(a_f)]$  and  $J_f^e = \frac{pc}{q_f}$ . The net effect is:  $\frac{\partial \alpha_r}{\partial \beta} = \frac{(r+\lambda)c\theta}{(1-\beta)^2} \frac{\eta(\theta) - \beta}{\beta\theta q(\theta)[1 - G(\alpha_r)] + (r+\lambda)\eta(\theta)}$ .

The slope of probability of leaving employment is given as:  $\frac{\partial q^w}{\partial \theta} = q(\theta)(1 - \eta)[1 - G(\alpha_r)] - \theta q(\theta)q(\alpha_r) \frac{\beta c}{1-\beta}$ , or in simplified terms:

*equation 16*

$$\frac{\partial q^w}{\partial \theta} = \frac{\partial q^w}{\partial \theta} \frac{\partial \theta}{\partial \beta} - \theta q(\theta)g(\alpha_r) \frac{c\theta}{(1-\beta)^2}$$

#### 4.8 The role of policy

We will follow a simple approach to the modeling of hiring and firing taxes by assuming that the firm that hires a worker whose initial (general) productivity is  $p$  receives a hiring subsidy of  $pH$ , and when the separation takes place, it has to pay a tax  $pT$ . Unemployed workers receive some compensation, which is policydetermined. We assume that the policy parameter is the after-tax replacement rate<sup>15</sup>, that is, the ratio of net unemployment benefit to average net income from work. We define the net unemployment benefit  $b$  by:

*equation 17*

$$b = \rho[w - T(w)]$$

<sup>14</sup> The net worth of unemployed worker  $i$  satisfies:  $rU_i = z + q_i^w(W_i^e - U_i)$   $rW_j = w_j + \lambda(U_i - W_j)$

<sup>15</sup> Here we introduce the possibility of progressive or regressive taxation by assuming that if the gross wage at a job  $j$  is  $w_j$  the net wage received by the worker is  $(1 - t)(w_j + \tau)$ . the net transfer from the worker to the tax authorities is  $T(w_j) = tw_j - (1 - t)\tau$ .

Where  $-T(w)$  is the average net wage rate,  $\rho$  is the policy parameter  $0 \leq \rho \leq 1$ . The firms net worth from a vacancy and from job paying  $w_j$  are given by:  $rV = -pc + q(\theta)(J + pH - V)$ ;  $rJ_j = p + a - w_j - \lambda(J_j + pF)$ . Hiring subsidy of  $pH$ , Employment is subsidized at the rate  $a$  per job, firing tax  $pF$ , tax subsidy  $\tau$ , the replacement rate  $\rho$ , marginal tax rate  $t$ . Therefore the initial wage is chosen to maximize the product:  $B_0 = (W_j - U)^{\beta}(J - j + pH - V)^{1-\beta}$ . But after the worker is taken on, the benefit to the firm from continuation of the contract is only  $J_j$  since no further hiring subsidies are received. In contrast, now the firing tax becomes operational, and if the firm fails to agree to a continuation wage, its loss will be  $J_j + pF$  and  $B(W_j - U)^{\beta}(J_j + pF - V)^{1-\beta}$ . Following the terminology introduced in the literature by Lindbeck and Snower (1988), we refer to  $w_{0j}$ , as the “outside” wage and to  $w_j$  as the “inside” wage:  $w_{0j}$ , is negotiated by those still outside the firm, before the firm gets locked in by turnover taxes, and  $w_j$ , is negotiated by those inside the firm, who benefit from the firing restrictions imposed on the firm. Given our assumptions, the outside (initial) wage solves:  $\beta \frac{\partial W_j}{\partial w_{0j}}(J_j + pH - V) + (1 - \beta) \frac{\partial J_j}{\partial w_{0j}}(W_j - U) = 0$  and the inside (continuation) wage solves  $\beta \frac{\partial W_j}{\partial w_{0j}}(J_j + pF - V) + (1 - \beta) \frac{\partial J_j}{\partial w_j}(W_j - U)$ . In the presence of taxes:  $\frac{\partial W_j}{\partial w_{0j}} = \frac{\partial W_j}{\partial w_j} = \frac{1-T'(w_j)}{r+\lambda}$  and  $\frac{\partial J_j}{\partial w_{0j}} = \frac{\partial J_j}{\partial w_j} = -\frac{1}{r+\lambda}$ . Imposing  $V = 0$  and  $w_j = w$  for all  $j$ —are:

equation 18

$$w_0 = \frac{1 - \beta}{1 - \rho(1 - \beta)} \left[ \frac{z}{1 - t} - (1 - \rho)\tau \right] + \frac{\beta}{1 - \rho(1 - \beta)} [(1 + c\theta - \lambda F + (r + \lambda)H)p + a]$$

$$w = \frac{1 - \beta}{1 - \rho(1 - \beta)} \left[ \frac{z}{1 - t} - (1 - \rho)\tau \right] + \frac{\beta}{1 - \rho(1 - \beta)} [(1 + c\theta - rF)p + a]$$

Equilibrium with policy now is given as:  $p + a + \tau - \lambda pF + (r + \lambda)pH = \frac{z}{(1 - \rho)(1 - t)} + \frac{pc}{(1 - \rho)(1 - \beta)} (\beta\theta[1 - (1 - \beta)\rho] \frac{r + \lambda}{q(\theta)})$  and  $u = \frac{\lambda}{\lambda + \theta q(\theta)}$ . Job destruction with policy is given as:  $rJ(x) = px - w(x) + \lambda \int_R^1 J(s) dG(s) - \lambda J(x)$ . The Nash wage bargaining equation is given as:  $w(x) = (1 - \beta)z + \beta(x + c\theta)p$ . Unemployment compensation with taxes is:  $b = \rho(1 - t)[E(w(x)|x \geq R) + \tau]$ ;  $b = \rho(1 - t)(p + \tau)$ . The outside wage is given as:  $w_0 = 1 - \beta \left[ \frac{z}{1 - t} - (1 - \rho)\tau + \rho p \right] + \beta[(1 + c\theta - \lambda F + (r + \lambda)H)p + \beta a]$ . The inside wage is:  $w = (1 - \beta) \left[ \frac{z}{1 - t} - (1 - \rho)\tau + \rho p \right] + \beta[(x + c\theta + rF)p + \beta a]$ . Reservation productivity is:  $J(R) + pF = 0$  and  $J(x) = (1 - \beta) \frac{p(x - R)}{r + \lambda} - pF$ . The job destruction rule with policy now becomes:

equation 19

$$R + \frac{a + (1 - \rho)\tau}{p} = \rho + rF - \frac{z}{p(1 - t)} - \frac{\beta c}{1 - \beta} \theta + \frac{\lambda}{r + \lambda} \int_R^1 (s - R) dG(s) = 0$$

To close the model, we need to derive the equation for market tightness (job creation). And it goes as follows:

equation 20

$$w_0 - w(R) = \beta[1 - R + (r + \lambda)(H - F)p]$$

$$(r + \lambda)[J^0 - J(R)] = (1 - \beta)(1 - R)p - \beta(r + \lambda)(H - F)p$$

$$J^0 = \frac{pc}{q(\theta)} - pH$$

$$(1 - \beta) \left( \frac{1 - R}{r + \lambda} - F + H \right) = \frac{c}{q(\theta)}$$

The net subsidy to hiring and firing,  $H - F$ , increases  $\theta$ . And the steady state employment is:  $u = \frac{\lambda G(R)}{\lambda G(R) + \theta q(\theta)}$ . Compensating Policy Changes follow:  $F = H$  and  $a + (1 - \rho)\tau - \rho p + rpF - \frac{z}{1-t} = -z$  :  $a + \tau = -rpF + \frac{t}{1-t}z + \rho(p + \tau)$ . Tax subsidy  $\tau$  should be chosen to :  $\tau = \frac{\rho}{1-\rho}p$  and  $\tau = \frac{t}{1-t}z + \rho(p + \tau)$  where  $\tau = \frac{tz+b}{1-t}$ . The net revenue raised by the government is :  
*equation 21*

$$T = [tw^e - (1 - t)\tau](1 - u) - ub$$

$w^e$  is conditional expectation pre-tax wage  $T = t(w^e - z)(1 - u) - b$  and pre-tax wage rate for given  $x$  is also:  
*equation 22*

$$w(x) = (1 - \beta) \left( \frac{z + b}{1 - t} - \tau \right) + \beta(x + c\theta + rF) + \beta a$$

$$w(x) = (1 - \beta)z + \beta(x + c\theta)$$

Optimal subsidy is given as:  $H = F + \left( \frac{1}{1-\beta} - \frac{1}{1-\eta} \right) \frac{c}{q(\theta)}$  and  $a + \tau = \rho(p + \tau) + \frac{t}{1-t}z - rpF + \left( \frac{\beta}{1-\beta} - \frac{\eta}{1-\eta} \right) cp\theta$ . It follows that the reservation productivity  $R$  with policy intervention is higher than in the policy-free environment if :  $a + (1 - \rho)\tau - \rho p + rpF - \frac{z}{1-t} < -z$  ;  $a + \tau < \frac{tz+b}{1-t} - rpF$  .). The effect on job creation is neutralized if hiring subsidies and firing taxes are chosen such that  
*equation 23*

$$-\frac{dR}{r + \lambda} - F + H = 0$$

On unemployment insurance see, for example, the papers in the Phelps(1970) volume which address the positive aspects of the question of unemployment compensation and search, as do numerous papers on partial models of search; see, for example, Mortensen (1977) and the other papers collected in the same issue of the journal<sup>16</sup>.

## 5. Labor and wages in Real business cycles model (RBC)

Kydland, and Prescott introduced three revolutionary ideas in their (1982) paper "Time to Build and Aggregate Fluctuations.", see Rebelo (2005). The first is the business cycles models can be studied in general equilibrium framework. Second, is the possible unification between growth theory and business cycles theory, and that business cycle models must be consistent with the empirical regularities of long-run growth. And the third is the possibility of calibration with the parameters drawn and generating the artificial data that can be compared with the original data. Here we will simulate RBC model with habits and RBC model presented by the IRF function and we will outline some characteristics of the wages and employment in relation to spending and productivity shocks.

### 5.1 Real business cycles with habits

Economy is populated by a large number of households  $j \in [0,1]$ , the utility function of the representative household  $j$  is given as:

<sup>16</sup> For empirical work on the effect of unemployment compensation on search activity, see the survey by Devine and Kiefer (1991), the book by Layard, Nickell and Jackman (1991), and the evaluations by Atkinson and Micklewright (1985, 1991).

equation 24

$$u(c_t(j), h_t(j)) = \frac{c_t(j)^{1-\sigma^c}}{1-\sigma^c} - \frac{h_t(j)^{1+\frac{1}{\sigma^L}}}{1+1/\sigma^L}$$

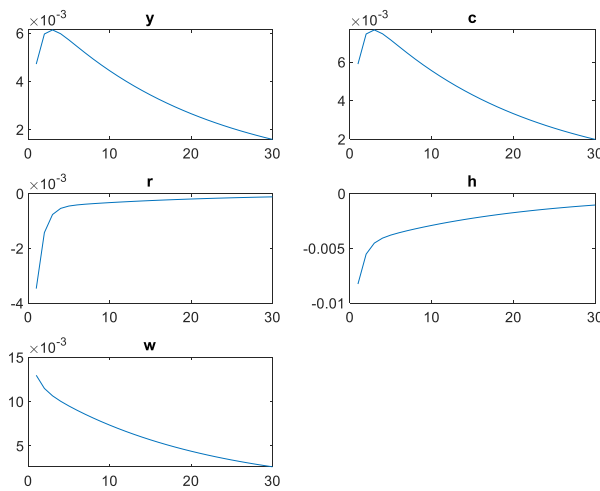
Where  $\sigma^c$  is the risk aversion and  $\sigma^L$  is the frischian elasticity of labour<sup>17</sup>,  $u(\cdot)$  is increasing in consumption  $c_t(j)$  and decreasing in hours worked  $h_t(j)$ . Welfare index is defined as a sum of current and expected utilities:

equation 25

$$\mathcal{W}_t(j) = \sum_{\tau=0}^{+\infty} \beta^\tau u(c_{t+\tau}(j), h_{t+\tau}(j))$$

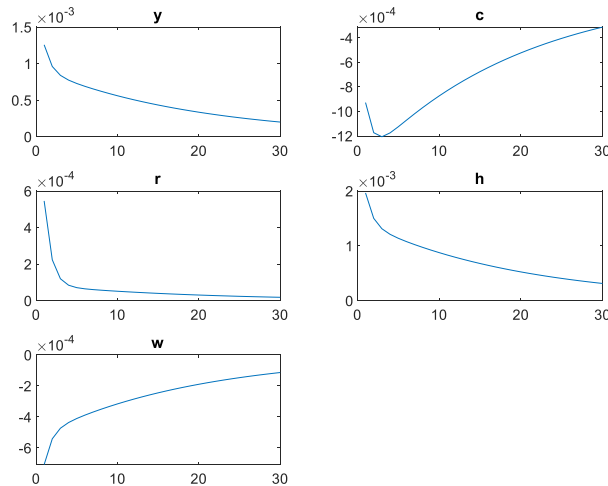
Additionally, the production technology follows a Cobb-Douglas technology:  $y_t(j) = e^{\varepsilon_t^A} h_t(j)^{1-\alpha}$ . Where  $\varepsilon_t^A \sim \mathcal{N}(0, \sigma_{A,t}^2)$  is an IID exogenous disturbance associated with a productivity shock. The resources constraint is given by the demand from households and authorities and it is equal to:  $y_t = c_t + g^y \bar{y} e^{\varepsilon_t^G}$ . Where  $\varepsilon_t^G$  is a IID normal shock,  $\bar{y}$  is the steady-state level of GDP, and  $g^y$  is the spending to GDP ratio. Basic parameters for RBC model are:  $\alpha = 0.36$  (capital factor);  $\beta = 0.99$ ,  $g^y = 0.2$ ;  $\sigma^c = 2.5$ ;  $\sigma^L = 0.5$ , and habit parameter

Figure 1 RBC model with consumption habits and productivity shock  $VC(1,1) = 0.01^2$



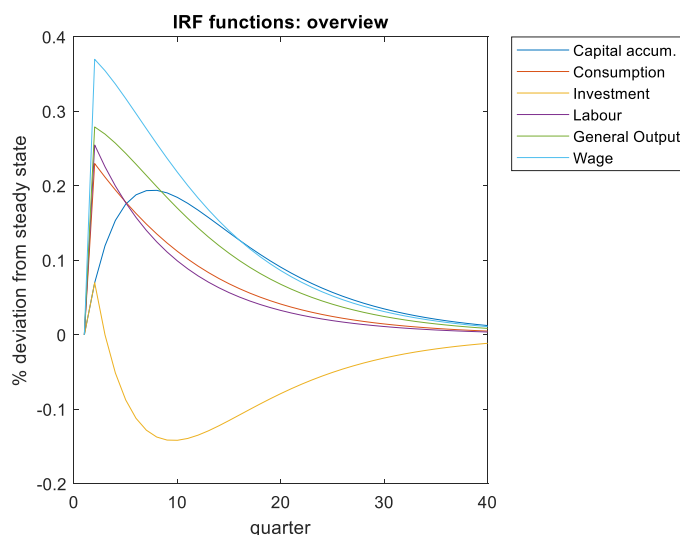
<sup>17</sup> The Frisch elasticity measures the relative change of working hours to a one-percent increase in real wage, given the marginal utility of wealth  $\lambda$ . In the steady-state benchmark model is given as:  $\frac{dh/h}{dw/w} = \frac{1-h}{h} \left( \frac{1-\eta}{\eta} \theta - 1 \right)^{-1}$

Figure 2 RBC model with consumption habits and spending shock  $VC(1,1) = 0.01^2$



Variance covariance matrix for shocks, for productivity shock  $VC(1,1) = 0.01^2$  and for spending shock  $VC(2,1) = 0.01^2$ . So in the fig.1 as production falls, real interest rate rises, same with labor hours or labor supply. Since the productivity shock many workers are unemployed. Also, real wage decreases with consumption decreasing also. Real interest rate here may be causing productivity fall, and downward real wage. However, with government spending shock(endogenous), productivity increases, also real wage is rising. While the real interest rate is failing. Next graph, shows the movement of 6 macro variables and their IRF functions<sup>18</sup> labor market in RBC framework is presented by two variables: Labour and wage.

Figure 3



The IRF functions of all six macro variables: Capital accumulation, Consumption, Investment, Labour, General output and Wage, shows that each of the six variables after the shock in the

<sup>18</sup> The irf function returns the dynamic response, or the impulse response function (IRF), to a one-standard-deviation shock to each variable in a VAR(p) model. A fully specified varm model object characterizes the VAR model.



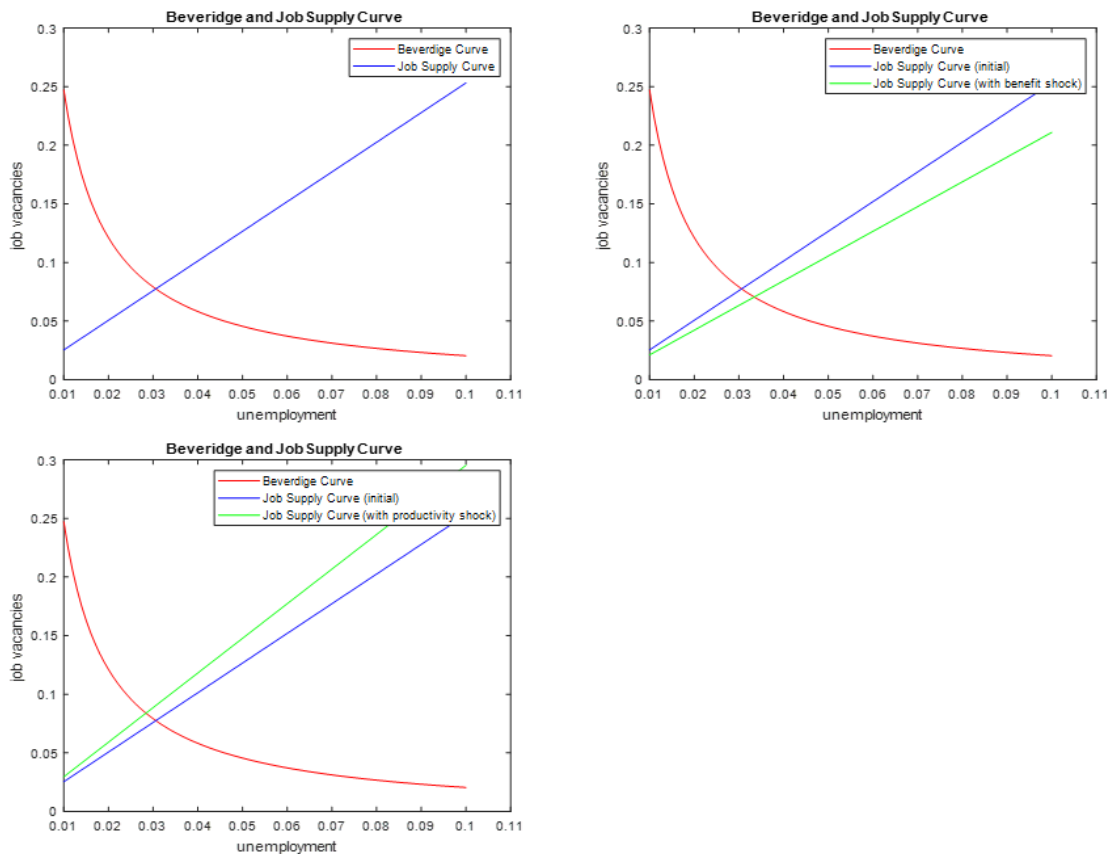
error term converges to steady-state (around 0% deviation from steady-state) in around 40 quarters.

## RESULTS AND DISCUSSION

### 6.Diamond-Mortensen -Pissarides (DMP) simulation

Diamond-Pissarides-Mortensen model is a dynamic version of labor market depiction. In the next three figures are presented: in fig.4 Beveridge curve and job supply curve, in fig.5 Beveridge curve and job supply curve with benefit shock, and in fig.6 Beveridge curve and job supply curve (initial) and job supply curve (productivity shock). Diamond-Mortensen-Pissarides search model and calculates a Beveridge curve (mathematical description of the labor market)

Figure 4 DMP model dynamics (with benefit shocks and productivity shocks)



The association between unemployment productivity and benefits in DMP framework is as follows. Constant returns matching function is  $M(uL, vL)$  where  $uL$ -are unemployed,  $vL$ -are vacancies and  $M(uL, vL) = vL \cdot M\left(\frac{u}{v}, 1\right)$  and  $\theta \equiv \frac{v}{u}$  and the vacancy filling rate is:  $q \equiv \frac{M}{vL} = M\left(\frac{u}{v}, 1\right) = M\left(\frac{1}{\theta}, 1\right) = q(\theta)$ . And unemployed exit hazard is:  $\theta q(\theta) = \frac{M}{uL}$ , where  $\theta q(\theta) \rightarrow 0$  as  $\theta \rightarrow 0$  and  $\theta q(\theta) \rightarrow \infty$  as  $\theta \rightarrow \infty$ . Value of job vacancy is:  $J = \frac{c}{q(\theta)}$ ,  $1/q(\theta)$  is the expected time to fill a vacancy, and  $c$  are the cost per period. Where  $c = y - w$ ; where  $y$  is output and  $w$  are the wages, and if we know that  $rV = -c + q(\theta)(J - V)$ , where  $J$ -is the value of filled vacancy, and  $V$  is the value of unfilled vacancy. Now if we assume that  $V = 0$  than  $J = c/q(\theta)$ . Now if we equate job creation  $\theta q(\theta) \times uL$  and job destruction rate  $\delta(1 - u)L$  we get equilibrium unemployment equation such as:

equation 26

$$u = \frac{\delta}{\delta + \theta q(\theta)} = \frac{\delta}{\delta + \left(\frac{v}{u}\right) q\left(\frac{v}{u}\right)}$$

Where  $q(\theta) = \frac{(r+\delta)c}{y-w}$  so that we can write:  $u = \frac{\delta}{\delta + \theta \frac{(r+\delta)c}{y-w}} \Rightarrow \frac{\delta(y-w)}{\delta(y-w) + \theta(r+\delta)c} = 1 + \frac{\delta(y-w)}{\theta(r+\delta)c}$ . If

we take logs from both sides:

equation 27

$$\begin{aligned} \ln(u) &= \ln\left(\frac{\delta(y-w)}{\theta(y-w) + \theta(r+\delta)c}\right) = \ln(\delta) + \ln(y-w) - \ln\theta(y-w) - \ln(\theta) - \ln(r+\delta) \\ &\quad - \ln(c) \\ &= \ln(\delta) + \ln(y) - \ln(w) - \ln(\theta)y - \ln(\theta) - \ln(r+\delta) - \ln(c) = \ln(\delta) + \ln(y) \\ &\quad - \ln(w) - \ln(\theta)y - \ln(\theta) - \ln(r+\delta) - \ln(y) + \ln(w) = \ln(\delta) - \ln(\theta)y \\ &\quad - \ln(\theta) - \ln(r+\delta) \end{aligned}$$

For the association benefits and unemployment, the solution might be straightforward, since the value of unemployment is  $rU = b + y(\theta)[W - U]$ , where  $w$  is intertemporal value of employment and  $u$ -is intertemporal value of unemployment and  $rW = w - \theta(W - U)$ , and  $b$  are unemployment benefits. And now from previous we know that following applies:

equation 28

$$\begin{aligned} W - U &= \frac{\beta}{1-\beta}(J - V) \Leftrightarrow (r+\delta)(W - U) = (r+\delta)(J - V) \\ &\Leftrightarrow (r+\delta)(w - b + \theta q(\theta)(W - U)) = y - w \end{aligned}$$

For a free entry we have  $J = \frac{c}{q(\theta)}$ , and  $W - U = \frac{\beta}{1-\beta} \frac{c}{q(\theta)}$ ; and the wage equation now becomes:  $w = (1-\beta)b + \beta(y + c\theta)$  where  $\beta$  is the bargaining power of labor. If  $\beta = 1$  real wage is equal to productivity + average search costs  $\frac{cv}{u}$ . If  $\beta = 0$  real wage is equal to unemployed income. Labor market equilibrium is established on the intersection between wage setting curve (labor supply curve) and free entry conditions (which is approximately equal to labor demand curve), and now:

equation 29

$$\begin{aligned} w &= (1-\beta)b + \beta(y + c\theta) \\ (1-\beta)(y - b) &= \frac{c}{q(\theta)}[\delta + r + \beta\theta q(\theta)] \end{aligned}$$

Or if we define unemployment to be supply minus demand for labor i.e  $u = w - (1-\beta)(y - b)$  and if we simplify  $u = w - (y - b - \beta y + \beta b) = w - y + b - \beta y - \beta b$  and :

equation 30

$$\begin{aligned} u &= (1-\beta)b + \beta(y + c\theta) - \left(\frac{c}{q(\theta)}[\delta + r + \beta\theta q(\theta)]\right) \\ &\Rightarrow b - \beta b + \beta y + c\beta\theta - \left(\frac{c\delta}{q(\theta)} + \frac{cr}{q(\theta)} + c\beta\theta\right) = b - \beta b + \beta y - c \frac{(\delta + r)}{q(\theta)} \end{aligned}$$

Since  $b - \beta b > 0$  since we know that labor bargaining power ideally is around  $\beta = \frac{1}{2}$ .

## 7. Bhattacharya et al. (2017) model version of DMP model

The number of new hires  $h_t$  is equal to :

equation 31

$$h_t = A\sqrt{u_t \cdot v_t}$$

Where  $A$  is the efficiency of the matching process;  $u_t$  are the unemployed and  $v_t$  are vacancies. The job finding rate  $f_t$  is equal to:

equation 32

$$f_t = \frac{h_t}{u_t} = \frac{A\sqrt{u_t \cdot v_t}}{u_t} = A \cdot \sqrt{\theta}$$

Where  $\theta = \frac{v_t}{u_t}$  is the market tightness. The number of unemployed workers in  $t + 1$  is  $u_{t+1} = (1 - f_t)u_t + \lambda e_t$ ,  $e_t$  are the employed workers but  $\lambda e_t$  are employed workers who became separated from their job at time  $t$ . The law of motion of unemployment can be written as:

equation 33

$$\dot{u} = (1 - A \cdot \sqrt{\theta} - \lambda)u_t + \lambda$$

And the Beveridge curve BC relationship is :

equation 34

$$u = \frac{\lambda}{A \cdot \sqrt{\theta} + \lambda}$$

The probability  $q$  that a firm fills a vacancy in a given period is found by using the matching function:

equation 35

$$q_t = \frac{h_t}{v_t} = \frac{A\sqrt{u_t \cdot v_t}}{v_t} = \frac{A}{\sqrt{\theta}}$$

An employed worker produces  $y$  units of output each period and is paid a wage  $w$  and so the period profit to a firm from a filled job is  $y - w$ . Firms incur a cost  $\kappa$  each period that they advertise a job vacancy.

equation 36

$$\kappa = q_t(y - w) \frac{1}{\lambda}$$

$\frac{1}{\lambda}$  is the expected life if vacancy, and the vacancy setting equation (curve) is :

equation 37

$$\theta = \left[ \frac{A}{\kappa} \left( \frac{y - w}{\lambda} \right) \right]^2$$

And finally, the wage setting relation WS is given as:

equation 38

$$w = \beta(y + \theta\kappa) + (1 - \beta)b$$

Where  $\beta$  is the labor bargaining power.

Figure 5 Increase in cost of advertising a vacancy  $\kappa$

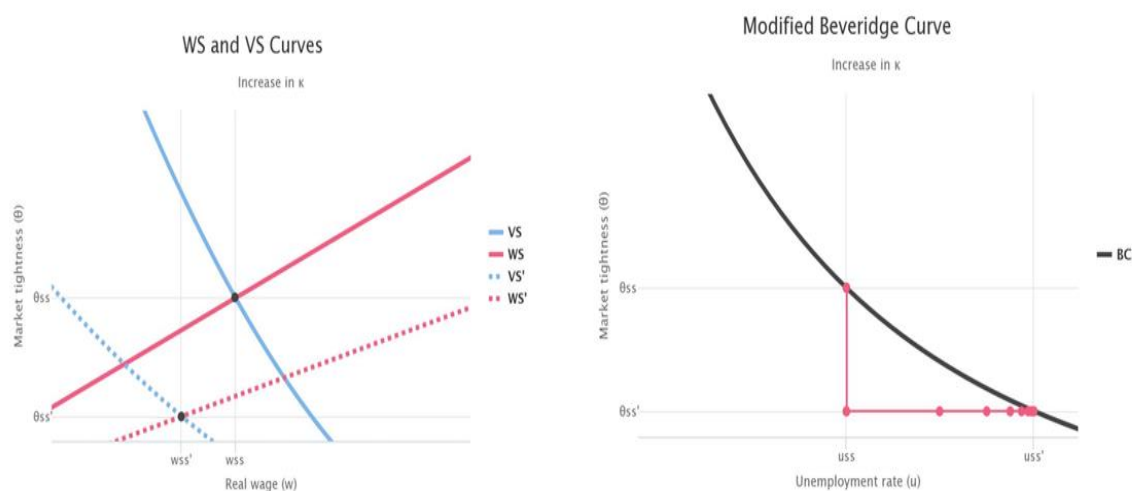


Figure 6 Increase in matching efficiency  $A$

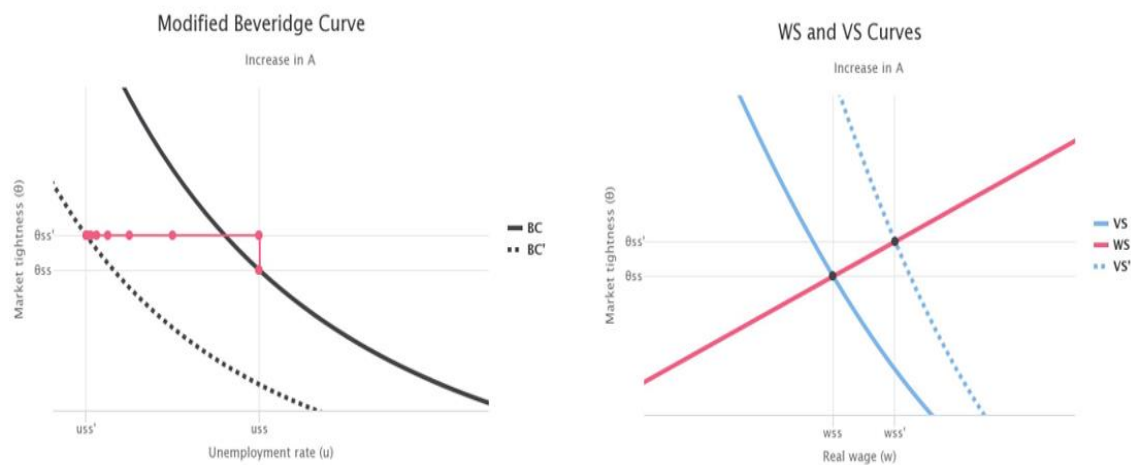


Figure 7 Increase in labor bargaining power  $\beta$

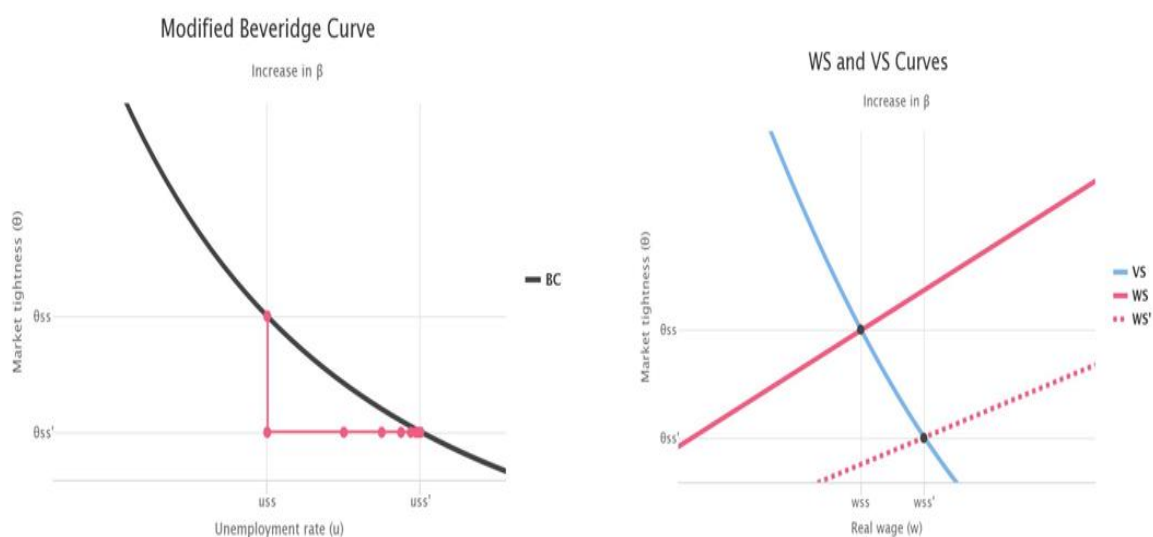


Figure 8 Increase in unemployment benefits  $b$  the value to the worker of not being employed

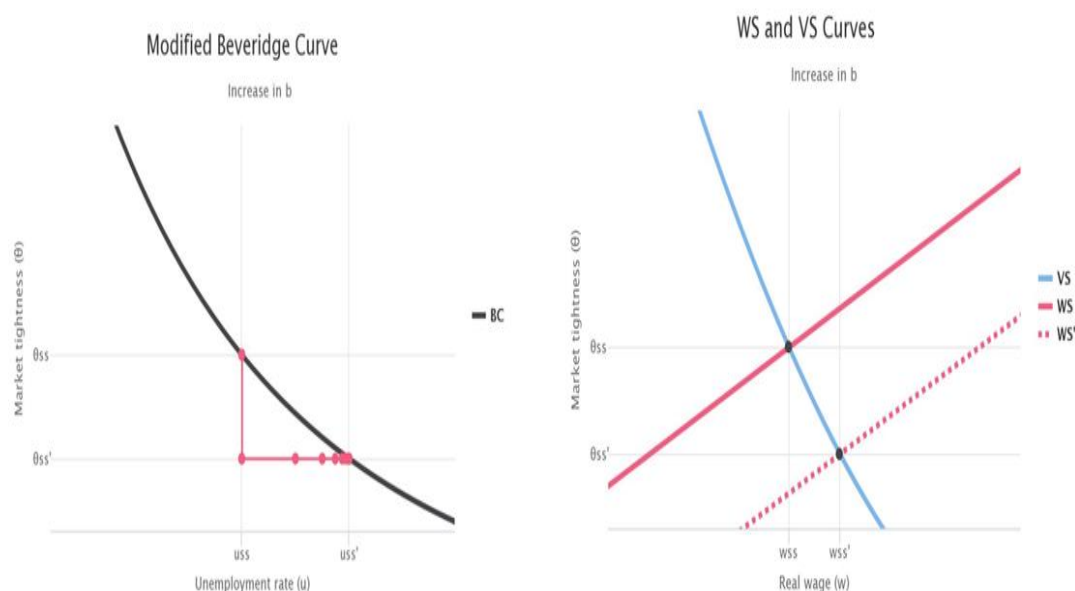




Figure 9 Increase in the exogenous separation rate  $\lambda$

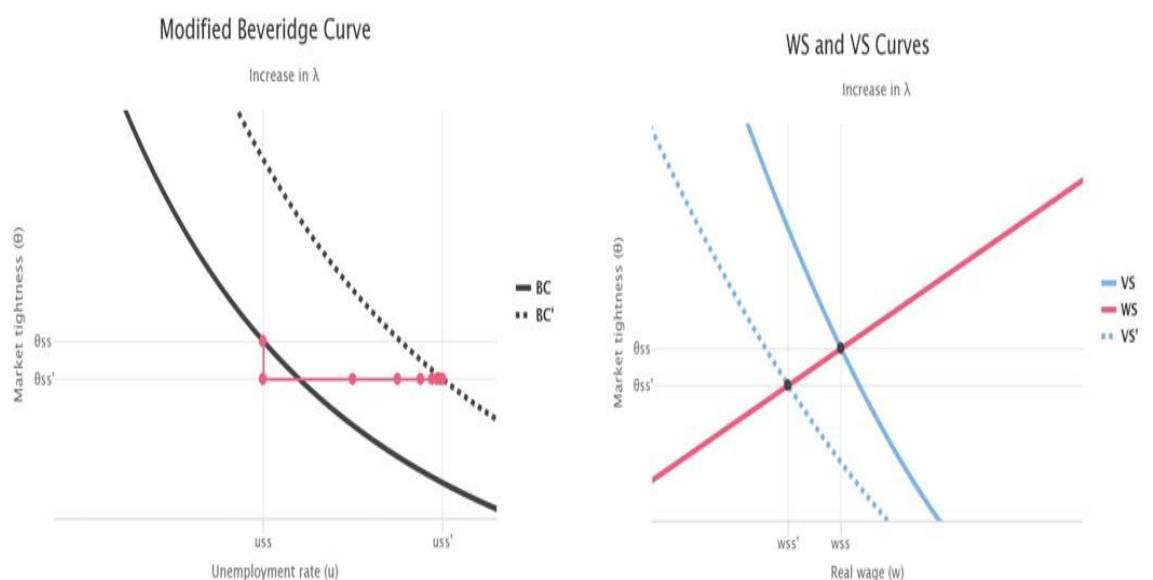
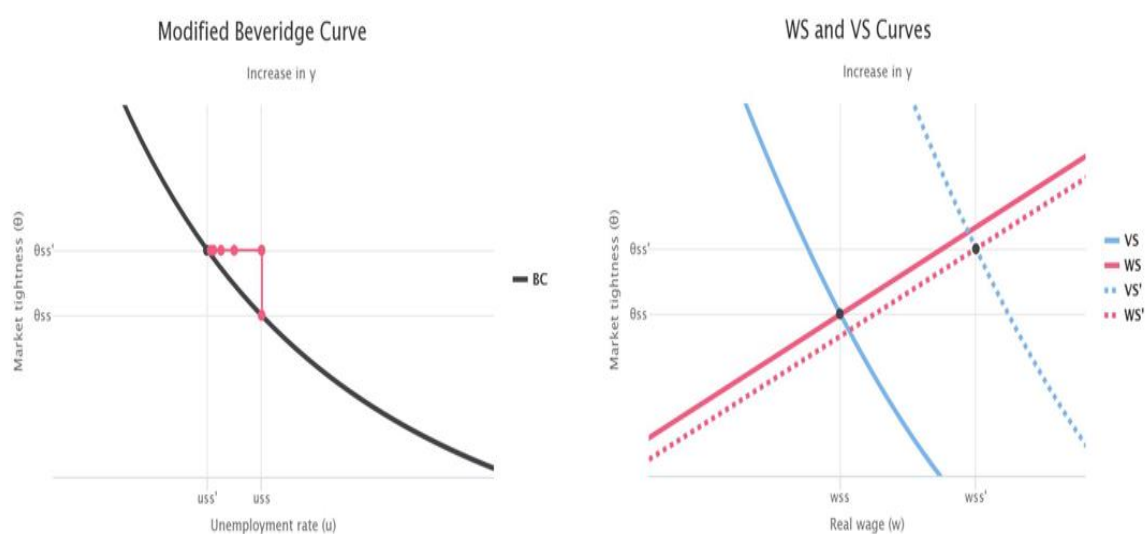


Figure 10 Increase in the units of output in each period  $y$



## 8.Conclusion

In the case of productivity shock or aggregate demand shock negative movement between vacancies and unemployment along the Beveridge curve exists, and this is due to aggregate demand shock. This is along with the traditional positive co-movement that is due to efficiency of the matching process. Opposite movements between Vacancy setting curve (VS) and unemployment rate( $u$ ) as well as market tightness  $\theta$  occur during negative productivity shock (recession). Other variables such as matching efficiency for instance also provides opposite movement between VS curve and  $\theta$ . Labor bargaining power case VS curve and  $\theta$  to move in same direction, though unemployment will rise but also real wage  $w$  will rise, and a rise (downward) movement along the vacancy setting VS-curve. With the increase of the vacancy advertising costs  $\kappa$ , vacancy setting curve moves to the left, market tightness  $\theta$  is decreasing, while the wage setting curve moves downwards and intersects now to the right with the VS curve, meaning growth in real wage. Bargaining power of labor  $\beta$ , increases real wage  $w$ , decreases market tightness  $\theta$  and causes downward movement along the vacancy setting curve VS. Exogenous separating rate  $\lambda$  moves vacancy setting curve VS to the left and causes negative movement towards the graph origin on to the wage setting curve WS, Also in this case market tightness decreases as unemployment rises. Some of the empirical paper draw similar conclusions e.g. Pater (2017). Unemployment benefits  $b$  on the other hand, cause a decrease in market tightness and increase in unemployment. In the MATLAB simulation of the DMP model benefit shock caused movement in the job supply curve downwards to the right which as consequence increased the unemployment rate and lowered the job vacancies curve. While in the same version of DMP model productivity shock caused job supply curve to move to the left and unemployment rate was decreasing, while the equilibrium vacancies were increased. RBC model proved that labor, and wages, along with 4 other macroeconomic variables: capital accumulation, consumption, investment and general output, converge to steady-state in 40 quarters. New- Keynesian models with habits was tested when in presence of productivity shock and consumption shock. In the former working hours  $h$  increased, as output per capita  $y$  declined, and interest rate  $r$  rose, while the real wages fall along with consumption per capita  $c$ . In the latter working hours  $h$  decreased, and interest rate  $r$  decreased, real wages  $w$  were increasing, along with consumption per capita  $c$ . Results from this paper are ambiguous at best to us whether unemployment is inadequate aggregate demand problem or mismatch problem. But the main conclusion is that DMP model as a central component of contemporary macroeconomics, also is most realistic account of unemployment. Its building blocks are three (see also Hall (2012)), namely: first it is a stochastic model of labor turnover, workers become unemployed (separate from jobs), and find new jobs, second it is a model of labor market tightness, where employers are choosing job creation volumes and are exerting recruiting efforts that control the job finding rate, in response to the payoff to job creation, and third it is a bargaining model of wage determination that sets incentive to create jobs because of the difference between workers' productivity and workers' wages.

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