UNIVERSITY OF NOVI SAD
TECHNICAL FACULTY
"MIHAJLO PUPIN"
ZRENJANIN

ITROCONFERENCE[10]

INFORMATION TECHNOLOGY AND EDUCATION DEVELOPMENT

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PROCEEDINGS

ZRENJANIN, June 2019

UNIVERSITY OF NOVI SAD
TECHNICAL FACULTY ”MIHAJLO PUPIN”
ZRENJANIN
REPUBLIC OF SERBIA

X INTERNATIONAL CONFERENCE OF

INFORMATION TECHNOLOGY AND DEVELOPMENT OF EDUCATION ITRO 2019

PROCEEDINGS OF PAPERS

X MEĐUNARODNA KONFERENCIJA

INFORMACIONE TEHNOLOGIJE I RAZVOJ OBRAZOVANJA ITRO 2019

ZBORNIK RADOVA

ZRENJANIN, JUNE 2019

Publisher and Organiser of the Conference:
University of Novi Sad, Technical faculty „Mihajlo Pupin", Zrenjanin, Republic of Serbia

For publisher:
Dragica Radosav, Ph. D, Professor,
Dean of the Technical faculty „Mihajlo Pupin", Zrenjanin, Republic of Serbia

Editor in Cheaf - President of OC ITRO 2019:
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Proceedings editor:
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Technical design:
Ivan Tasic, Ph. D, Professor;
Dusanka Milanov MSc, Assistant
Dragana Draskovic MSc, Assistant

Circulation: **50**
ISBN: 978-86-7672-322-5

By the resolution no. 142-451-530/2019-01/01, Autonomous Province of Vojvodina, Provincial Secretariat For Science and Technological Development donated financial means for publishing this Conference Proceedings.

The Conference is supported by the Autonomous Province of Vojvodina

CIP - Каталогизација у публикацији
Библиотеке Матице српске, Нови Сад

37.01:004(082)
37.02(082)

INTERNATIONAL Conference of Information Technology and Development of Education ITRO (10 ; 2019 ; Zrenjanin)
Proceedings of papers [Elektronski izvor] / X International Conference of Information Technology and Development of Education ITRO 2019 = Zbornik radova / X međunarodna konferencija Informacione tehnologije i razvoj obrazovanja ITRO 2019, Zrenjanin, June 2019. - Zrenjanin : Technical Faculty "Mihajlo Pupin", 2019. - 1 elektronski optički disk (CD-ROM) : tekst, slika ; 12 cm

Nasl. sa naslovnog ekrana. - Bibliografija uz svaki rad.

ISBN 978-86-7672-322-5

а) Информационе технологије - Образовање - Зборници б) Образовна технологија - Зборници

COBISS.SR-ID 329889287

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With this publication, the CD with all papers from the International Conference on Information Technology and Development of Education, ITRO 2019 is also published.

INTRODUCTION

International Conference on Information Technology and Education Development (ITRO 2019), was held the jubilee tenth time. Since the very beginning, the conference has been connecting science, profession and experiences in education. Information technologies influence educational processes and student achievements. Contemporary topics relate to Interactive EBooks and electronic Teachers logbooks. Thematic fields of the conference are alined with general, but olso with national trends in education:

- Theoretic and methodology questions of contemporary pedagogy
- Digital didactics of media
- Modern communication in teaching
- Curriculum of contemporary teaching
- E-learning
- Education management
- Methodic questions of natural and technical sciences subject teaching
- Information and communication technologies
- Dual education.

The conference work was contributed by plenary lectures covering various aspects of ICT in education development:

- *Digital transformation of educational system in Higher Education*, Branko Perišić, Faculty of Technical Sciences, University of Novi Sad;
- *Security issues of e-learning system*, Igor Franc, E-security, Belgrade;
- *From E to ES teacher logbooks*, Žarko Mušicki, primary school "Žarko Zrenjanin", Novi Sad;
- *Canvy, The Thrue Story of Mobile App*, Marius Marcu, Politechnica University of Timisoara, Romania.

The Proceedings containes 59 articles based on research and scientific work in the field of information technologies in education.

The conference was financially supported by the Provincial Secretariat for Higher Education and Scientific Research, Novi Sad. The Technical Faculty "Mihajlo Pupin" has provided the necessary technical support.

The ITRO Organizing Committee would like to thank to the authors of articles, reviewers and participants in the Conference who have contributed to its tradition and successful realization.

Regards until the next ITRO Conference,

Chairman of the Organizing Committee

Jelena Stojanov

We are very grateful to:

Autonomous Province of Vojvodina

for donated financial means, which supported publishing of the Conference Proceedings and organizing of the Conference.

CONTENTS

SCIENTIFIC PAPERS

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Conceptual Errors - Serious Barrier in Math Education

M. Miteva, L. K. Lazarova, M. Ljubenovska
University "Goce Delcev", Faculty of Computer Science, Stip, Republic of Macedonia
marija.miteva@ugd.edu.mk, limonka.lazarova@ugd.edu.mk, marija.101967@student.ugd.edu.mk

Abstract - Math teachers in the high schools, and furthermore, the university math teachers, can testify that in general, the quality in mathematical training of students graduating high schools, and the acquired math knowledge, is decreasing. On the other side, many studies can point out that the most frequent phenomena in mathematical training of students is wrong acceptance of the concepts, which results in inability for dealing with those concepts and is serious barrier in math education and further application of mathematics. The perception created during our work with students over a period of ten years indicate that the students have distorted and incorrect knowledge of basic mathematical terms and concepts. In order to describe the situation, a test with basic mathematical question has been made for the students in the first year on different faculties. The main aim of this paper is to analyze the situation about such formal concept acceptance and to give conclusions and recommendations to cross over it.

I. INTRODUCTION

We know that mathematics is very exact subject and studying it needs thoroughly studying all its concepts and their relations. Making 'small' mistakes in math considerations and calculations can cause big difference between the obtained result and the correct one. The problem gets bigger if the wrong result is not simple math task or exercise, but it is needed for some application in concrete problem from everyday life or other field in science and society, and we know that mathematics is essential for solving different problems in physics, chemistry, biology, ecology, economy, engineering, etc.

Students usually find difficult the process of learning mathematics. One reason for this situation is the close relations among math concepts and inability to deal with those concepts if only one thing is not understood correctly. The wrong accepted concepts usually become serious barrier in students' math education. As they pass in each next grade the gaps in their math knowledge are getting larger and larger and students' confidence while solving math tasks is getting lower and lower. This situation very often results with students' giving up from mathematics, which further becomes the main reason why students avoid to study faculties where mathematics appears as essential subject, like natural sciences and engineering.

The problem with the lack of profiles possessing good math skills then becomes serious problem for the society. The science and society very often need people who can formulate certain practical problem and will be able to solve that problem. Overcoming the situation with the decreasing math knowledge and math skills among students in primary and secondary schools, seems to be difficult. The solution of this problem obviously could not be found easy, but obviously its beginning should start where the roots of the problem are: the primary school, and both students and teachers should be involved in overcoming the problems with mathematics. The first step in this, not simple and easy process, should be deep analysis about students' difficulties and the errors they do.

In [1] we can read about three types of math mistakes and how to prevent them. The math errors that students usually make could be classified in three categories: careless errors, computational errors and conceptual errors. The first type of errors, careless errors, occurs usually because students do not pay enough attention on the math problem they are solving, or the students are working too fast. This type of mistakes is not considered as serious barrier in learning mathematics, but they still cause wrong results. The second type, computational errors, occurs when mathematical operations are not applied correctly. One computational mistake in a work with many computational steps means that the rest of the work and the final result will be wrong. The third type of mistakes, maybe the seriously one, are conceptual mistakes. This type of mistakes occurs because students have not understood correctly math concepts and relations among the concepts. They use these concepts formally, paying attention on the form how something is written, not on the content of the problem that is presented, i.e. what is written. We consider this as serious barrier in further mathematical training of students and moreover, in the possibility for applying math knowledge anywhere. Thus, the importance of correct concept acceptance is significant. Similar point of view about math mistakes could be found in [2, 3].

Many research studies have been done addressing the mistakes that students make in mathematics, especially pointing out conceptual errors or misconceptions. These research studies are also emphasizing the importance and the impact of the conceptual knowledge which, in general, is difficult for the students to acquire, because the new concepts could only be built upon already well comprehend knowledge [4-14]. Research studies concerning conceptual errors in specific math area are presented in [15, 16].

II. EXPERIMENTAL RESULTS

In order to deal with a complex problem as the one explained in the previous section, it is useful for the first step to analyze carefully concrete examples of mistakes that students make, and then try to make theoretical conclusions for a general case. We have started to do such kind of analysis and we will present the current results in this paper.

During our work with students over a period of ten years, we have concluded that many of them have misunderstood variety of basic concepts in mathematics and are disabled to deal with those concepts. For example, students can easily find the value of the unknown variable x in the linear equation when the only variable x is on the left hand side, but if we change the sides in the same equation, many of the students are not sure what will be the answer (they easily solve in their minds $5x-1=4$ but if we set the equation $4=1-5x$, some of them are getting confused). Also, students can solve a system of two equations with variables x and y, but if we set a system with variables j and k, some of the students will also become confused. Students can learn to plot the graph of a certain function, but those who have accepted this concept formally, are not able to answer which is the value of the function in a given point, looking at the graph. Many of the students who can define real function with one variable and its graph, cannot recognize whether given curve in the plane is graph of a function or not (for example very often they consider circle as a graph of function). Students can easily plot the curve $y=x^2$, but they don't know how to plot $x=y^2$. Most of the students know that they can calculate square root only from non-negative number (working with the real numbers), i.e. $\sqrt{-8}$ doesn't exist in $\mathbf{R}$, but some of them are not sure whether $\sqrt[3]{-8}$ exists or not, and some of them will answer that $\sqrt[3]{-8}$ equals ± 2. There are also students that write $9x-1=8x$. Almost all of them know that $0:2=0$, but some of them also write $2:0=0$. Students can learn and perform operations with vectors given with coordinates in the space, but some of them are not able to answer what does the coordinates tell about the position of the vector in the space (actually, it is not determinate with the coordinates). Many students integrating $\ln x$ write as result $1/x$. These are minor number of examples about students' conceptual errors and it is not difficult to enlarge the list.

Wanting to recognize the level of students' conceptual knowledge in mathematics, when they have started first year on a faculty, and to receive a perception about students' math skills after finishing the high school (this will help us on the other side to determine the structure of the group we are working with), we have made on-line math test. Testing was done at the beginning of the academic year, for the students on Faculty of Computer Science at ours University. The questions with the offered choices and received answers will be presented in continuation.

About 85 students have answered the test with 15 questions with triple choice. The average grade of students' responses is 62,30% right answers. The standard deviation is 21,04%.

We have the top 5 questions with more than 50% wrong answers. The analysis for each question is given in continuation: the question with the lowest score, i.e. most wrong answers, was "$\ln 2x$ is equal to" and only 23,81% of the students choose the correct answer $\ln 2+\ln x$; 42,86% answered that it is equal to $2\ln x$ and 33,33% answered that it is equal to $\ln 2\cdot\ln x$.

The question "$\sin 2x$ is equal to" have 40,48 correct answers, $2\sin x\cos x$, but 38,10% choose the answer $\sin x+\sin x$ and 21,43% answered $2\sin x$, thus more than a half of the students have wrong responses.

The great numbers of students also have misconceptions regarding roots and negative powers. About the question "the term $\sqrt{2(a+b)}$ is equal to" most of the students, 48,81%, have chosen the wrong answer $\sqrt{2a}+\sqrt{2b}$; 41,67% answered correctly, $\sqrt{2}\cdot\sqrt{a+b}$, and there were also students, 9,52%, who choose $2\sqrt{a}+2\sqrt{b}$ as an answer.

About the question "$5x^{-2}$ is equal to" , 46.43% of the students choose the wrong answer $\frac{1}{5x^2}$, 42,86% of the them choose the correct answer $\frac{5}{x^2}$ and 10,71% answered $\frac{x^2}{5}$.

Regarding the term e^{ab}, less than a half, i.e. 46,43% have correctly answered that it is equal to $\left(e^a\right)^b$; 40,48% thought it is equal to $e^a \cdot e^b$ and 13,10% thought $e^{ab}=e^{a+b}$, which is surprisingly bed result (unable to compare the powers).

We have received exceptional bed responses about students' conceptual knowledge for cutting terms in algebraic fractions. Although the most of the students, 55,95% have correctly answer that $\frac{ab+c}{b}=a+\frac{c}{b}$, it is not small the percent of the students, 36,90%, that have cut b in the given term and answered $\frac{ab+c}{b}=a+c$. The rest 7,14% of the students answered $\frac{ab+c}{b}=a$.

Misconceptions regarding cutting terms in the fractions are very often phenomena. Students had three offered choices for the question "$\frac{\ln x}{2x}$ is equal to" and 32,14% have cut x and answered that the term is equal to $\frac{\ln}{2}$(here we can also recognize misconceptions for the function $\ln x$ and inability to notice that $\ln$ without x makes no sense); 61,90% have the right answer $\frac{1}{2}\cdot\frac{\ln x}{x}$ and 5,95% have answered that the term is equal to $\frac{\sqrt{\ln x}}{x}$.

Determining equivalent term with $\left(x+3\right)^2$, 27,38% have the wrong answer x^2+9; 69,05% have the correct answer x^2+6x+9 and 3,57% of the students choose it is equal to x^2+6.

Even through the other seven questions were answered with approximately 75% correct answers, having in mind its basicness and simplicity, we can say that it is inadmissibly for graduated high school students ambitious for academic degree, to make such conceptual mistakes.

About 26,19% of the students that have done the test have misconceptions for simplifying terms like $x^2 \cdot x^2$; 15,48% of them have responded $2x^4$ is the simplification for the previous term, 10,71% have chosen $2x^2$ and the other 73,81% have the right response.

About the term $4+2x^2$, there still are students thinking it is equal to $6x^2$ (21,43% have chosen this answer as the correct one); 77,38% have chosen the right equivalent term $2\left(2+x^2\right)$ and insignificant 1,19% thought $12x^2$ is the correct equal term.

The same number of correct answers have the question searching an equivalent term for $\frac{3}{5x}$. 14,29% of the students find out that $\frac{3}{5}x$ is the same term as the previous one, and 8,33% answered that $5\cdot\frac{3}{x}$ is the right one. The rest 77,38% answered correctly choosing the term $3\cdot\frac{1}{5x}$ as the right one.

The question asking for the solution of the equation $2x=0$, have 77,38 right answers, but some of the students think that $x=-2$ is the solution (17,86%) and the rest 4,76% have responded that the given equation has no solution.

The last three question were: recognizing an equivalent term to $3x^2+2x$ with 20,24% wrong answers (14,29% have answered that it is equal to $5x^3$ and 5,95% have as a response $6x^3$); finding the solution of the equation $-4+x=-8$ which also has 20,24% wrong responds (14,29% have chosen $x=-12$ as solution, 4,76 have chosen $x=2$ as solution; the others had no answer) and calculating the value of the term $\frac{1}{3}+\frac{3}{4}$, 13,10% have given wrong answer, some of them writing it is $\frac{4}{7}$ and the others $\frac{1}{4}$.

We will mention here that only 2 students, which is 2,35% of the whole number of students who have done the test, achieve the score 15, i.e. all answers given correctly; 7 students (8,24%) achieve the score 14; then 9 students (10,59%) achieve the score 13; all other students have done mistake on three and more questions.

The general perception, after receiving the responses of the questions in the test, is that the numbers of students that have incorrectly accepted many mathematical concepts and make different conceptual errors, is not small at all. It is not easy to exceed such problem. It is in practice impossible to achieve a situation without mistakes in students doing math, but with a serious effort, it could be add up to smaller number then the one in the result of previously described test. The process should start in the earlier grades. The problem based learning [17] can have contribution in it. Paying greater attention what is the content of the formula and certain procedure, explained parallel with many examples, could also contribute to get better students results in learning math. Setting real life problem first and then introducing new concepts and relations among them, needed to solve such real problem, could results with clearness in understanding mathematics. This cam also increases the motivation for learning mathematics. However, the process is not easy and is searching deep dedication of the teachers teaching math.

III. CONCLUSION

This analysis based on a test with 15 questions is maybe not large and explicit, but it could serve as good starting point in an attempt to overcome as more as possible problems with wrong accepted concepts in math education and to decrease the number of conceptual errors that students make while solving mathematical tasks and other problems which need application of mathematics in their consideration. We have introduced many misconceptions that students are facing up with, which appear as serious barrier in further studying mathematics and ability to apply misunderstood math concepts in different everyday situations and different problems while studying other subjects at school, and in certain problems in the science, techniques and society after that. The necessity and the importance to get rid of those problems are obvious, but not easy. Increasing motivation for learning mathematics is closely related with better conceptual acceptance in mathematics. In the last years many researchers are interested in this topic, but the attempts for crossing over such situation are obviously not enough yet.

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