СНМД

Српско научно математичко друштво

Kongres mladih matematičara u Novom Sadu

03 – 05. oktobar 2019.
Novi Sad, Srbija

Knjiga sažetaka

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Srpska akademija nauka i umetnosti, Ogranak u Novom Sadu
Departman za matematiku i informatiku, PMF, Univerzitet u Novom Sadu
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Plan rada

Četvrtak, 03. oktobar 2019.

Srpska akademija nauka i umetnosti, Ogranak u Novom Sadu, Nikole Pašića 6, Novi Sad

14:00-15:00		Registracija učesnika
15:00-15:15		Otvaranje Kongresa
15:15-16:00	Dušan Jakovetić	*Distributed optimization and learning*
16:00-16:45	Luka Milićević	*Multilinear maps on multilinear varieties*
16:45-17:00		pauza za kafu
17:00-17:45	Marko Petković	*Iterative methods and neural networks for the computation of generalized inverses*
17:45-18:30	Bojan Prangoski	*Quasi-analytic representation theory of* $(\mathbb{R}^d;+)$ *over quasi-complete locally convex spaces*
19:30		svečana večera u restoranu "Fontana"

Petak, 04. oktobar 2019.

Departman za matematiku i informatiku, Trg Dositeja Obradovića 4, Novi Sad

08:30-09:00	Registracija učesnika

Amfiteatar I, predsedavajući: Bojan Prangoski

09:00-09:20	Suzana Aleksić	*Frames for Hilbert and Banach spaces*
09:20-09:40	Sanja Atanasova	*Characterization of wave front sets via Stockwell transform*
09:40-10:00	Pavel Dimovski	*Modulation spaces related to translation-invariant Banach spaces of quasi-analytic ultradistributions*
10:00-10:20	Lenny Neyt	*Characterizing the nuclearity of Gelfand-Shilov spaces*
10:20-10:40	Zorica Milovanović Jeknić	*Nonlocal boundary value problem*

Amfiteatar VII, predsedavajući: Marko Petković

09:00-09:20	Marija Krstić	*Stability of the Stochastically Perturbed Tumor-Immune Interaction Model with Delay*
09:20-09:40	Jasmina Đorđević	*A stochastic analysis of the impact of fluctuations in the environment on pre-exposure prophylaxis for HIV infection*
09:40-10:00	Marija Milošević	*Comparison of the Euler-Maruyama and backward Euler methods for a class of pantograph stochastic differential equations*
10:00-10:20	Dušan Đorđević	L^p *and almost sure convergence of an approximate method for stochastic differential equations*
10:20-10:40	Vuk Vujović	*Stohastički Heroinski model*

10:40-11:00	pauza za kafu

Amfiteatar I, predsedavajući: Bojan Bašić

11:00-11:20	Lazar Milenković	*Aproksimacioni algoritmi za minimizaciju uskog grla u asimetričnoj verziji problema trgovačkog putnika*
11:20-11:40	Anna Slivková	*Hešov broj je neograničen za* $d \to \infty$
11:40-12:00	Kristina Ago Balog	*O jako palindromičnim rečima: ternarni slučaj*
12:00-12:20	Stefan Hačko	*O nekim aritmetički interesantnim kolekcijama permutacija*
12:20-12:40	Danijela Mitrović	*Emulaciona ekvivalencija kombinatornih igara*

Amfiteatar VII, predsedavajući: Luka Milićević

11:00-11:20	Martin Ljubenović	*Some majorization relations and their linear preservers on* $\ell^p(I)$
11:20-11:40	Maja Obradović	*A class of neutral stochastic differential equations with time-dependent delay and Markovian switching and the Euler-Maruyama approximation*
11:40-12:00	Miljana Stanković	*Stochastic competition model with herd behavior and Allee effect*
12:00-12:20	Stefan Tošić	*Generalized Polynomial Chaos and Stochastic Galerkin Method*
12:20-12:40	Ersin Gilić	*Some new fixed point results for convex contractions in B-metric spaces*

12:40-14:00	pauza za ručak

Amfiteatar I, predsedavajući: Srboljub Simić

14:00-14:20	Milana Pavić-Čolić	*Kinetic monatomic gas mixture models: on the Cauchy problem and* L^p *theory for the system of Boltzmann equations*
14:20-14:40	Enes Kačapor	*The Strongest Inverted Compressed Column*
14:40-15:00	Srđan Trifunović	*Interakcija nestišljivog viskoznog fluida sa termoelastičnim pločama*
15:00-15:20	Nevena Dugandžija	*Generalized solution to multidimensional cubic Schrodinger equation with delta potential*
15:20-15:40	Sanja Ružičić	*Approximate solution to pressureless gas dynamics model and shadow wave tracking procedure*

Amfiteatar VII, predsedavajući: Dušan Jakovetić

14:00-14:20	Nataša Krklec Jerinkić	*Distributed Fixed Point Methods for Solving Systems of Linear Algebraic Equations*
14:20-14:40	Tijana Ostojić	*On an Inexact Restoration Subgradient Method*
14:40-15:00	Jovana Dedeić	*Encoding Compensable Processes Into Adaptable Processes*
15:00-15:20	Davor Kumozec	*Numerical solution of transport equation by means of the method of characteristics*
15:20-15:40	Aleksandra Delić	*Analysis and numerical approximation of boundary value problems with fractional derivatives*

15:40-16:00	pauza za kafu

Amfiteatar I, predsedavajući: Filip Tomić

16:00-16:20	Milica Žigić	*Wick-type nonlinearities in stochastic evolution equations with randomness*
16:20-16:40	Katarina S. Kostadinov	*Existence and asymptotic behavior of q-regularly varying solutions of nonlinear second order q-difference Thomas-Fermi equation*
16:40-17:00	Valentina Timotić	*Logarithmic (translationally) rapidly varying sequences and selection principles*
17:00-17:20	Nevena Petrović	*Anti-Gaussian quadrature rule for trigonometric polynomials*
17:20-17:40	Tatjana V. Tomović	*Multiple Orthogonality and Applications in Numerical Integration*

Amfiteatar VII, predsedavajući: Ivana Đurđev

16:00-16:20	Kristina Asimi	*Obećanja svode konačne probleme na beskonačne*
16:20-16:40	Vladica Andrejić	*Algoritmi za ostatke levog faktorijela i Kurepina hipoteza*
16:40-17:00	Miloš Milovanović	*Intuicionističko zasnivanje matematike i primene u muzici, arhitekturi, obrazovanju...*
17:00-17:20	Simona Kašterović	*Kripkeove semantike za lambda račun sa parovima i sumama*
17:20-17:40	Nenad Stojanović	*Metric logics*

20h	druženje u klubu "Giardino", Bulevar Mihajla Pupina 1, poslednji sprat

Subota, 05. oktobar 2019.

Departman za matematiku i informatiku, Trg Dositeja Obradovića 4, Novi Sad

Amfiteatar I, predsedavajući: Kristina Ago Balog

09:00-09:20	Boriša Kuzeljević	*Globalna teorija ultrafiltera*
09:20-09:40	Samir Zahirović	*O obogaćenom stepenom grafu grupe*
09:40-10:00	Emir Zogić	*Laplacian resolvent energy of graphs*
10:00-10:20	Irena M. Jovanović	*Spectral distances of graphs*
10:20-10:40	Milan Bašić	*On the spread of integral circulant graphs*

Amfiteatar VII, predsedavajući: Đorđe Vučković

09:00-09:20	Snežana Gordić	*Stacionarni Kolomboovi stohastički procesi sa primenama u rešavanju jednačina*
09:20-09:40	Limonka Koceva Lazarova	*Some compositions of distributions in neutrix calculus*
09:40-10:00	Marija Miteva	*Products of Distributions in Colombeau Algebra*
10:00-10:20	Daniel Velinov	*On the frequently hypercyclic* C_0*-semigroups*
10:20-10:40	Radoslav Božić	*The Application of Dynamic Software in the Examining Functions with Parameters and Their Derivatives*

10:40-11:00	pauza za kafu

Amfiteatar I, predsedavajući: Nenad Teofanov

11:00-11:20	Filip Tomić	*Paley-Wiener theorems and wave front sets*
11:20-11:40	Ivana Vojnović	*Continuity of pseudodifferential operators on mixed-norm Lebesgue spaces*
11:40-12:00	Đorđe Vučković	*Toroidal pseudodifferential operators in spaces of ultradistributions on* $\mathbb{T}^n$
12:00-12:20	Snježana Maksimović	*A sequential approach to ultradistribution spaces*

Amfiteatar VII, predsedavajući: Boriša Kuzeljević

11:00-11:20	Đorđe Baralić	*Small covers over neighborly polytopes*
11:20-11:40	Milan Zlatanović	*Koneksije na nesimetričnim Rimanovim mnogostrukostima*
11:40-12:00	Nenad O. Vesić	*IMPOSSIBLE: Whether, Why, How*
12:00-12:20	Jovana Nikolić	*Parcijalni kvazimorfizmi na grupi Hamiltonovih difeomorfizama kotangentnog raslojenja*
12:20-12:40	Anika Njamcul	*Maximal topologies obtained via ideals*

12:40-13:20	pauza za kafu i posluženje	

Amfiteatar I, predsedavajući: Tijana Ostojić

13:20-13:40	Stefan Ivković	*Semi-Fredholm theory on Hilbert C^*-modules*
13:40-14:00	Miloš Cvetković	*Decompositions of bounded linear operators*
14:00-14:20	Marija Cvetković	*Generalized Ulam-Hyers stability of integral and operator equations*
14:20-14:40	Nebojša Č. Dinčić	*Solving the Sylvester Matrix Equation* $AX - XB = C$ *when* $\sigma(A) \cap \sigma(B) \neq \emptyset$
14:40-15:00	Bogdan Đorđević	*On some properties of singular Sylvester operator equations*

Amfiteatar VII, predsedavajući: Danijela Mitrović

13:20-13:40	Ivana Đurđev	*Sendvič polugrupe u lokalno malim kategorijama*
13:40-14:00	Edin Glogić	*On Kirchhoff index, Laplacian energy, number of spanning trees of graphs and their relations*
14:00-14:20	Dragan S. Rakić	*Partial orders based on generalized inverses and annihilators*
14:20-14:40	Dragan Jočić	*Some notes on distributivity equations and aggregation operations*
14:40-15:00	Dušan J. Simjanović	*Fuzzy Relation Equations and Fuzzy Rough Approximation Operators*

Amfiteatar I

15:00	Zatvaranje Kongresa

Na osnovu Curry-Howardove korespondencije ovaj sistem je ekvivalentan sistemu prirodne dedukcije za intuicionističku iskaznu logiku ([2]). Poznat je rezultat o saglasnosti i potpunosti intuicionističke iskazne logike i Kripkeovih semantika ([3]). Prateći ideju Kripkeovih semantika za intuicionističku iskaznu logiku ([3]) i Kripkeovih semantika za lambda račun sa osnovnim tipskim sistemom ([4]), definišemo Kripkeove semantike za lambda račun sa parovima i sumama. Saglasnost lambda računa sa predloženim semantikama smo dokazali indukcijom po dužini izvođenja (tipiziranja). Za dokaz potpunosti koristili smo konstrukciju kanoničkog modela. Kanonički modeli su posebna klasa modela koja se najčešće definiše na osnovu nekog skupa tako da zadovoljava samo formule koje pripadaju tom skupu. Potpunost je direktna posljedica postojanja kanoničkog modela i gore navedene osobine.

Cilj je da proširimo ovaj sistema na sistem u kome ćemo moći zapisati rečenicu „vjerovatnoća da term M ima tip σ je veća ili jednaka $\frac{2}{3}$". Osnova ovog sistema će biti lambda račun sa parovima i sumama za koji smo definisali Kripkeove semantike i dokazali saglasnost i potpunost tipiziranja i navedenih semantika.

Literatura

[1] Ghilezan, S., Ivetić, J., Kašterović, S., Ognjanović, Z., and Savić, N., *Probabilistic Reasoning about Simply Typed Lambda Terms*. In Logical Foundations of Computer Science - LFCS 2018, volume 10703 of Lecture Notes in Computer Science, pages 17–189, 2018.

[2] Howard, W. A., *The Formulae-as-Types Notion of Construction*, pp. 479–490 in To H.B. Curry: Essays on Combinatory Logic, Lambda Calculus and Formalism, London : Academic Press, 1980 (originally circulated 1969).

[3] Mints, G. E., *A Short Introduction to Intuitionistic Logic*, Kluwer Academic / Plenum Publishers, 2000.

[4] Mitchell, J. C., and Moggi, E., *Kripke-style Models for Typed Lambda Calculus*, Annals of Pure and Applied Logic, vol. 51, pp. 99–124, 1991.

[5] Ognjanović, Z., Rašković, M., and Marković, Z.,*Probability Logics: Probability-Based Formalization of Uncertain Reasoning*, Springer, 2016.

Some compositions of distributions in neutrix calculus

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In this presentation some new contributions in neutrix calculus will be presented. The obtained results reffer to some pairs of distributions for which the composition and the convolution product can not be calculated in normal sense. The main objective is the application of the basic definitions in the neutrix calculus, in order to calculate composition and convolution product of distributions which not exist in usual sense. On this way the set of pairs of distributions for which these types of products can be calculated, is extended.

Some compositions of distributions in neutrix calculus

Limonka Koceva Lazarova

University "Goce Delcev", Stip, Macedonia

Congress of Young Mathematicians
Novi Sad, 03-05 October 2019

NEUTRIX CALCULUS

- 1932, Jacques Hadamard

$$\int_0^1 \frac{A(x)}{x^{p+1/2}}dx, \tag{1}$$

$p \in \mathbb{Z}^+$, $A(x)$ is infinitely differentiable function.

$$\int_\epsilon^1 \frac{A(x)}{x^{p+1/2}}dx = F(\epsilon) + I(\epsilon),$$

$F(\epsilon) = \int_\epsilon^1 \frac{A(x)-B(x)}{x^{p+1/2}}dx$, $I(\epsilon) = \int_\epsilon^1 \frac{B(x)}{x^{p+1/2}}dx$, $B(x) = \sum_{i=0}^{p-1} \frac{A^{(i)}(0)}{i!}x^i$.

NEUTRIX CALCULUS

Van der Corput has used the Hadamard finite part in his asymptotic researches.
In 1959 Van der Corput in **Introduction to the neutrix calculus** has established the Neutrix calculus.

-In the 80's in the previous century, **Brian Fisher** has extended the definitions about products, compositions and convolution products to larger class of distributions by using of the Neutrix calculus.

NEUTRIX CALCULUS

Application of the Neutrix calculus

- In the theory of quantum fields in order to obtain finite results for the coefficients in perturbation series, by Jack Ng and van Dam.
- Slawomir Sorek has used the neutrix product of the distributions in non-linear systems in electronics, signal processing and also in the telecommunications.

NEUTRIX CALCULUS

By **van der Corput**:

Definition

Let N' be a set and let N be a commutative, additive group of functions mapping N' into a commutative, additive group N''. If N has the property that the only constant function in N is the zero function, then N is said to be a **neutrix** and the functions in N are said to be **negligible.**
The condition that the constant function in N is only zero function is called **neutrix condition.**

NEUTRIX CALCULUS

Definition

Let $f(\xi)$ be a real (or complex) valued function defined on N' and suppose it is possible to find a constant l such that $f(\xi) - l$ is negligible in N. Then l is called the neutrix limit or N-limit of $f(\xi)$ as ξ tends to b and we write

$$N - \lim_{\xi \to b} f(\xi) = l.$$

If the neutrix limit l exists, then it is only one.

Neutrix compositions of distributions

In the Schwartz' theory of distributions:

$F(f(x)) = ?$, when F and f are arbitrary distributions.

Example: $\delta^2 = ?, \sqrt{\delta} = ?$

Antosik and Fisher have made many tries in order to extend the definitions for composition of distribution F and locally integrable function f in the same way like composition of two distributions is defined.

Neutrix compositions of distributions

Antosik:
The regular Temple's sequences are defined as δ-sequences $\delta_n(x) = n\rho(nx)$ for $n = 1, 2, \ldots$ and they converge to Dirac δ function (as distribution).

$\rho(x)$ is fixed infinitely differentiable function on $\mathbb{R}$, with the following properties:

- $\rho(x) = 0, \quad |x| \geq 1,$
- $\rho(x) \geq 0,$
- $\rho(x) = \rho(-x),$
- $\displaystyle\int_{-1}^{1} \rho(x)\, dx = 1.$

Neutrix compositions of distributions

Let F is distribution in $\mathcal{D}'$ and if $F_n(x) = \langle F(x-t), \delta_n(x) \rangle$, then $F_n(x)$ is regular sequence of infinitely differentiable functions which converges to $F(x)$.

Definition

Let $f, g \in \mathcal{D}'$. We said that the distribution $g(f(x))$ exists and it is equal to $h(x)$ on $\mathbb{R}$ if the sequence from compositions $\{g_n(f_n)\}$ converge to the distribution $h(x)$.

Neutrix compositions of distributions

Many compositions of distributions are calculated by using of the previous definition:

(i) $\sqrt{\delta} = 0$

(ii) $\sqrt{\delta^2 + 1} = 1 + \delta$

(iii) $\log(1 + \delta) = 0$

(iv) $\sin\delta = 0$

(v) $\cos\delta = 1$

(vi) $\frac{1}{1+\delta} = 1.$

Neutrix compositions of distributions

We used the following definition:

Definition

Let F is a distribution in $\mathcal{D}'$ and let f is locally integrable function. It is said that the neutrix composition $F(f(x))$ exists and is equal to h on the open interval (a, b) if

$$N - \lim_{n\to\infty} \int_{-\infty}^{+\infty} F_n(f(x))\varphi(x)dx = \langle h(x), \varphi(x)\rangle \tag{2}$$

for all test functions φ in $\mathcal{D}[a, b]$, where N is a neutrix with domain $N' = \mathbb{Z}^+$, range $N'' = \mathbb{R}$ and negligible functions

$$n^\lambda \ln^{r-1} n, \ln^r n : \lambda > 0, r = 1, 2, \dots \tag{3}$$

and all functions which converge to 0 in the usual sense, when n tends to the infinity.

1. L.Lazarova, B.Jolevska-Tuneska, I.Akturk, E.Ozcag, *Note on the distribution composition* $(x_+^{\mu})^{\lambda}$, Bulletin of the Malaysian Mathematical Sciences Society, Springer, (2016), pp.1-13;
2. E.Ozcag, L.Lazarova, B.Jolevska-Tuneska, *Defining compositions of the distributions* $x_+^{\mu}, |x|^{\mu}, x^{-s}$ *and* $x^{-s} \ln |x|$ *as a neutrix limit of regular sequences*, Communications in Mathematics and Statistics, Volume 4, Issue 1, Springer Berlin Heidelberg, (2016), pp.63-80.

Neutrix composition of the distributions x^λ and x_+^μ for $\lambda = -1, -2, \dots, \mu > 0$ and $\lambda\mu \in \mathbb{Z}^-$

The local integrable functions $x_+^\lambda, x_-^\lambda, |x|^\lambda$ for $\lambda > -1$ are defined with:

$$x_+^\lambda = \begin{cases} x^\lambda, & x > 0, \\ 0, & x < 0 \end{cases}, \quad x_-^\lambda = \begin{cases} |x|^\lambda, & x < 0, \\ 0, & x > 0 \end{cases}, \qquad (4)$$

and $|x|^\lambda = x_+^\lambda + x_-^\lambda$.
The distributions x_+^λ and x_-^λ are defined for $\lambda < -1$, $\lambda \neq -2, -3, \dots$ with $(x_+^\lambda)' = \lambda x_+^{\lambda-1}$ and $(x_-^\lambda)' = -\lambda x_-^{\lambda-1}$ and distribution $|x|^\lambda$ is defined with $|x|^\lambda = x_+^\lambda + x_-^\lambda$.

Neutrix composition of the distributions x^λ and x_+^μ for $\lambda = -1, -2, \ldots, \mu > 0$ and $\lambda\mu \in \mathbb{Z}^-$

The distributions x_+^r and x_-^r are defined by:

$$x_+^r = \frac{(-1)^{r-1}(\ln x_+)^{(r)}}{(r-1)!}, \quad x_-^r = -\frac{(\ln x_-)^{(r)}}{(r-1)!} \tag{5}$$

for $r = 1, 2, \ldots,$

Neutrix composition of the distributions x^λ and x_+^μ for $\lambda = -1, -2, \ldots, \mu > 0$ and $\lambda\mu \in \mathbb{Z}^-$

Lemma: Let $\rho(x)$ is infinitely differentiable function. For $s \in \mathbb{Z}^+$ we have:

$$\int_{-1}^{1} v^i \rho^{(s)}(v)dv = \begin{cases} 0, & 0 \leq i < s \\ (-1)^s s!, & i = s \end{cases}, \tag{6}$$

$$\int_{-1}^{0} v^r \rho^{(s)}(v)dv = \int_{0}^{1} v^s \rho^(s)(v)dv = \frac{1}{2}(-1)^s s! \tag{7}$$

$$\int_{0}^{1} v^s \ln|v|\rho^{(s)}(v)dv = \frac{1}{2}(-1)^s s!\phi(s) + (-1)^s s! c(\rho) \tag{8}$$

$$\int_{0}^{1} v^s \ln(1-v)dv = -\frac{\phi(s)}{s}. \tag{9}$$

Neutrix composition of the distributions x^λ and x_+^μ for $\lambda = -1, -2, \ldots, \mu > 0$ and $\lambda\mu \in \mathbb{Z}^-$

In the previous lemma the integrals are calculated for $s = 0, 1, 2, \ldots,$ where

$$c(\rho) = \int_0^1 \ln t \rho(t)\, dt,\ \phi(s) = \begin{cases} \sum\limits_{k=1}^{s} \frac{1}{k}, & s \geq 1 \\ 0, & s = 0 \end{cases}$$

Neutrix composition of the distributions x^{λ} and x_+^{μ} for $\lambda = -1, -2, \ldots, \mu > 0$ and $\lambda\mu \in \mathbb{Z}^-$

The following theorem is proved:

Theorem

The distribution $(x_+^{\mu})^{-m}$ exists and

$$(x_+^{\mu})^{-m} = x_+^{-s} - (-1)^s \frac{(-1)^m m![2c(\rho) + \phi(m-1)] + s\phi(s-1)}{s!} \delta^{(s-1)}(x) \tag{10}$$

for $\mu > 0$, $m = 1, 2, \ldots$ and $\mu m = s (s \in \mathbb{Z}^+)$.

Neutrix composition of the distributions x^{λ} and x_+^{μ} for $\lambda = -1, -2, \ldots, \mu > 0$ and $\lambda\mu \in \mathbb{Z}^-$

Corollary: The distribution $(x_-^{\mu})^{-m}$ exists and

$$\left(x_-^{\mu}\right)^{-m} = x_-^{-s} + \frac{(-1)^m m!\,[2c(\rho) + \phi(m-1)] + s + \phi(s-1)}{s!}\delta^{(s-1)}(x) \tag{11}$$

for $\mu > 0, m = 1, 2, \ldots$ and $\mu m = s \in \mathbb{Z}^+$.
Corollary: Let by $F_r(x)$ we denote the distribution x_-^{-r}, then the distribution $F_r(x_+^{1/r})$ exists and

$$\left(x_+^{\frac{1}{r}}\right)^{-r} = x_+^{-1} - (-1)^r r![2c(\rho) + \phi(r-1)]\delta(x) \tag{12}$$

for $r = 1, 2, \ldots,$ where $\phi(r)$ and $c(\rho)$ are defined in the previous lemma.

Neutrix composition of the distributions x_+^μ, $|x|^\mu$, x^{-s} and $x^{-s}\ln|x|$

We have proved that:

Theorem

The composition of the distributions $(x_+^\mu)_-^s$ *exists and*

$$(x_+^\mu)_-^{-s} = \frac{(-1)^{m+s}c(\rho)}{\mu(m-1)!}\delta^{(m-1)}(x) \tag{13}$$

for $\mu > 0, s = 1, 2, \ldots,$ *where* $\mu s = m \in \mathbb{Z}^+$
Special case:

$$\left(x_+^{\frac{1}{s}}\right)_-^{-s} = (-1)^{s+1}sc(\rho)\delta(x).$$

Neutrix composition of the distributions x_+^μ, $|x|^\mu$, x^{-s} and $x^{-s}\ln|x|$

The result is given in:

Theorem

The composition of the distributions $(|x|^\mu)_-^s$ *exists and*

$$(|x|^\mu)_-^{-s} = \frac{2(-1)^{m+s}c(\rho)}{\mu(m-1)!}\delta^{(m-1)}u(x) \tag{14}$$

for $\mu > 0, s = 1, 2, \ldots,$ *where* $\mu s = m = 1, 3, 5, \ldots,$ *and*

$$(|x|^\mu)_-^{-s} = 0 \tag{15}$$

for $\mu > 0, s = 1, 2, \ldots,$ *where* $\mu s = m \neq 1, 3, 5, \ldots.$
Special case $(|x|^{\frac{1}{s}})_-^{-s} = 2sc(\rho)\delta(x)$.

Neutrix composition of the distributions x_+^μ, $|x|^\mu$, x^{-s} and $x^{-s}\ln|x|$

By using of the theorem which refers to the compositions $(x_+^\mu)^{-s}$, the corollaries follow:

Corollary: The compositions of the distributions $(x_+^\mu)_+^{-s}$ exists and

$$(x_+^\mu)_+^{-s} = x_+^{-m} - (-1)^m [L_{m,s}^* + \frac{c(\rho)}{\mu(m-1)!}]\delta^{(m-1)}(x) \qquad (16)$$

for $\mu > 0, s = 1, 2, \ldots$ and $\mu s = m \in \mathbb{Z}^+$, where

$L_{m,s}^* = \frac{(-1)^s s![2c(\phi)+\phi(s-1)]+m\phi(m-1)]}{m!}$.

Neutrix composition of the distributions x_+^μ, $|x|^\mu$, x^{-s} and $x^{-s}\ln|x|$

Corollary: The composition of the distributions $(|x|^\mu)_+^{-s}$ exists and

$$(|x|^\mu)_+^{-s} = |x|^{-m} + [L_{m,s}^* - \frac{2(-1)^m c(\rho)}{\mu(m-1)!}]\delta^{(m-1)}(x), \tag{17}$$

for $\mu > 0, s = 1, 2, \dots$ and $\mu s = m = 1, 3, 5, \dots$, and

$$(|x|^\mu)_+^{-s} = |x|^{-m} + L_{m,s}^* \delta^{(m-1)}(x) \tag{18}$$

for $\mu > 0, s = 1, 2, \dots$ and $\mu s = m \neq 1, 3, 5, \dots$.

Neutrix composition of the distributions x_+^μ, $|x|^\mu$, x^{-s} and $x^{-s}\ln|x|$

By Fisher and Nicholas the following theorem is proved:

Theorem

The composition of the distributions $(x_+^r)^{-s}$ *exists and*

$$(x_+^r)^{-s} = x_+^{-rs} + K_{r,s}\delta^{rs-1)}(x) \tag{19}$$

for $r, s = 1, 2, \ldots,$ *where* $K_{r,s} = (-1)^{rs-1}\frac{(-1)_s s![2c(\rho)+\phi(s-1)]+rs\phi(rs-1)}{(rs)!}$.

Lemma:

If φ is arbitrary function in $\mathcal{D}[-1,1]$. Then:

$$\langle x_+^{-s}, \varphi(x)\rangle = \int_0^1 x^{-s}[\varphi(x) - \sum_{i=0}^{s-1} \frac{\varphi^{(i)}(0)}{i!} x^i]dx - \\ - \sum_{i=0}^{s-2} \frac{\varphi^{(i)}(0)}{i!(s-i-1)} - \frac{\phi(s-1)\varphi^{(s-1)}(0)}{(s-1)!}, s = 1, 2, \ldots, \quad (20)$$

$$\langle x_+^{-s}\ln x_+, \varphi(x)\rangle = \int_0^1 x^{-s}\ln x[\varphi(x) - \sum_{i=0}^{s-1} \frac{\varphi^{(i)}(0)}{i!} x^i]dx - \\ - \sum_{i=0}^{s-2} \frac{\varphi^{(i)}(0)}{i!(s-i-1)^2} - \frac{\phi_1(s-2)\varphi^{(s-1)}(0)}{(s-1)!}, s \geq 2, \phi_1(s) = \sum_{i=1}^{s+1} \frac{\phi(i)}{i}. \quad (21)$$

Neutrix composition of the distributions x_+^μ, $|x|^\mu$, x^{-s} and $x^{-s}\ln|x|$

We have proved the following theorem:

Theorem

The composition of the distributions $x^{-s}\ln|x|$ and x_+^r exists and

$$(x_+^r)^{-s}\ln|x_+^r| = rx_+^{-rs}\ln x_+ + K_{r,s}^*\delta^{(rs-1)}(x), \tag{22}$$

for $s = 1, 2, \dots$, *where* $c_1(\rho) = \int\limits_0^1 \ln^2 t\rho^{(s-1)}(t)dt$

$K_{r,s}^* = \frac{(-1)^{rs-1}}{(rs-1)!}\{\frac{[1+(-1)^{s+1}]c_1(\rho)}{2(s-1)!} + \phi(s-1)[K_{r,s} + \phi(rs-1)]\}.$

Thank You For Your Attention