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COMPUTER SIMULATION FOR STOCHASTIC INVENTORY MANAGEMENT

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Abstract:
In this paper, the importance of inventory management for gaining a competitive advantage of a certain enterprise is shown. Furthermore, the basic – deterministic model of inventories is presented and the methodology for determining the optimal quantity of order is given. In addition, the stochastic inventory model has been developed with a detailed analysis of the uniform and normal distribution of demand probabilities. In order to introduce the reader to the computer simulation of inventory management, a brief overview of the general principles of simulation methods is given, with their advantages and disadvantages. Moreover, a computer simulation of the management of inventories for a specific case was performed by developing a software application in the Python programming language. The results of the simulation modeling are shown in tabular and graphical form. In the conclusion, an interpretation of the obtained results from the execution of the program is given and proposed guidelines for further research are given by simulating other parameters of the inventory, when the following occur as random variables: order quantities, the level to be re-ordered, the cost of inventory and delivery time. Validation of the mathematical model has been performed and it has been compared with similar mathematical models based on which it has been shown that the results are realistic and expected.

Key words:
inventory management, normal distribution, stochastic model, optimal order quantity, inventory costs, delivery time, mathematical model, computer simulation

INTRODUCTION

Inventory management, inventory planning and control to meet the competitive priorities of the enterprise are an important issue for managers in all types of businesses [4]. Effective inventory management is the basis for understanding the full potential of any supply chain. For companies operating at relatively low profit margins, poor inventory management can seriously weaken the business. The challenge is not to reduce inventory to the core to reduce the costs or to have a lot to meet all the demands, but to have the right quantity to most effectively reach the competitive priorities of the business. This type of efficiency can only occur if the right amount of inventory moves through the value chain – through suppliers, companies, warehouses or distribution centers, and consumers.

Inventory management is a process that requires information on expected demand, order quantities, backup or security inventory, reorder time, shortage of inventory and all costs associated with these parameters.

1 LITERATURE REVIEW

Methodologies for stochastic inventory modelling are proposed in the papers [1], [8], [4]. More detailed scientific research on modelling and simulation of the inventory management system has been performed in the scientific paper [1]. Therefore, a brief theoretical overview of these researches will be given here, to which our paper is complemented. Paper [1] is focused on the application of different model approaches in inventory management with random/uncertain demand, i.e. inventory models, simulation models and optimization model. From inventory models it analyses: ABC analysis and demand analysis. The author states that two main modelling techniques are used in supply chain, namely simulation and optimization. The simulation should answer the question *What if?*, while the optimization should answer the question What is best?. The simulation model was created in Microsoft Excel with a min-max strategy and 100% level of service. Market demand is described by beta distribution. A period of 52 weeks with 32 repetitions was simulated. With stochastic programming 100 demand scenarios are simulated for 20 periods. Empirical data on the monthly demand of Coffee 32 are described by a triangular distribution. The purpose of the optimization model was to minimize total procurement costs for a period of 20 months for 100 scenarios. The AMPL language for CPLEX software was used for the simulation modelling. The following conclusions were obtained from the research:

- If the company has a number of suppliers delivering more than one product, the solution is the need to consolidate demand from the same suppliers to reduce the shipping costs.
- If it is concluded that the company operates with dependent products, then it is necessary to review the inventory models that deal with dependent products.
- To respond quickly to rapid environmental change (for example, demand) forecasting methods need to be developed before inventory planning.

So far, the demand forecasting is based only on managerial competencies.

2 THEORY OF STOCHASTIC INVENTORY MODEL

The stochastic stock model includes at least one stochastic variable. The values of that variable are represented by the probability distribution – this is a way of defining the range of possible values that the variable can take. Stochastic models are closer to reality than deterministic ones.

In business economics and business decision making there are often problems in which the cause-effect relationships between the variables in the model are not precisely determined, as not all the values of the parameters needed to obtain a single solution are known [8].

Problems in which there is a risk when making decisions, then accidental influences and lack of information are very common. Stochastic modelling is an important area of modelling that will be explained below through a simple example in business decision making. However, in practical terms, not only in complex problems, it is very difficult to specify a mathematical model, as well as to find an analytical solution [6].

There may be different sources of random variability in inventory models, such as:

- delivery time,
- various costs, etc.
- however, the most significant is the demand that we took as an average value in the deterministic model.

Demand variability is not very important until the moment it is ordered. If demand rises sharply at that point, inventory will fall faster than expected. Inventory may be depleted before new arrives. As a result, many companies maintain backup inventory (see Figure 1). Spare / security inventory is used in case of demand growth during the delivery period.

The question arises: How big should those reserves be? If inventory is too small, production may stop, and if it is too large, capital is tied to unnecessary inventory. We will get the answer to the question if we know the probability distribution of demand.

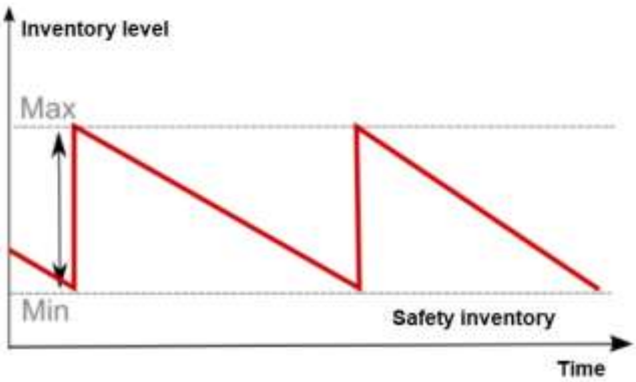

Fig.1 *Moving inventory by maintaining a reserve*

3 METHODOLOGY FOR PERFORMING INVENTORY MANAGEMENT SIMULATION

3.1 General principles of simulation methods

Simulation methods are applied when the system for which a model is required to be developed is too complex for an analytical approach. Here we will present only the basic procedure of this technique and apply it to the inventory management area [6].

Simulation is most often used for systems that involve behavioural uncertainty and make business decisions risky in some way. Uncertainty in mathematical models is represented by the formation of specimens with an appropriate distribution of probabilities. This type of simulation is often called the Monte Carlo simulation. These are the simplest types of simulation models. Random system characteristics are defined by random variables. The input values of such variables are defined by the distribution of the probabilities that best represent them. Given that these are random values, the output values of the model are calculated as average indicators of a sufficient number of iterations implemented on the model.

Simulation modelling consists of the following basic steps:

- For the random variable we choose the type of probability distribution and its parameters that best reflect the behaviour of that random variable.
- We perform a large enough number of iterations, experiments in which the corresponding random variable appears.
- For each experiment we record the output values of the model and finally for the results we calculate the mathematical expectation, the standard deviation and other statistical parameters.

Basically all simulation methods contain random number generators that simulate values for random variables.

3.2 Description of the methodology through a simulation of inventory management for a specific case

In this paper we will present a simulation model of stocks of a store that sells packages of milk, and for which data on demand for 64 days have been collected (see Table 1).

Tab. 1 *Empirical distribution of milk consumption probabilities (in packages)*

Number of packages of milk sold per day	**Average**	**Frequency**	**Relative frequency** (%)	**Cumulative frequency** (%)
10 ÷ 20	15	10	16	16
20 ÷ 30	25	18	28	44
30 ÷ 40	35	24	37	81
40 ÷ 50	45	7	11	92
50 ÷ 60	55	5	8	100
total		64	100	

The quantity of packages of milk (Q) that is ordered is a fixed / constant value. Critical inventory of milk packages (R) represent the level of inventory when goods need to be re-ordered. This means that if the inventory level at the end of the day is equal to or less than the critical inventory, the goods are ordered. Total costs include ordering costs and inventory costs, as well as the costs of lack of inventory. Input data in the model:

- the fixed costs per order are 180 monetary units (mu).
- the cost of storing one package per day is 0.45 m.u.
- the cost of inventory shortages is the loss that occurs when a consumer cannot buy milk in that store because there is no inventory, and it is estimated at 5 m.u. per package.
- the time required for the ordered goods to arrive from the moment of ordering is 2 days.

It is necessary to determine:

- the order quantity Q, (packages / order)
- critical inventory R (packages / day), for a given demand D (packages / day) which is a random quantity
- total costs (m.u)
- inventory costs (m.u)
- costs of lack of inventory (m.u).

We will solve the problem with simulation method. The demand will be generated by empirical distribution (see Table 1).

Simulation initial conditions are:

- Inventory is initially equal to 0 and on the first day reach the ordered quantity Q.
- We can determine the demand from the probability distribution (Table 1), i.e. from the collected data for 64 days of operation of the store. The expected demand is:

$$D = E(x) = \sum_{i=1}^{n} x_i \cdot fr_i = 31.70 \approx 32\ packages \tag{1}$$

Since this is a stochastic variable with an empirical probability distribution, we do not have an analytical solution to the problem. We will get the approximate values using the analytical solution of the deterministic model, which means that we will assume that the consumption is a fixed value and equal to the expected one.

$$D = 32\ packages/day$$

$$h = 0.45\ m.u./day$$

$$C = 180\ m.u.$$

Based on the input data we calculate the order quantity (Q), storage costs (Cs), order costs (Cn) and order time (Z):

$$Q_o = \sqrt{\frac{2CD}{h}} = \sqrt{\frac{2\cdot 180\cdot 32}{0.45}} = 160\ packages/order \tag{2}$$

$$C_s = h \cdot \frac{Q}{2} = \frac{0.45\cdot 160}{2} = 36\ m.u/day \tag{3}$$

$$C_n = \frac{C\cdot D}{Q} = \frac{180\cdot 32}{160} = 36\ m.u/day \tag{4}$$

$$Z_0 = \sqrt{\frac{2C}{hD}} = \sqrt{\frac{2\cdot 180}{0.45\cdot 32}} = 5\ days \tag{5}$$

Critical inventory level (R):

The time required for the supplier to deliver the goods from the moment of ordering is 2 days, and the daily expected demand is 32 packages, so it is *R = 2 * 32 = 64 packages / day*. This is how we got the basic simulation parameters:

- Quantity of order: Q = 160 packages / order,
- Critical inventory level: R = 64 packages / day,
- The demand D is generated by a triangular distribution closest to the empirical distribution given in Table 1, with parameters (a, c, b) = (15; 32; 55), shown in Fig. 2.

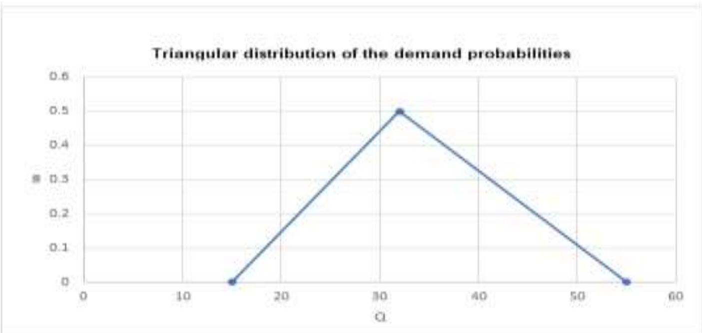

Fig.2 *Triangular distribution of the demand probabilities of milk packages*

We will perform the simulation in 30 days. Immediately after the first day, the first shipment of 160 packages arrives and there is no inventory. The new order starts when the inventory falls below 64 packages (re-order level) and it takes 2 days to arrive. The demand is generated according to a discrete empirical distribution. The simulation is implemented with the Python programming language.

Proposed algorithm:

1. initialize: recorded demand, sum of frequencies, probabilities, cumulative probabilities, h, C, n, L.
2. for all frequency items calculate the probabilities.
3. for all probability items calculate cumulative probabilities.
4. calculate the expected demand.
5. calculate Q, Cs, Cn, Z, R.
6. initialize: demand, orders, initial level of inventory, end level of inventory, lack of inventory, order decision, number of orders, inventory costs, costs for lack of inventory, order costs, total costs.

7.
```
currentQ = Q
    for index, value in demand.items():
    if index == 0:
        orders.append(Q)
        initialLevelOfInventory.append(Q)
    else:
        orders.append(currentQ)
        initialLevelOfInventory.append(endLevelOfInventory[index-1] + currentQ)

    endLevelOfInventory.append(initialLevelOfInventory[index]-demand[index])
    inventoryCosts.append(endLevelOfInventory[index] * h)
    lackOfInventory.append(R - endLevelOfInventory[index])

    if endLevelOfInventory[index] >= R:
        orderDecision.append("NO")
        numberOfOrders.append(0)
    elif endLevelOfInventory[index] < R:
        orderDecision.append("YES")
        numberOfOrders.append(1)

    if lackOfInventory[index] < 0:
        currentQ = 0
        lackOfInventory[index] = 0

    else:
        currentQ = 160

    costsForLackOfInventory.append(lackOfInventory[index] * n)
    orderCosts.append(numberOfOrders[index] * C)
    totalCosts.append(inventoryCosts[index] + costsForLackOfInventory[index] + orderCosts[index])
```

8. Calculate the sums of: inventory costs, costs for lack of inventory, order costs, total costs
9. Plot graphs

3.3 Results

Suppose the store operates according to $Q = 155; R = 64$. Then the simulation we performed is called a historical simulation. It simulates the business system according to the real parameters. The purpose of this simulation model is to perform experiments. By changing the input values for the parameters we want to gain important knowledge about the system, which will help us make quality business decisions. In the specific example, we simulated the daily demand for milk packages for 30 days and controlled the inventory, so that when the level of spare inventory falls below 64 packages / daily, a re-order is made, taking into account the current inventory and calculating the following costs (Fig. 4):

- inventory costs,
- costs for lack of inventory,
- order costs, and
- total costs

The diagrams for the movement of demand, orders, initial and end level of inventory are shown in Fig. 3, and the diagrams for the costs of: inventory, lack of inventory, orders, and total costs are shown in Fig. 4.

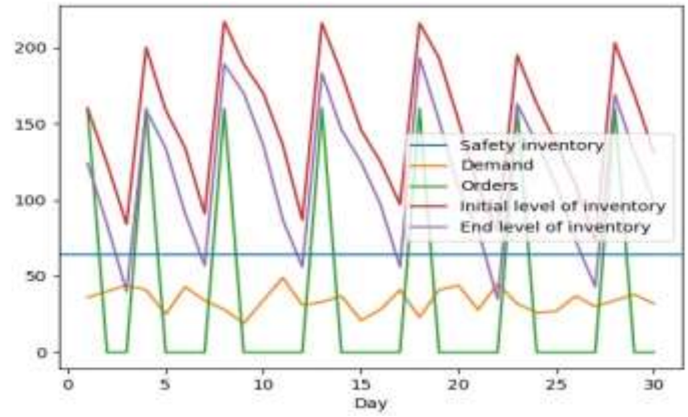

Fig.3 *Diagrams of demand, orders, critical inventory, initial and end level of inventory*

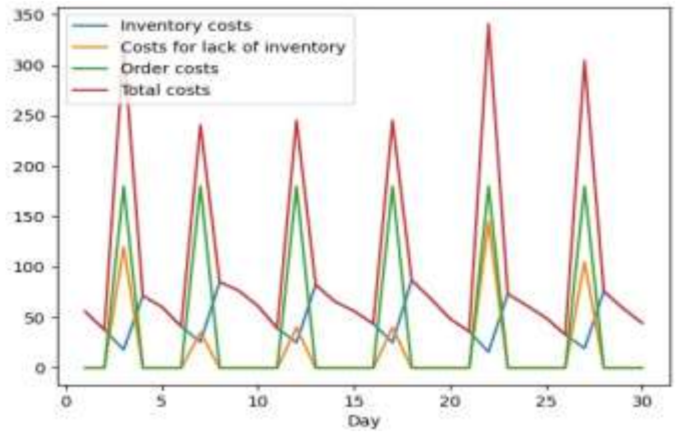

Fig.4 *Diagrams of inventory costs, lack of inventory, orders and total costs*

4 DISCUSSION

The developed mathematical model for inventory management in stochastic demand is an excellent tool that gives the manager of the procurement department important information about the quantity of orders, the critical level of inventory, and the moment when it is necessary to make an ordering decision. Inventory costs, costs of lack of inventory, and total costs depend from the values of these parameters.

From the diagram in Fig. 4 it can be seen that with the increase of the shortage of inventory, the total costs increase sharply, which is very unfavourable for the operation of the enterprise. This tells us how negative it is for the business of the company that consumers cannot get the goods at the moment they ask for it.

5 CONCLUSION

The proposed simulation model enables successful inventory management simulation when the order quantity has random character.

Computer simulation can also be used for simulation modelling and when necessary to determine the effects of changes in some other parameters in the model. They can be:

- The level at which it needs to be re-ordered,
- Inventory costs,
- Delivery time.
- Changes in the structure of the model can also be examined, for example changes in the distribution of demand, variable delivery time instead of fixed, etc.

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