DEFINITE INTEGRAL FOR CALCULATING VOLUME OF REVOLUTION THAT IS GENERATED BY REVOLVING THE REGION ABOUT THE X (Y) - AXIS AND THEIR VISUALIZATION

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Abstract

If we know the function that is limit of some shape, then by the help of a definite integral we can calculate the volume of that shape. Students learn the procedure about the definite integral calculating, but they never understand what exactly they calculate.

For visually presenting the shape of which we calculate the volume and explaining the procedure about getting the shapes, we will use the free software GeoGebra. This way the students can calculate the volume of the shapes and at the same time, they can present the shape visually and see what they calculate.

That way we will have students who know how to apply their knowledge in mathematics about solving practical problems.

Keywords: *definite integral, integral, GeoGebra, applied mathematics*

1. INTRODUCTION

Technology has become more important at schools everyday due to the computers, the mobile devices and the software that is available everywhere. Students can use the advantages of alternative way of studying with the integration of technology at schools and in studying. Especially in teaching mathematics, use of technology can contribute in solving mathematics problems and skills development for creative thinking, which will contribute to increase the mathematics thinking of students. In addition, different types of software for teaching mathematics are created, which it is much easier to perform mathematical calculations, to represent graphs, to design dynamic geometric problems, etc., which provides an easy access to multiple representations of mathematical content, which improves student achievement (Ahmet Emin Tatar, Dilek Düs, 2020).

GeoGebra is a free software for mathematics, which encourages multiple presentations and research learning, which has an easy – to – use interface. What makes GeoGebra software practical and important to the curriculum is its superior ability to think geometry to algebra in mathematics education (Markus Hohenwarter, 2010). For these reasons, GeoGebra was used in this study.

2. GROUPS IN THIS RESEARCH

At the University "Mother Teresa" – Skopje, at the Faculty of Technical Sciences, one of the topics covered in Mathematics 2 is the application of a definite integral. In the academic year 2018/2019 the teaching material was presented in a classic way (Anton Davis, 2012), and in the academic year 2019/2020 online teaching was realized by using online platforms and free software Geogebra was used to visualize the problems in overcoming them, which was available for use by the students themselves.

Two groups of students were involved in this research. In the first group there were students who attended to Mathematics 2 in the academic year 2018/2019, and in the second group there were students who attended to Mathematics 2 in the academic year 2019/2020.

The purpose of this study was to determine the effect of using GeoGebra software on students' achievement on the topic Application of a certain integral and to obtain an opinion about the use of computers and GeoGebra in mastering not only this topic, but also other Mathematics content.

In addition, for the assigning tasks to be solved by the students, the study also covered the following issues:

- What is the impact of using GeoGebra on students' achievement on the topic Application of a definite integral?

- What are the students' views about the use of GeoGebra for a conceptual understanding of mathematical concepts?

3. MATERIALS

### 3.1. Material prepared for classic processing of teaching contents

A rotating body is obtained by rotating a plane face around a line. The line is called the axis of rotation. Some of the figures obtained by rotation are:
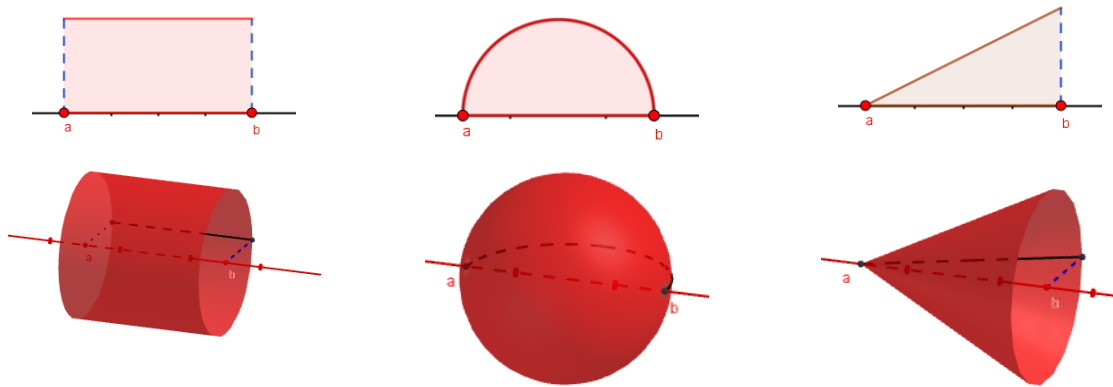
Fig. 1. Some Familiar Solids of Revolution

A solid of revolution is also obtained by rotating a curvilinear trapezoid about an axis. Suppose that the function $y = f(x)$ is continuous and non-negative on the interval *[a, b]* and let there be a solid obtained by rotating a curvilinear trapezoid, bounded by the graph of the function $y = f(x)$, the *x*-axis and make $x = a$, $x = b$ about the *x* - axis.

To determine the volume of the resulting solid of revolution, first we divide the interval [a, b] as follows: $a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$. Then we choose numbers $z_1, z_2, \ldots, z_n$ such as $z_i \epsilon [x_{i-1}, x_i]$, for each $i = 1, 2, \ldots, n$. Rotating the rectangle about the x-axis gives the following cylinder:
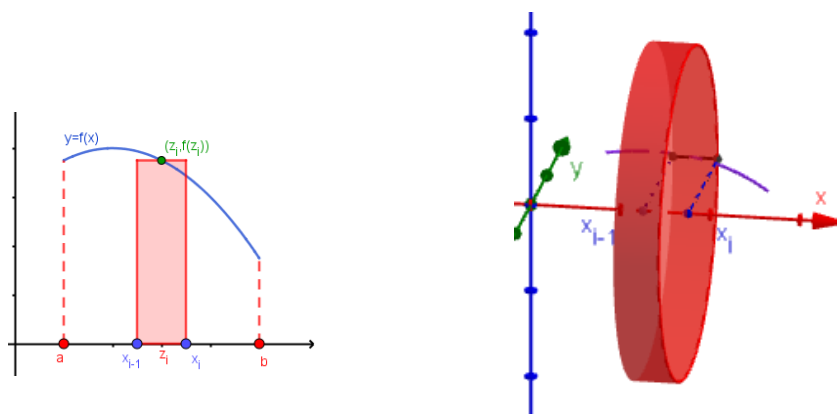
Fig. 2. Rotate a rectangle about the *x*-axis

The volume of the solid of revolution obtained is approximately equal to the sum of the volumes of the cylinders with radius $f(z_i)$ and height $\Delta x_i = x_i - x_{i-1}$, for each $i = 1,2,\dots,n$.

The volume of one single cylinder is equal to

$V_i = \pi \cdot f^2(z_i) \cdot (x_i - x_{i-1}) = \pi \cdot f^2(z_i) \cdot \Delta x_i$ for $z_i \epsilon [x_{i-1}, x_i]$, for each $i = 1,2,\dots,n$.

By summing the volumes V_i of the cylinders for each $i = 1,2,\dots,n$ the volume of the solid of revolution is estimated approximately,

$\sum_{i=1}^{n} \pi \cdot f^2(z_i) \cdot \Delta x_i$.

This is the Riemann sum for the function $\pi \cdot f^2(x)$ of the segment [a,b]. If we allow the norm of division to weigh to 0, we get that the volume of the solid of revolution is:

$V = \lim_{\substack{n \to \infty \\ \Delta x \to 0}} \sum_{i=1}^{n} \pi \cdot f^2(z_i) \cdot \Delta x_i$.

The last sum is the integral sum for the function $\pi \cdot f^2(x)$, from which we obtain a formula for calculating the volume of a solid of revolution obtained by rotating a curvilinear trapezoid bounded by the graph of the function *y=f(x)*, *x* - axis and lines *x=a, x=b* around *x* - axis.

Theorem: If *f (x)* is a non-negative continuous function of the segment [a, b], then the volume of the solid of revolution obtained by rotating about the *x*-axis of the curvilinear trapezoid enclosed by the *x*-axis and the graph of the function *f (x)* of the segment [a, b] is

$$V = \pi \cdot \int_{a}^{b} f^2(x)dx$$

If the solid of revolution is obtained by rotating about *y* - axis of the curvilinear trapezoid, bounded by the curve *x = g (y)*, *y* - axis and makes *y = c, y = d*, where *g (y)* is a continuous and non-negative function of segment [c, d], then the volume of the resulting rotating body is calculated by the formula:

$$V = \pi \cdot \int_{c}^{d} g^2(y)dy$$

3.2. Material prepared with GeoGebra

For visual representation of the rotating figures for which volume is determined and conceptual understanding of the term it will be used an applet in Geogerba.

The procedure for making the GeoGebra applet will be described. The link where the applet is available is https://www.geogebra.org/m/avk7k3ed
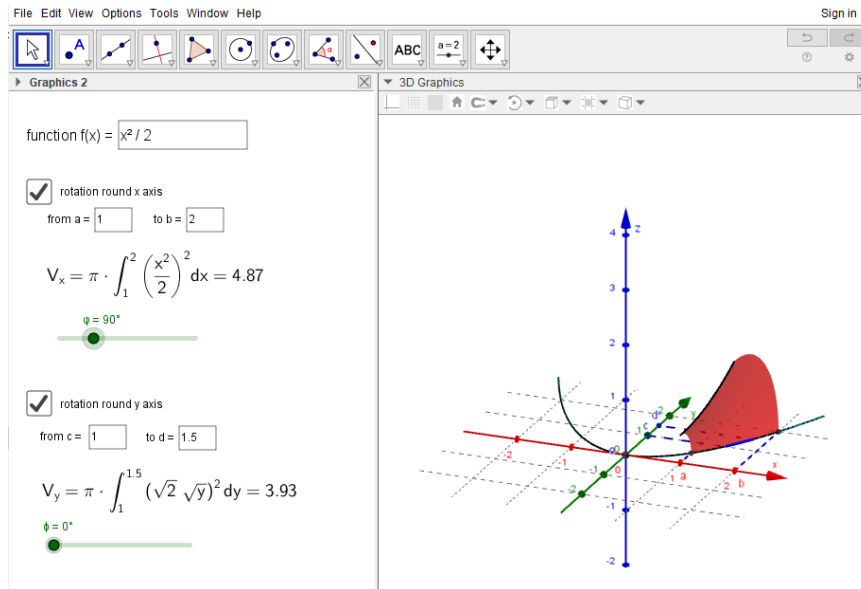
Fig. 3. GeoGebra applet for representing a rotating body

In the beginning, it will be opened GeoGebra with Graphics and 3D graphic display (3D Graphics).

Initial values for the borders are applied: a=1, b=2, c=1, d=2.

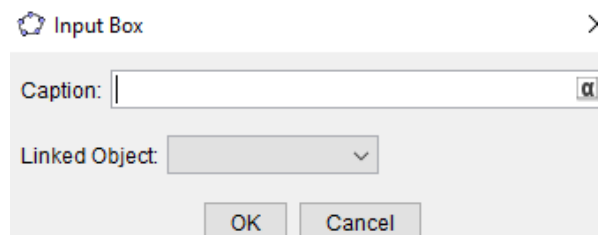In the input field a function is typed $f(x) = x^2/2$ (arbitrary value for the function)

In the drawing pad (Graphics) sliders are applied (Slider) for the angles φ and ϕ from 0° to 360°.
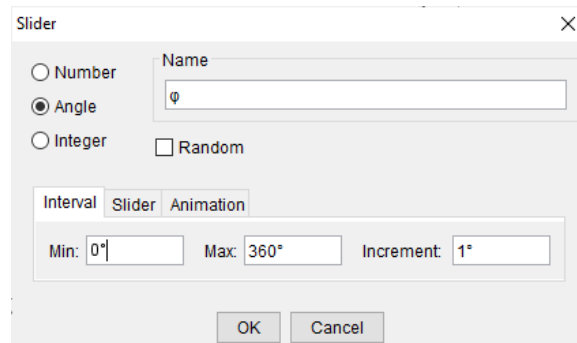
Fig. 4. Slider application window

Then input fields are entered (Input Box).

Then in the input field (Caption) "function $f(x) =$" is written and from the drop-down menu (Linked Object) the function $f(x)$ is chosen.

Fig. 5. Input fields

The same procedure is repeated for the interval limit values.

Input Box – with a title "from a =" and from the drop-down menu choose a,

Input Box – with a title "to b =" and from the drop-down menu choose b,

Input Box – with a title "from c =" and from the drop-down menu choose c,

Input Box – with a title "to d =" and from the drop-down menu choose d.

Construct segments that define the borders of a curvilinear trapezoid.

In the input field the following point commands are entered individually:

A=(a,0,0); B=(b,0,0); C=(0,c,0); D=(0,d,0); E=(a,f(a),0); F=(b,f(b),0).

Then in the input window commands for segments are entered and constructed:
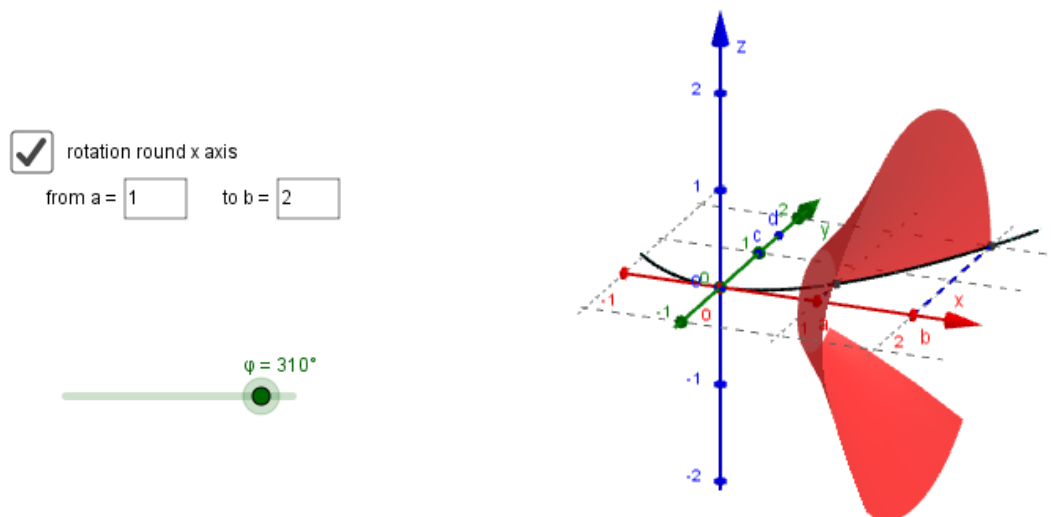
Segment(A,E) and Segment(B,F), which are the borders of the plain area, are constructed with dashed lines.

The curve can now be constructed by rotating and draws the solid of revolution.

In the input window a parametric curve is entered:

Surface(u, f(u) cos(w), f(u) sin(w), u, a, b, w, 0, φ).

Moving the slider, it gives a solid of revolution.

Fig. 6. Construct the solid of revolution by moving the slider

To determine the volume of a solid of revolution when the plane figure rotates about the y-axis, we will first determine the inverse function of the given function f(x).

With the command Invert (f), we get the inverse function g.

The points G = (g (c), c, 0) and H = (g (d), d, 0) are applied.

Then in the input window commands for sections are entered and the following is constructed:

Segment(C,G) and Segment(D,H), which are the borders of the plain area, are constructed with dashed lines.

We enter a parametric curve in the input window, but care must be taken if the inequality g (c) <g (d) is met. If the condition is met, the first part of the parametric curve will be applied, and if the condition is not met, the second part of the parametric curve will be applied.

If(g(c) < g(d), Surface(u_1 cos(w_1), f(u_1), u_1 sin(w_1), u_1, g(c), g(d), w_1, 0, ϕ), Surface(u_1 cos(w_1), f(u_1), u_1 sin(w_1), u_1, g(d), g(c), w_1, 0, ϕ))

Finally, just add formulas to calculate the volume with a definite integral and display fields to choose which of the given values should be displayed.

In the input window enter:

$V_x = \pi$ Integral(f²(x), a, b)

$V_y = \pi$ Integral(g²(y), c, d)

With text windows in the drawing area (Graphics) is applied

V_x =\pi \cdot \int_{**a**}^{**b**}{ \left(**f** \right)^2dx} = **V**$_x$

V_y =\pi \cdot \int_{**c**}^{**d**}{ \left(**g** \right)^2dy} = **V**$_y$

Add in Check Box to Show/Hide Objects, where the title about what solid of revolution should be displayed is written and which objects should be displayed by checking the button, ending with the applet construction.

4. RESULTS

After the realization of the teaching of mathematics in the both two school years, a final exam was realized.

One type of the task, which was included in the final exam in Mathematics 2 in both academic years, was determining the volume of a rotating body.

**Exercise:** So as to determine the volume of the solid of revolution, it is obtained:

a) with rotation about x - axis of the plane character enclosed by the graph of $f(x) = \sqrt{8x}$ and by the lines $x = 1$ and $x = 3$.

b) with rotation about y - axis of the plane character enclosed by the graph of $f(x) = \sqrt{8x}$ and by the lines $y = 0.5$ and $y = 4$.

After the exams, a table with the obtained results was made, but for this research the result was taken only from this type of task.

The number of students who solved the solid of revolution volume task is presented in the tables.

	Number of students
Exercise is solved	8
Exercise is not solved	18
Summary	26
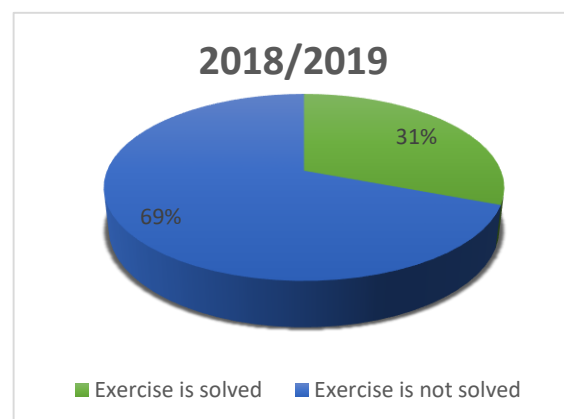
Table 1. Number of students in the academic year 2018/2019

	Number of students
Exercise is solved	12
Exercise is not solved	11
Summary	23

2019/2020

48% 52%

■ Exercise is solved ■ Exercise is not solved
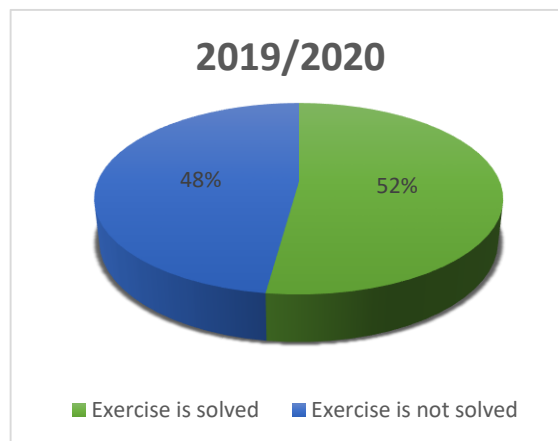
Table 2. Number of students in the academic year 2019/2020

During the realization of the exam, the students in the academic year 2018/2019 solved the task on a piece of paper, 31% of them had calculated the volume accurately, but only a small percentage could imagine the figure about which they calculated the volume. The check whether they knew what the figure looked like, was realized by asking a question during the consultation and offering 5 pictures from which they had to choose and explain which of the pictures showed the solid of revolution.

In the academic year 2019/2020, the exam was realized online. While solving the exam tasks, the students solved the tasks on sheets but they were allowed to use the GeoGebra applets available on the following link https://www.geogebra.org/m/avk7k3ed.

Students visually presented the figure about which they calculated volume and 52% of them had calculated the value of the task accurately. All students who calculated the volume of the figure had an idea of what kind of figure they performed volume calculation, i.e. at the consultations held after the exam all students, who solved the task from the offered 5 pictures, determined and explained correctly on which picture is represented the solid of revolution.

5. CONCLUSIONS

Using the advanced technology, various mathematical software can be introduced as learning environments. In this study, the free software GeoGebra was used to study the topic Application of a certain integral and the volume calculation of a solid of revolution.

It has been found that the use of GeoGebra in the study of a certain integral has a positive effect on the students' success. Dynamic software also keeps mathematics away from abstraction and helps to visualize a solid of revolution, which has contributed positively to students' understanding of the concept of a definite integral and volume of a solid of revolution. Dynamic software also allows you to learn math by doing exercises. With GeoGebra you can visualize and specify math concepts and construct lessons with the proper visual environment.

The students who were involved in the research had positive attitudes about the use of GeoGebra in teaching mathematics. With the applets help created in GeoGebra, the new concepts in mathematics can be understood and the teaching material can be mastered. The students said the software improves visualization and helps them notice the relationship among the mathematical concepts of the content they study. The findings of this research are the same as the findings in the papers of (Tatar, E. 2013); and (Zengin,Tatar, 2015).

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