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Application of Newton's Backward Interpolation Using Wolfram Mathematica

Zoran Trifunov[#1], Liridon Zenku[*2], Teuta Jusufi-Zenku[#3]

[#]*Mother Teresa Univerisy, Faculty of Technical Sciences, R.N Macedonia*

Abstract – *Interpolation is one of the most basic and most useful numerical techniques. It constitutes an irreplaceable tool during work with tabular or graphical functions. The Newton's backward interpolation is one of most important numerical techniques which have huge application in mathematics, computer science and technical science. This paper provides an analytical description of Newton's backward interpolation and how Wolfram Mathematica software can be used to solve the problems from Newton's backward interpolation.*

Keywords – *Backward, Interpolation, Mathematics, Wolfram.*

I. INTRODUCTION

Interpolation is a very important and very useful technique in numerical analysis. If a function whose analytic form is either completely unknown or inconvenient for calculation, then it is desirable to replace it with another function, convenient and simple to calculate, and "close enough" to the given. That operation of replacing one function to another, simpler, is the main task of interpolation [2].

To construct a polynomial of interpolation, there are many techniques, including linear interpolation, Lagrange's interpolation formula, Divided differences, Spline interpolating, Newton's forward and backward interpolation, Stirling interpolation, Bessel's interpolation etc. [3].

This paper describes the analytic form of solving problems with Newton's backward interpolation formula and solving the same problem with use of Wolfram Mathematica software. We decided to use this method because it is simpler to evaluate and we can write code in Wolfram Mathematica and other software to determine the required value, unlike some other methods that are appropriate only in theoretical studies. This paper is organized as follows. Section 2 provides an analytical description of Newton's backward interpolation formula. Section 3 provides an application of this interpolation and how it can be solved with Wolfram Mathematica. Finally, Section 4 concludes the paper and provides future work directions.

II. NEWTON'S BACKWARD INTERPOLATION FORMULA

Let be given the points $x_0, x_1, x_2, \dots, x_n$ and the function f on $\left[x_0, x_n\right]$.

Suppose now that the points $x_0, x_1, x_2, \dots, x_n$ are equidistant, i.e., $x_{i+1} - x_i = t$ for $i = 0, 1, 2, \dots, n-1$.

Definition 1: The finite first order backward difference for the function f in relation to x (or x_i) is called the expression

$$\nabla f\left(x\right) = f\left(x\right) - f\left(x-t\right) \text{ or}$$

$$\nabla f\left(x_i\right) = f\left(x_i\right) - f\left(x_{i-1}\right)$$

and the k^{th} order backward difference is defined as

$$\nabla^k f\left(x\right) = \nabla\left(\nabla^{k-1} f\left(x\right)\right) \text{ or}$$

$$\nabla^k f\left(x_i\right) = \nabla\left(\nabla^{k-1} f\left(x_i\right)\right).$$

These differences are often presented in a tabular format as in Table 1.

x_i	$f(x_i)$	$\nabla f(x_i)$	$\nabla^2 f(x_i)$	$\ldots$	$\nabla^n f(x_i)$
x_0	$f(x_0)$				
x_1	$f(x_1)$	$\nabla f(x_1)$			
$\vdots$	$\vdots$	$\vdots$			
x_{n-1}	$f(x_{n-1})$	$\nabla f(x_{n-1})$	$\nabla^2 f(x_{n-1})$		
x_n	$f(x_n)$	$\nabla f(x_n)$	$\nabla^2 f(x_n)$	$\ldots$	$\nabla^n f(x_n)$

Table 1 The finite backward difference [1]

Newton's backward interpolation formula for interpolation is obtained from the Definition given above.

For $s = \dfrac{x_p - x_n}{t}$ we get the polynomial:

$$P_n(x_p) = f(x_n) + \nabla f(x_n)s + \nabla^2 f(x_n)\frac{s(s+1)}{2!} + \ldots + \nabla^n f(x_n)\frac{s(s+1)\ldots(s+n-1)}{n!}$$

This formula is useful when the value of f is required at point x_p near the end of the segment.

The error in this case is

$$E_n(x) = t^{n+1}s(s+1)\ldots(s+n)\frac{f^{(n+1)}(\xi)}{(n+1)!} \approx s(s+1)\ldots(s+n)\frac{\nabla^{n+1} f(x_n)}{(n+1)!}$$

III. APPLICATION OF NEWTON'S BACKWARD INTERPOLATION

Now let's apply the Newton's backward difference by analytically solving a concrete example to find the number of graduated students in 2017 in the Republic of North Macedonia. The available data for the number of graduated students for different years from 2012 to 2018 at higher school and faculties – first cycle of studies (undergraduate studies) is taken from the State Statistical Office. This data is given in Table 2.

Year (x_i)	Number of graduated students $(f(x_i))$
2012	10392
2014	9863
2016	8247
2018	7698

Table 2 Number of graduated students at higher schools and faculties **-** first cycle of studies [5]

Solution of the Problem:

First, it is necessary to create the backward difference table. This is given in Table 3.

x_i	$f(x_i)$	$\nabla f(x_i)$	$\nabla^2 f(x_i)$	$\nabla^3 f(x_i)$
2012	10392			
2014	9863	-529		
2016	8247	-1616	-1087	
2018	7698	-549	1067	2154

Table 3 Backward difference table for the concrete problem

Next, for $t = 2$, and for $x = 2017$, for s the following value is obtained:

$$s = \frac{2017 - 2018}{2} = -0.5.$$

From Table 3 and Newton's backward formula, we get

$$P_3(2017) = f(x_3) + \nabla f(x_3)s + \nabla^2 f(x_3)\frac{s(s+1)}{2!} + \nabla^3 f(x_3)\frac{s(s+1)(s+2)}{3!}$$

$$P_3(2017) = 7698 + (-549)(-0.5) + 1067\frac{(-0.5)(-0.5+1)}{2!} + 2154\frac{(-0.5)(-0.5+1)(-0.5+2)}{3!}$$

$$P_3(2017) = 7698 + 274.5 - 133.375 - 134.625$$

$$P_3(2017) = 7704.5$$

The error will be

$$E_3(x) = s(s+1)(s+2)\frac{\nabla^3 f(x_3)}{3!}$$

$$E_3(x) = (-0.5)(-0.5+1)(-0.5+2)\frac{2154}{3!} = -134.625$$

$$E_3(x) = -134.625$$

Next is given a code in Wolfram Mathematica for Newton's backward interpolation to calculate the number of graduated students in 2017, which is given in Figure 1. The code is written and adapted according to the definitions used in this paper.

In[54]:= Print["The Backward Difference table is"];

$$\text{Print}\left[\text{TableForm}\left[\begin{pmatrix} 0 & x_0 & f[x_0] & "" & "" & "" \\ 1 & x_1 & f[x_1] & \nabla f[x_1] & "" & "" \\ 2 & x_2 & f[x_2] & \nabla f[x_2] & \nabla\nabla f[x_2] & "" \\ 3 & x_3 & f[x_3] & \nabla f[x_3] & \nabla\nabla f[x_3] & \nabla\nabla\nabla f[x_3] \end{pmatrix}, \text{TableHeadings} \to \{\text{None}, \{"i", "x_i"\}\}\right]\right];$$

The Backward Difference table is

i	x_i				
0	2012	10 392			
1	2014	9863	-529		
2	2016	8247	-1616	-1087	
3	2018	7698	-549	1067	2154

In[65]:= $$f[x_3] + \nabla f[x_3] \times q + \nabla\nabla f[x_3] \times \frac{q \times (q+1)}{2!} + \nabla\nabla\nabla f[x_3] \times \frac{q \times (q+1) \times (q+2)}{3!};$$

In[66]:= $$\text{Print}\left["P_3[2017] = ", f[x_3] + \nabla f[x_3] \times q + \nabla\nabla f[x_3] \times \frac{q \times (q+1)}{2!} + \nabla\nabla\nabla f[x_3] \times \frac{q \times (q+1) \times (q+2)}{3!}\right]$$

$$P_3[2017] = \frac{15\,409}{2}$$

In[61]:= $$q \times (q+1) \times (q+2) \times \frac{\nabla\nabla\nabla f[x_3]}{3!};$$

In[62]:= $$\text{Print}\left["E_3[x] = ", q \times (q+1) \times (q+2) \times \frac{\nabla\nabla\nabla f[x_3]}{3!}\right]$$

$$E_3[x] = -\frac{1077}{8}$$

Fig. 1 Wolfram Mathematica code for Newton's backward interpolation formula

IV. CONCLUSIONS

From the above calculations and the outputs of the code, the same values for the table, error and for the polynomial are obtained. However, the main advantage of using Wolfram Mathematica is that the it calculates much more quickly and much more correctly both, the backward difference table and Newton's backward interpolation polynomial. This code is convenient and easy to use. Also, it can be used code generated in other software such as MATLAB, Maple, etc.

REFERENCES

[1] Fatmir Hoxha, Metoda të analizës numerike, Infbotues, Tiranë, 2008.

[2] Трпеновски Б., Целакоски Н., Елементи од нумеричката математика, Просветно Дело, Скопје, 1992.

[3] Chakrabarty, Dhritikesh. (2017). Backward Divided Difference: Representation of Numerical Data by a Polynomial Curve. 2. 1 – 6.

[4] Wolfram Mathematica, link: https://en.wikipedia.org/wiki/Wolfram_Mathematica [online accessed on 06.01.2020].

[5] Key Indicators from theme: Education and Science, State Statistical Office, link: http://www.stat.gov.mk/IndikatoriTS_en.aspx?id=5 [online accessed on 25.06.2019]

[6] Richard L. Burden, J. Douglas Faires, Numerical Analysis, Brooks Cole Pub., 2011.

[7] Stojanovska L. Trifunov Z. (2010) „Constructing and Exploring Triangles with GeoGebra". Anale Seria Informatica, Vol VIII, Fac.2, România, pp. 45-54.

[8] Trifunov Z.**,** Karamazova E., … (2015) „Introduction of discrete and continuous random variable". LAP LAMBERT Academic Publishing. ISBN: 978-3-659-79405-6

[9] Biswajit Das, Dhritikesh Chakrabarty (2016) „ Newton's backward interpolation: Representation of numerical data by a polynomial curve", International Journal of Applied Research 2016; 2(10): 513-517, pp 513-517.

[10] Robert J Schilling, Sandra L Harries. „Applied Numerical Methods for Engineers", Brooks /Cole, Pacific Grove, CA, 2000

[11] A. Favieri, (2018), „Hypertext on laplace transform using wolfram mathematica", INTED2018 Proceedings, **ISBN:** 978-84-697-9480-7, **ISSN:** 2340-1079, pp. 4979-4986