Feedback

On 'Iterating circum-medial triangles': Martin Lukarevski writes:

The authors of [1] show that the iterating process for circummedial and circummidarc triangles ends with an equilateral triangle and ask if there are other circumcevians which lead to equilaterality as well. We may additionally ask for a short iterating process which will reach the equilateral circumcevian triangle in minimum steps. Does only one step ever suffice? Yes, for two remarkable triangle centres.

The circumcevian triangles of the first and second isodynamic points, J_1 and J_2 respectively, are equilateral, see [2, p. 274], [3, p. 296].

References

1. H. Humenberger and F. Embacher, Iterating circum-medial triangles, *Math. Gaz.* **103** (November, 2019) pp. 480-487.
2. G. Leversha, *The geometry of the triangle*, United Kingdom Mathematics Trust (2013).
3. R. Johnson, *Advanced Euclidean Geometry*, Dover (1960).

On 'An arithmetical question related to perfect numbers': Roger Heath-Brown writes:

The proof of Theorem 3 in [1] was quite long and intricate and the purpose of this Feedback is to give an alternative proof.

Proof: Let $\sigma(n) = u^2$, $u \in \mathbb{N}$. Then u is a factor of $2n$ and so $2n = uv$, $v \in \mathbb{N}$. Then $u^2 = \left(u^2 - uv\right)^2 = u^2(u - v)^2$ and so $u = v \pm 1$.

Let U and V be the odd parts of u and v, respectively, then $n = PUV$ and either $u = 2PU$ or $v = 2PV$, where P is a non-trivial power of 2.

Let q be any prime factor of $\sigma(P) = 2P - 1$ and note that q is then a factor of U. Then

$$\frac{uv}{V} = 2PU = \left(\sigma(P) + 1\right)U \leqslant \sigma(P)\left(U + \frac{U}{\sigma(P)}\right)$$

$$\leqslant \sigma(P)\left(U + \frac{U}{p}\right) \leqslant \sigma(P)\sigma(U)$$

with equality only if $U = q = \sigma(P)$ is prime. So, since P, U and V are pairwise coprime

$$\sigma(P)\sigma(U)\sigma(V) = u^2 = u\left(v \pm 1\right) = uv\left(1 \pm \frac{1}{v}\right) \leqslant \sigma(P)\sigma(U)\left(V \pm \frac{V}{v}\right),$$

Therefore $u + v + 1$ and $\sigma(V) \leqslant V + \dfrac{V}{v}$.

If $V \neq 1$: Then $\sigma(V) \geqslant V + 1 \geqslant V + \dfrac{V}{v}$. Equality must therefore hold throughout and so $v = V$ is an odd prime and $u = 2PU$.

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