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Problems and Solutions

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PROBLEMS AND SOLUTIONS

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Proposed problems should be submitted online at
americanmathematicalmonthly.submittable.com/submit.
Proposed solutions to the problems below should be submitted by May 31, 2020, via the same link. More detailed instructions are available online. Proposed problems must not be under consideration concurrently at any other journal nor be posted to the internet before the deadline date for solutions. An asterisk (*) after the number of a problem or a part of a problem indicates that no solution is currently available.

PROBLEMS

12153. *Proposed by Omran Kouba, Higher Institute for Applied Sciences and Technology, Damascus, Syria.* For a real number x whose fractional part is not 1/2, let $\langle x\rangle$ denote the nearest integer to x. For a positive integer n, let

$$a_n = \left(\sum_{k=1}^{n} \frac{1}{\langle \sqrt{k}\rangle}\right) - 2\sqrt{n}.$$

(a) Prove that the sequence $a_1, a_2, \ldots$ is convergent, and find its limit L.
(b) Prove that the set $\{\sqrt{n}(a_n - L) : n \geq 1\}$ is a dense subset of $[0, 1/4]$.

12154. *Proposed by Martin Lukarevski, University "Goce Delcev," Stip, North Macedonia.* Let r_a, r_b, and r_c be the exradii of a triangle with circumradius R and inradius r. Prove

$$\frac{r_a}{r_b + r_c} + \frac{r_b}{r_c + r_a} + \frac{r_c}{r_a + r_b} \geq 2 - \frac{r}{R}.$$

12155. *Proposed by Albert Stadler, Herrliberg, Switzerland.* Let $f : [0, \infty) \to [0, 1]$ be the function that satisfies $f(0) = 1$, is differentiable on $(0, \infty)$, and has the following property: If A is a point on the graph of f and B is the x-intercept of the line tangent to the graph of f at A, then $AB = 1$.
(a) Prove $\int_0^\infty f(x)\,dx = \pi/4$.
(b) For $n \in \mathbb{N}$, prove that $\int_0^\infty x^{2n} f(x)\,dx$ is a rational polynomial of π.

12156. *Proposed by Hideyuki Ohtsuka, Saitama, Japan.* For positive integers m and n and nonnegative integers r and s, prove

$$\sum_{0\leq j_1\leq\cdots\leq j_m\leq r} \frac{\binom{n+s}{n}\binom{n+j_1}{n}\binom{s+j_1}{s}}{\prod_{i=1}^{m}(n+j_i)} = \sum_{0\leq j_1\leq\cdots\leq j_m\leq s} \frac{\binom{n+r}{n}\binom{n+j_1}{n}\binom{r+j_1}{r}}{\prod_{i=1}^{m}(n+j_i)}.$$

12157. *Proposed by Nick MacKinnon, Winchester College, Winchester, UK.* Show that there are infinitely many positive integers that are neither the sum of a cube and a prime nor the difference of a cube and a prime (in either order).

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