

The American Mathematical Monthly

Letter from the Editor	3
Geometry of Elliptic Numbers and Sums of Powers of Consecutive Natural Numbers	4
Pythagorean Triples	13
Hypergeometric Functions	23
Hypergeometric Functions Subsumed by ${}_2F_1$ Multiplier and ${}_2F_1$ Transform	33
Multiple Proofs of Some Classical Results about Critical Point Location	49
How Substitution and Not \ln Generalized the Chebyshev Identity	84
NOTES	
The Best Way to Resonate Your Post-It Equatorially	44
Collisions and Angular Momentum for Singular Potentials	70
A New Proof of Euler's First Theorem	70
A Probabilistic Proof of a Wallis-type Formula for the Gamma Function	79
Generalization of a Ramanujan Identity	80
PROBLEMS AND SOLUTIONS	
PROBLEMS	
The Best of Both: The New Science of Cause and Effect in Complex and Chaotic Systems	94
SOLUTIONS	
MAINTENANCE	
MA: A Probabilistic Proof of a Lemma That Is Not Bernoulli's, but Is	
Errata: Proof of the Irregularity of e	

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Problems and Solutions

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PROBLEMS AND SOLUTIONS

Edited by **Daniel H. Ullman, Daniel J. Velleman, Douglas B. West**

with the collaboration of Paul Bracken, Ezra A. Brown, Zachary Franco, Christian Friesen, László Lipták, Rick Luttmann, Frank B. Miles, Lenhard Ng, Kenneth Stolarsky, Richard Stong, Stan Wagon, Lawrence Washington, Elizabeth Wilmer, Fuzhen Zhang, and Li Zhou.

Proposed problems should be submitted online at

americanmathematicalmonthly.submittable.com/submit.

Proposed solutions to the problems below should be submitted by May 31, 2020, via the same link. More detailed instructions are available online. Proposed problems must not be under consideration concurrently at any other journal nor be posted to the internet before the deadline date for solutions. An asterisk () after the number of a problem or a part of a problem indicates that no solution is currently available.*

PROBLEMS

12153. *Proposed by Omran Kouba, Higher Institute for Applied Sciences and Technology, Damascus, Syria.* For a real number x whose fractional part is not $1/2$, let $\langle x \rangle$ denote the nearest integer to x . For a positive integer n , let

$$a_n = \left(\sum_{k=1}^n \frac{1}{\langle \sqrt{k} \rangle} \right) - 2\sqrt{n}.$$

(a) Prove that the sequence a_1, a_2, \dots is convergent, and find its limit L .

(b) Prove that the set $\{\sqrt{n}(a_n - L) : n \geq 1\}$ is a dense subset of $[0, 1/4]$.

12154. *Proposed by Martin Lukarevski, University "Goce Delcevo," Stip, North Macedonia.*

Let r_a, r_b , and r_c be the exradii of a triangle with circumradius R and inradius r . Prove

$$\frac{r_a}{r_b + r_c} + \frac{r_b}{r_c + r_a} + \frac{r_c}{r_a + r_b} \geq 2 - \frac{r}{R}.$$

12155. *Proposed by Albert Stadler, Herrliberg, Switzerland.* Let $f : [0, \infty) \rightarrow [0, 1]$ be the function that satisfies $f(0) = 1$, is differentiable on $(0, \infty)$, and has the following property: If A is a point on the graph of f and B is the x -intercept of the line tangent to the graph of f at A , then $AB = 1$.

(a) Prove $\int_0^\infty f(x) dx = \pi/4$.

(b) For $n \in \mathbb{N}$, prove that $\int_0^\infty x^{2n} f(x) dx$ is a rational polynomial of π .

12156. *Proposed by Hideyuki Ohtsuka, Saitama, Japan.* For positive integers m and n and nonnegative integers r and s , prove

$$\sum_{0 \leq j_1 \leq \dots \leq j_m \leq r} \frac{\binom{n+s}{n} \binom{n+j_1}{n} \binom{s+j_1}{s}}{\prod_{i=1}^m (n+j_i)} = \sum_{0 \leq j_1 \leq \dots \leq j_m \leq s} \frac{\binom{n+r}{n} \binom{n+j_1}{n} \binom{r+j_1}{r}}{\prod_{i=1}^m (n+j_i)}.$$

12157. *Proposed by Nick MacKinnon, Winchester College, Winchester, UK.* Show that there are infinitely many positive integers that are neither the sum of a cube and a prime nor the difference of a cube and a prime (in either order).

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