

4484. Proposed by Leonard Giugiuc and Michael Rozenberg.

Let $a, b, c \in [0, 2]$ such that $a + b + c = 3$. Prove that

$$4(ab + bc + ac) \leq 12 - ((a - b)(b - c)(c - a))^2$$

and find when the equality holds.

4485. Proposed by Jonathan Parker and Eugen J. Ionascu.

For every square matrix with real entries $A = [a_{i,j}]_{i=1..n, j=1,2..n}$, we define the value

$$GM(A) = \max_{\pi \in S_n} \{\min\{a_{1\pi(1)}, a_{2\pi(2)}, \dots, a_{n\pi(n)}\}\}$$

where S_n is the set of all permutations of the set $[n] := \{1, 2, 3, \dots, n\}$.

Given the 6×6 matrix

$$A := \begin{bmatrix} 20 & 9 & 7 & 26 & 27 & 13 \\ 19 & 18 & 17 & 6 & 12 & 25 \\ 22 & 24 & 21 & 11 & 20 & 11 \\ 20 & 8 & 9 & 23 & 5 & 14 \\ 22 & 17 & 4 & 10 & 36 & 33 \\ 21 & 16 & 23 & 35 & 15 & 34 \end{bmatrix}$$

find the value $GM(A)$.

4486. Proposed by Marian Cucoaneş and Marius Drăgan.

Let $a, b > 0$, $c > 1$ such that $a^2 \geq b^2c$. Compute

$$\lim_{n \rightarrow \infty} (a - b\sqrt[n]{c})(a - b\sqrt[3]{c}) \cdots (a - b\sqrt[n]{c}).$$

4487. Proposed by Martin Lukarevski.

Let a, b, c be the sides of a triangle ABC , m_a, m_b, m_c the corresponding medians and R, r its circumradius and inradius respectively. Prove that

$$\frac{a^2}{m_b^2 + m_c^2} + \frac{b^2}{m_c^2 + m_a^2} + \frac{c^2}{m_a^2 + m_b^2} \geq \frac{4r}{R}.$$

4488. Proposed by George Apostolopoulos.

Let ABC be an acute-angled triangle. Prove that

$$\sqrt{\cot A} + \sqrt{\cot B} + \sqrt{\cot C} \leq \sqrt{\cot \frac{A}{2} \cdot \cot \frac{B}{2} \cdot \cot \frac{C}{2}}.$$