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Preface

Vladimir Andrunachievici Institute of Mathematics and Computer Science (IMCS), in this spring celebrates its 55th anniversary from the foundation. With all certainty we can say that IMCS is a centre of excellence for advance research in the domains of pure mathematics, the applied one and computer science.

Founded by the illustrious mathematician Vladimir Andrunachievici, it is an organization with a valuable human potential and with remarkable results in all the mentioned domains. Currently the famous scientific schools in the theory of algebraic rings (founded by acad. V. Andrunachievici), in the qualitative theory of differential equations (founded by acad. C. Sibirschi), in the theory of quasigroups (founded by Professor V. Belousov), in functional analysis (founded by cor.m. I Gorhberg) and in mathematical logic (founded by Dr. A. Kuznetov) continue their successful work and development. More than 30 doctors in habilitations and 400 PhDs were trained in the Institute.

During these 55 years over 3500 scientific papers and about 155 monographs have been published. The Institute publishes the journal "Buletinul Academiei de Stiinte a Republicii Moldova. Matematica" (from 1989 to this day 90 volumes were out), also the journal "Computer Science Journal of Moldova" (from 1993 to this day 80 volumes were out). Other journal in collaboration with colleagues from Poland, "Quasigroups and related systems", has published 27 volumes.

In the last 25 years the Institute took part in executing more than 60 projects with partners from advanced research centres in different countries of Europe, Asia, America.

The **Fifth Conference of the Mathematical Society of Moldova** is the international conference dedicated to this anniversary

and is a continuation of the tradition of reviewing new achievements in mathematics, informatics and information technology and information exchange with colleagues from major research centers across borders.

The organizers of the IMCS-55 edition are: Mathematical Society of the Republic of Moldova, Vladimir Andrunachievici Institute of Mathematics and Computer Science in cooperation with Tiraspol State University and Information Society Development Institute.

During the IMCS-55, the solutions for the most current problems that appear in various fields of mathematics and computer science as well as the socio-economic aspect of modern society are also addressed. The conference considers discussions between the research groups of universities, research institutes and the commercial IT field. In this regard, it is planned to establish scientific priorities for applying the solutions found by researchers to specific problems in society, national economy and abroad as well.

The volume includes 77 papers, covering topics in major areas of mathematics and informatics. It is divided in 3 chapters: Pure Mathematics, Applied Mathematics and Computer Science.

We express our sincere thanks to all people who contributed to the success of this event, especially from the Scientific Committee, who spent time carefully reviewing all the proposals submitted to IMCS-55 to insure a qualitative improvement of the papers, and from the Organizing Committee which ensured its high level.

Mitrofan Choban
Inga Titchiev

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Four color theorem – map solver

Natasha Stojkovikj, Mirjana Kocaleva, Jordan Miovski and Biljana Zlatanovska

Abstract

Today graph theory has grown into a significant area of mathematical research, with applications almost everywhere (in chemistry, operations research, ..). In this paper, the problem of map coloring with four colors with theory of graphs is considered. We considered the map of metropolitan area of Skopje. For coloring, the software "Four color theorem – map solver" is used.

Keywords: graph, graph theory, maps, map coloring, algorithms.

1 Introduction

Graphs are very practical mathematical model and they are used in various areas of science and everyday life. We are considering the problem of map coloring with four colors. The problem consisted of the possibility of coloring the world map with only four colors, but two neighboring countries must not be colored with the same color [1].

This problem can be considered with the theory of graphs. Graphs are very closely related to maps, as every map can be treated as a graph. A graph $G(V, E)$ consists of a finite non-empty set V whose elements are called nodes, vertices, or points and a finite set E of unordered pairs of distinct nodes, i.e. links of G [4], [5], [6].

2 Chromatic number on graph

A nodes coloring (or simply coloring) of a graph G, is a labelling $f : V(G) \rightarrow \{1, 2, ...\}$. The labels are called colors, such that no two adjacent nodes get the same color and each node gets one color. A k - coloring of a graph G consists of k different colors and G is then called k - colorable. It follows from the definition that the k - coloring of a graph $G(V, E)$ partitions the set of nodes V into k independent sets $V_1, V_2, ..., V_k$ such that $V = V_1 \cup V_2 \cup ... \cup V_k$. The independent sets $V_1, V_2, ..., V_k$ are called the color classes and the function $f : V(G) \rightarrow \{1, 2, ..., k\}$ such that $f(v) = i$, where $v \in V_i$, $1 \leq i \leq k$, is called the color function [2], [3].

For map coloring, we will use the following greedy algorithm. The algorithm is as follows:

Algorithm 1

1. Define Graph G.
2. Sort the nodes in descending order by degrees.
3. Assign color B_1 to the first node, and then to all nodes that are not neighbors to the previous node, but are not neighbors to each other.
4. Repeat the step 2, with color B_2 for the next uncolored node.
5. Repeat the step 3 while there are nodes without colors.
6. Finish.

3 Four color theorem - map solver

"Four color theorem – map solver" is a software which paints map with 4-coloring algorithm. In the Skopje there will be taken the following Municipalities: Novo Selo, Lepenec, Bardovci, Vizbegovo, Butel, Staro Skopje, Gazi Baba, Chair, Gorce Petrov, Centar 1, Karposh, Zlocucani, Vlae, Gorce Petrov 6, Mirce Acev, Sisevo, Saraj, Gorce Petrov 3. To establish a relationship with the original definition of a 4-color graph coloring algorithm, here the nodes of the graph will be the cadastral municipalities themselves, and the links between the nodes, in fact will be the borders by which they are separated (Fig.1.).

Regarding the color grading algorithm in 4 colors, to paint the metropolitan area of Skopje, 4 colors are needed. The steps of the

Figure 1. Map of the municipalities of Skopje, properly painted with 4 colors.

algorithm in the concrete case are the following (Fig.1.):

Step 1: Novo Selo has 3 neighbors (Gorce p.3, Mirce A., Lepenec); Lepenec has 5 neighbors (New V., Gorce p.6, Vlae, Zlocucani, Bardovci); Bardovci has 3 neighbors (Lepenec, Zlocucani, Vizbegovo); Vizbegovo has 4 neighbors (Bardovci, Zlocucani, Chair, Butel); Butel has 4 neighbors (Vizbegovo, Chair, Gazi B., Staro S.); Staro Skopje has 2 neighbors (Butel, Gazi B.); Gazi Baba has 4 neighbors (Staro S., Butel, Chair, Centar 1); Centar 1 has 3 neighbors (Karpos, Cair, Gazi B.,); Chair has 6 neighbors (Vizbegovo, Butel, Gazi B., Centar 1, Karposh, Zlocucani); Karposh has 5 neighbors (Centar 1, Chair, Zlocucani, Vlae, Gorce p.6); Zlocucani has 6 neighbors (Karposh, Chair, Vizbegovo, Bardovci, Lepenec, Vlae); Vlae has 4 neighbors (Gorce p.6, Karposh, Zlocucani, Lepenec); Gorce Petrov 6 has 3 neighbors (Vlae, Lepenec, Mirce A.); Mirce Acev has 4 neighbors (Gorce p.6, New V., Gorcep.3, Sisevo); Sisevo has 3 neighbors (Mirce A., Gorce p.3, Saraj); Saraj has 2 neighbors (Sisevo, Gorce p.3); Gorce Petrov 3 has 4 neighbors (Saraj, Sisevo, Mirce A., New V.)

Step 2 : We sort the municipalities in descending order by the number of neighboring municipalities they have.

Step 3 and Step 4: Chair and Zlocucani have the same number of neighbors, so we choose one of them. Chair is painted with one color (red) and all of its non-neighboring municipalities (Staro S., Lepenec, Gorce P.3) will be painted with the same color. After that, we paint Zlocucani with another color (blue) and all of its non-neighboring municipalities (Centar 1, Butel, New V., Sisevo, Gorce p.6) will be painted with the same color. Because Lepenec is already painted, we take the next municipality Karposh, and we paint it with a third color (green) and all of its non-neighboring municipalities (Gazi B., Vizbegovo, Saraj,

Mirce A.) will be painted with the same color. Because Vizbegovo, Butel and Gazi Baba are already colored, we take the next municipality of the list, and it is Vlae; Vlae is painted with a fourth color (yellow) and it is only one non-neighboring, no painted municipality and it is Bardovci, so Bardovci will be painted with the same color as Vlae. In the end, four colors are needed for coloring the map of Skopje. Like as example of municipality Shtip, also for this case we can find lower and upper bounds of chromatic number of the graph (Fig. 7). This graph is neither complete nor regular, so from corollary 1, we have that $3 \leq \chi(G)$. From the other side, we have that $\Delta(G) = max\{2, 3, 4, 5, 6\} = 6, \chi(G) \leq \Delta(G) = 6$, and $\chi(G) \in \{3, 4, 5, 6\}$. Using the Algorithm 1 we will find that chromatic number of this graph is 4.

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