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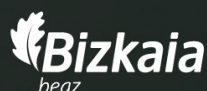
BILBAO **15-19** MAY 2017

ESGI {131} European
Study Group
with Industry

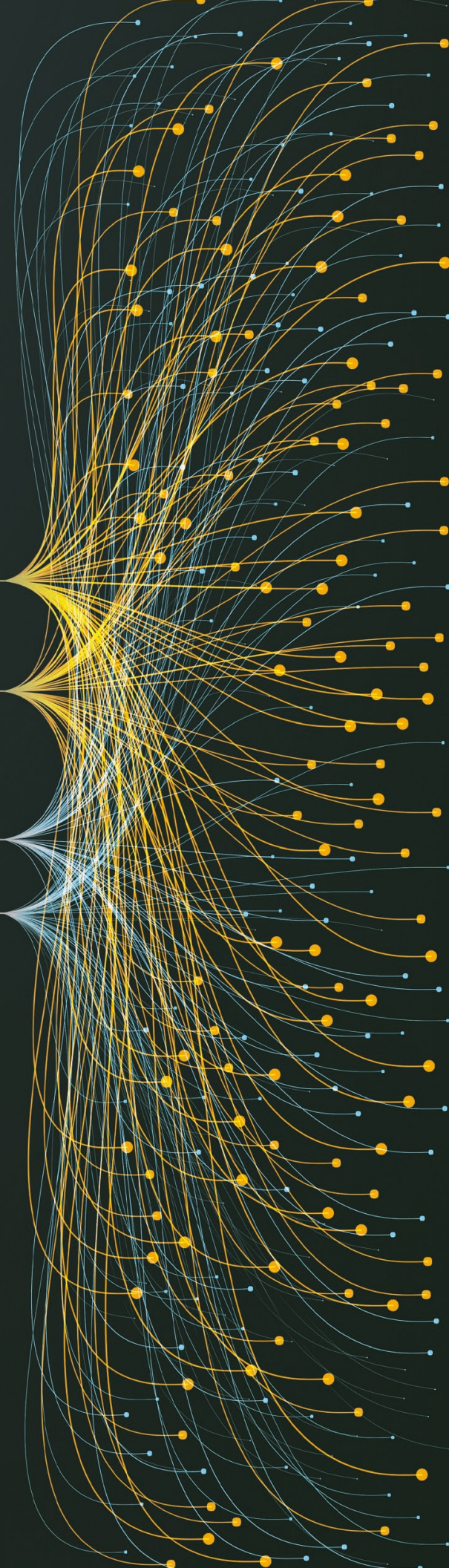
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IN COLLABORATION WITH



PARTICIPATING COMPANIES



**PROCEEDINGS OF THE
131st EUROPEAN STUDY GROUP WITH INDUSTRY**

BCAM – Basque Center for Applied Mathematics
Bilbao, 15–19 May 2017

ESGI {131}

EDITED BY

Ainara Gonzalez, Dae-Jin Lee and Nagore Valle (Basque Center for Applied Mathematics)
Contact: esgi131@bcamath.org

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Preface

The 131st European Study Group with Industry (131 ESGI) was held in Bilbao, Spain, from 15-19th May 2017. It was organized by BCAM - Basque Center for Applied Mathematics, in collaboration with BEAZ (public company of the Provincial Council of Bizkaia), MATH-IN (Spanish Network for Mathematics and Industry) and UPV/EHU (University of the Basque Country).

In addition, ESGI 131 was financially supported by the Basque Government, Severo Ochoa Excellence Accreditation and the Mathematics for Industry Network (MI-NET), COST Action funded project TD1409, which aims to facilitate more effective widespread application of mathematics to all industrial sectors, by encouraging greater interaction between mathematicians and industrialists.

Study Groups with Industry are an internationally recognized method of technology transfer between academia and industry. These one-week long workshops that started in Oxford in 1968 provide an opportunity for engineers and industrial developers to work together with academic mathematicians, students and young professional mathematicians on problems of direct practical interest. Their main objectives are to increase activity in Industrial Mathematics, spread awareness of the benefits of Mathematics, encourage the interaction between researchers from different areas and promote collaborative R&D projects between research groups from the academic sector and companies, addressing problems that can be solved with mathematical models and computational techniques. The Scientific Committee of ESGI 131 selected four problems to work on:

1. Parametric Design by Computational Fluid Dynamics Simulation (CIE Automotive)
2. Improvement of the Contact Center Performance (Eroski)
3. Self-Organized Networks (Fon Labs)
4. Big Data in Sports: Predictive Models for Basketball Player's' Performance (Xpheres Basketball Management)

At the beginning of the week, the 35 participants were divided into groups. Each group worked as a team on one of the problems proposed by the companies mentioned above. On the last day of the workshop, the working groups presented their progress in solving the problems and the recommended approaches. The study cases are assembled in this Study Group Proceedings Report, which provides a formal record of the work for both the industrial and the academic participants. The description of the problems, and the final reports of each working group, as well as a copy of this document, are posted on the website of ESGI 131: <https://wp.bcamath.org/esgi131/>

The Scientific Committee of ESGI 131 was formed by the following members:

- Elena Akhmatskaya, BCAM – Ikerbasque
- Laureano Escudero, Rey Juan Carlos University
- Luca Gerardo-Giorda, BCAM
- Carlos Gorria, UPV/EHU
- Dae-Jin Lee, BCAM
- Mikel Lezaun, UPV/EHU
- Jose Antonio Lozano, BCAM – UPV/EHU
- Ali Ramezani, BCAM

Challenges description

Challenge 1: Parametric Design by Computational Fluid Dynamics Simulation



DESCRIPTION

In the great majority of industrial problems involving fluid mechanics, turbulence effects must be taken into account. The physics of turbulence is extremely complex since non-linear effects lead to chaotic motion of the fluid, involving very different scales of space and time (multi-scale problem). In order to predict the fluid's mechanical behaviour, it is necessary to resolve the different scales as much as possible. Computational Fluid Dynamics (CFD) simulations of this type typically require considerable computational effort and time (in the order of weeks), for both designing the computational meshes and running the computer calculations.

Design optimization in the automotive industry is an interesting application of these type of numerical simulations; however, because they are currently unfeasible (due to their high computation cost), much faster prediction of the fluid flow is needed in order to run a design of experiments (DOE) with a combination of several parameters and multiple simulations.

OBJECTIVE

Implement effective techniques for accelerating the solution by:

- Identifying regions of the domain where the fluid flow characteristics are more influential for the problem, while saving computation cost by lowering the resolution in regions with lower turbulence intensity;
- Reducing the number of necessary numerical simulations with model reduction techniques (e.g. Proper Orthogonal Decomposition, Fast Fluid Dynamics, Scale-decoupling, etc.);
- Considering other effective techniques proposed by the participants.

Challenge 2: Improvement of the Contact Center Performance



DESCRIPTION

Since 2013, The Eroski Group has been immersed in the SIEC (Integral, Efficient Customer Service) Project, the objectives of which are:

- To provide a multipurpose contact point to internal customers.
- To define and implement the processes, technologies and equipment necessary to carry out that mission.

An important component of this project is the Contact Center, a specialized customer care team which carries out the tasks of contact with the customer, logging of incidents, correct escalation of incidents to specialist teams, and proper case closure.

OBJECTIVE

So that the Contact Center is efficient (i.e., it attends to customers with the appropriate measures according to the established service parameters), and taking into account that said contact center has a two-tiered structure (TIER 1 consisting of agents providing service in the first instance and TIER 2 teams composed of specialists in each area served), we seek a sizing model that allows us to properly manage the flow of incidents. Up to this point, we have only been able to use (ERLANG) sizing methods for TIER 1, having found no model for TIER 2. The proposed improvement project is directly related to optimizing resource allocation, reducing problem resolution time, and streamlining the process between the identification of a problem and the search for solutions. Some of the points and techniques that should be considered are:

- Proposal of a model that describes the operation of the Contact Center service, along with the flowchart indicating the order of stages, priorities, etc.
- Review of the indicators that are currently used to catalogue incidents and their level of resolution, and definition of new measurable variables in the model that are useful for examining solutions to the problem.
- Analysis of the efficiency of processes, software and technologies currently used.
- A study of some aspects of the theory of Operational Research, particularly in network flow models, queuing theory or stochastic programming to formulate a deterministic version of the model that is capable of applying optimization methods.

Challenge 3: Self-Organized Networks



DESCRIPTION

Fon is the world's leading carrier WiFi provider. Pioneers of residential WiFi sharing, we revolutionised carrier WiFi with our technology, creating a globally connected WiFi network. Today, we continue to innovate through two leading business areas. Fon Solutions offers best-in-class WiFi products and services. Our cutting-edge management solutions enable service providers to configure, deliver and operate their own WiFi services. Fon Network aggregates residential and premium carrier WiFi footprints creating one coherent global WiFi network. We facilitate WiFi interconnection between carriers, provide access deals to interested parties, and enable seamless user roaming. Fon's global clients include British Telecom, the Deutsche Telekom Group, SFR, Proximus, KPN, Cosmote, MWEB, SoftBank, Telstra, and Vodafone.

Designed specifically with Communications Service Providers (CSPs) in mind, Fon's cutting-edge WiFi Service Management Solution allows these companies to deliver WiFi services to subscribers and manage them just like cellular and fixed services, in a secure, scalable and flexible way.

WiFi networks are currently one of the main access technologies to the Internet, thanks to their low cost and easy deployment. However, their high density of WiFi access points may impact performance as the deployment is often unmanaged, unplanned, not coordinated in any way and consequently, far from optimal.

When a large number of WiFi hotspots are located within the same coverage area, it is likely that they operate in interfering frequencies with varying power levels. This has a severe impact on user performance due to the medium access mechanism defined in the 802.11 standard (CSMA/CA), whereby each user first listens to the medium and then only transmits if the listened channel is unoccupied.

OBJECTIVE

To develop an intelligent optimization algorithm to coordinate the frequency selection at the back end (for radio resource management purposes), in unmanaged, partially cooperative urban environments where not all the hotspots can be configured. The expected outcome of the algorithm is to:

- Minimize the number of interfering transmitters in the same contention domain, in areas where the spectrum is particularly crowded.
- Provide frequency channel planning at an urban district level.

Challenge 4: Big Data in Sports: Predictive Models for Basketball Player's' Performance



DESCRIPTION

Data analytics in professional sports has experienced rapid growth in recent years [1]. Development of predictive tools and techniques began to better measure both player and team performance. Statistics in basketball, for example, evaluate a player's and/or a team's performance [1,2].

Xpheres Basketball Management is one of the leading basketball player representation agencies in Spain and Europe. A database with men's professional basketball statistics from the last 16 seasons in more than 25 professional leagues and 71 FIBA tournaments has been obtained from **Aryuna**[®], a platform that allows performing advanced data analytics of men's professional basketball Statistics. The complete database consists of more than 37,000 games and upwards of 20,000 players.

OBJECTIVE

Based on a database, we aim to:

1. Characterize the performance curve, peak and optimal age in professional men's basketball using performance ratings of players in top European leagues.
2. Determine a rating correction factor for different basketball leagues, which accounts for intra-league and cross-league variability as well as for player characteristics (position, age, player ratings, etc.).
3. Determine which are the most important factors for predicting future outcomes (a successful professional career) of a basketball player.
4. Study statistical models to evaluate the performance of a player based on position, age, skills, league and other characteristics, and their influence in the game.

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[2] https://en.wikipedia.org/wiki/Basketball_statistics

Challenge 3: Self-Organized Networks

Academic coordinator Javier Del Ser (jdels@bcamath.org)

Institution BCAM, TECNALIA, UPV/EHU

Participants Aleksandra Stojanova¹, Dusan Bikov¹, Gorka Kobeaga³, Mirjana Kocaleva¹, Thimjo Koca⁴, Thomas Ashley⁴, Todor Balabanov⁶

Institutions Goce Delchev University, Macedonia¹, UPV/EHU – University of the Basque Country², BCAM – Basque Center for Applied Mathematics, Spain³, Autonomous University of Barcelona, Spain⁴, University of Seville, Spain⁴, Bulgarian Academy of Sciences, Bulgaria⁶.

Business coordinators José Pablo Salvador, David Valerdi

Company Fon Labs

ABSTRACT:

During recent years the number of WiFi networks has experienced rapid growth. The ever-growing number of wireless communications systems have made the optimal assignment of a limited radio frequency spectrum a problem of primary importance. With the common 802.11 wireless technology, few non-overlapping channels are available, and there is no standard mechanism for the access points to dynamically select the channel to be used in order to minimize interference with other access points. This has resulted in a situation where many WiFi networks use default or suboptimal channel assignments, leading to poor performance, and uneven spectrum usage. In this paper, we introduce three different approaches for the frequency assignment problem based on graph coloring that can significantly improve frequency distribution and reduce the number of collisions in the network.

Introduction

WiFi networks are one of the main access technologies to the Internet thanks to their low cost and easy deployment. However, this may impact performance as the deployment is often unmanaged, unplanned and clearly not optimal. Having a large number of WiFi hotspots within the same coverage area, increases the chance that they may operate in interfering frequencies with different power levels. This affects user performance due to the medium access mechanism imposed by the 802.11 standard (CSMA/CA), where each user first listens to the medium and only transmits if it is idle [1].

To access the medium, 802.11 employs a CSMA/CA (Carrier Sense Multiple Access with collision avoidance) MAC protocol. Briefly this protocol works the following way: a device that has a packet to transmit, first monitors the channel activity, and if the channel is idle for a predefined period of time, the device will transmit their packet. If the channel is sensed as busy, the device will defer its transmission (for a more detailed explanation on how 802.11 MAC protocol works, please check [7]). As transmissions take place in a shared medium there are two reasons for poor WiFi performance: co-channel interference (CCI) and adjacent channel interference (ACI). The first one, CCI, is when transmissions occur in the same frequency channel. The second source for poor WiFi performance, namely ACI, occurs when transmissions are sent on adjacent or partially overlapping channels. This second effect might not only defer ongoing transmissions, as mentioned above, but also corrupt transmitted frames, leading to an increased number of retransmissions and reduce the WiFi performance of the network. For that reason CCI is preferred over ACI [1].

The literature on frequency assignment problems (FAPs), also called channel assignment problems, has grown quickly over recent years. This is mainly due to the fast implementation of wireless telephone networks (e.g., GSM networks) and satellite communication projects. The renewed interest in other applications like TV broadcasting and military communication problems also inspires new research. Frequency assignment problems arise with application specific characteristics and draw heavily on ideas from graph coloring problems (GCPs) and both are NP-hard problems [5]. Frequency assignment problems are closely related, with the colors now being replaced by frequencies, and edges indicating where interference might occur if the same frequency were used at both ends. In FAPs, numerical labels are used to represent the frequencies, so that more general conditions can be handled than simply the requirement that pairs of frequencies at adjacent vertices be different [5, 6].

Researchers have developed different modeling ideas for handling interference among radio signals or the availability of frequencies, and the optimization criterion. There are different solution methods, which can be divided into two parts. Optimization and lower bounding techniques on the one hand, and heuristic search techniques on the other hand [2, 3, 4, 5, 6].

Our problem was proposed by FON (world's leading carrier WiFi provider), who were looking to develop an intelligent optimization algorithm to coordinate the frequency selection at the back end for RRM purposes (radio resource management), in unmanaged, partially cooperative urban environments where not all the hotspots can be configured.

The expected outcome of the algorithm was to minimize the number of interfering transmitters in the same contention domain, in areas where the spectrum is particularly crowded and provide frequency channel planning at an urban district level.

We proposed three different algorithms and their implementation in Python as a solution to this problem. These algorithms are: Generic algorithm [4], Iterated Local Search [2] and Reinforcement learning algorithm [3].

Iterated Local Search consists of the iterative application of a local search procedure to starting solutions that are obtained from the previous local optimum through a solution perturbation. Local search for the Graph Coloring Problem starts with some initial, infeasible, color assignment and iteratively moves to neighboring solutions, trying to reduce the number of conflicts until a feasible solution is found or a stopping criteria is met. The main goal of ILS is to build a biased randomized walk in the space of the local optima with respect to some local search algorithm. This walk is built by iteratively perturbing a locally optimal solution, applying a local search algorithm to obtain a new locally optimal solution, and finally using an acceptance criterion for deciding from which of these solutions to continue the search. The perturbation must be sufficiently strong to allow the local search to effectively escape from local optima and to explore different solutions, but also weak enough to prevent the algorithm from reducing to a simple random restart algorithm, which is known to typically perform poorly [2].

Reinforcement learning is a learning pattern, which aims to learn optimal actions from a finite set of available actions through continuously interacting with an unknown environment. In contrast to supervised learning techniques, reinforcement learning does not need an experienced agent to show the correct way, but adjusts its future actions based on the obtained feedback signal from the environment. There are three key elements in a RL agent, i.e., states, actions and rewards. At each instant a RL agent observes the current state, and takes an action from the set of its available actions for the current state. Once an action is performed, the RL agent changes to a new state, based on transition probabilities. Correspondingly, a feedback signal is returned to the RL agent to inform it about the quality of its performed action [3].

Reinforcement learning based local search (RLS) combines reinforcement learning techniques with descent-based local search. This approach, also can be used for solving well-known graph coloring problem (GCP) [3].

Summary

This challenge is related to development of an intelligent algorithm to coordinate the configuration of the frequency selection at the backend side for RRM purposes, in unmanaged partially cooperative urban environments where not all the hotspots will be configurable. The algorithm is based on the neighboring hotspots' list, their signal level and their frequency of operation. According the input data, the algorithm generates a frequency channel selection for interference mitigation leading to an optimized deployment of hotspots. It also optimizes the spectrum usage and overall improves user satisfaction given by a better quality of experience and overall higher bandwidth when accessing the network.

Depending on the specific WiFi technology, WiFi devices transmit in two different frequency bands: i) 2.4 GHz, which is typically crowded and ii) 5 GHz, and transmission in each band has different characteristics. Transmissions are configured to take place in a single channel. A WiFi channel has a bandwidth of 20 MHz. The 2.4 GHz band (802.11b/g/n) is divided into 13 channels separated 5 MHz, to that aim only 3 of them are non-overlapping (1, 6, and 11). This frequency channel separation is depicted in Fig. 1, where non-overlapping channels (1, 6 and 11) are highlighted in green.

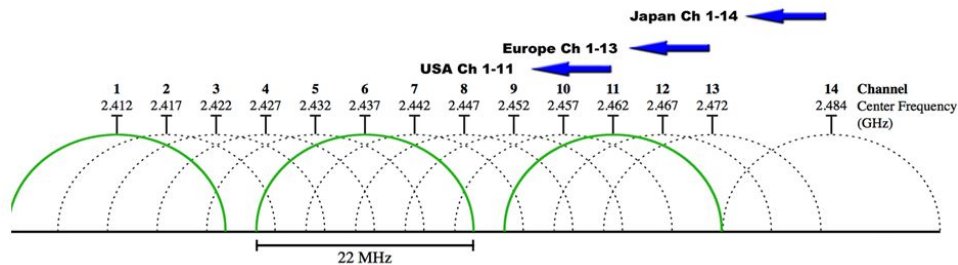


Figure 4: Frequency channel distribution for the 2.4 GHz band.

In the 5GHz band (11a/n/ac) the number of available channels depends on the channel bandwidth, which varies, according to the technology, among 20, 40, 80 or 160 MHz. In this band channels are separated 10 MHz, thereby there are 24 non-overlapping channels when the hotspots operate with a channel bandwidth of 20 MHz. Broader channels are also exposed to more noise or interference but this band is often less crowded than the 2.4 GHz, which allows for wider channels.

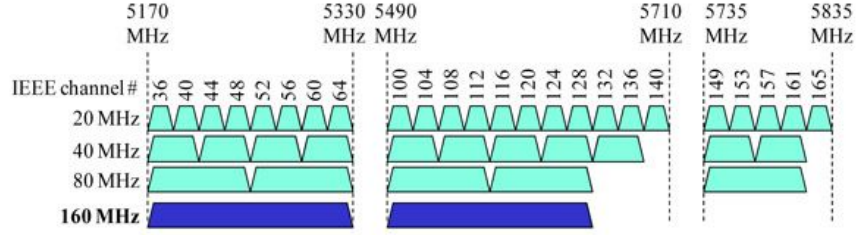


Figure 5: Frequency channel distribution for the 5 GHz band.

Taking everything into account, the objective function used to evaluate the solutions quality has been the following:

$$f(x) = 10 * \log_{10}(\sum II_i + BI_i)$$

where II_i and BI_i are the inner and the background interferences in Watts of the i -th AP, respectively. The inner interference for the i -th AP can be calculated in the following way:

$$II_i = \sum_{i \in N} \sum_{j \in N \setminus i} x_{i,j} I_{i,j} + \sum_{i \in N} \sum_{j \in N \setminus i} y_{i,j} I_{i,j}^*$$

where

$$I_{i,j} = \max_{f \in F} 10^{\frac{s_{i,j,f}}{10}}$$

$$I_{i,j}^* = (1 - \frac{1}{5} |ch(f_i) - ch(f_j)|) \cdot \max_{f_1, f_2 \in F} 10^{\frac{s_{i,j,f_1,f_2}}{10}}$$

$s_{i,j,f}$ and s_{i,j,f_1,f_2} are the signal power in decibels and

$$x_{i,j} = \begin{cases} 1, & \text{if the APs } i \text{ and } j \text{ have the same frequency } f \\ 0, & \text{otherwise} \end{cases}$$

$$y_{i,j} = \begin{cases} 1, & \text{if the channels of } i \text{ and } j \text{ satisfy } |ch(f_i) - ch(f_j)| \leq 4 \\ 0, & \text{otherwise} \end{cases}$$

In this work we have use the worst case interference to evaluate the frequency selection, but other variants of the evaluation function should be studied.

Algorithms

Iterated Local Search

The algorithm proposed in this section considers one solution during the search process and two parameters for the tuning of the algorithm. In the first step of the algorithm, a solution is initialized considering only the information of the background interference. For each FON AP we select the frequency that minimizes the interference with the non FON APs. Once we have an initial solution, this is iteratively modified towards better solutions. At each iteration of the algorithm, *param2* FON

APs are selected and their frequency setting is modified. The number of APs to be changed, $param2$, is an input parameter of the algorithm. The selection of the APs is done based on the interference proportional selection, the APs with a higher interference have a higher probability to be selected.

Algorithm 1 Iterated Local Search

```

for each FON node do
    Select frequency  $F_i$  that minimizes the background interference;
end for
Evaluate the total interference  $I$  related to the frequency selection  $F$ ;
 $it = 0$ 
while  $it \leq param1$  or  $I \neq 0$  do
    Update the probability of selecting a node,  $p_i = I_i / \sum I_i$ ;
    Select  $param2$  number of nodes using distribution  $(p_1, \dots, p_N)$ ;
    for each selected node do
        Define  $p_f = I_i^f / \sum I_i^f$  as the probability of selecting frequency  $f$ .
        Calculate the inverse,  $p_f = (1 - p_f) / \sum (1 - p_f)$ ;
        Sample ones distribution  $(p_1, \dots, p_{|F|})$  to select one frequency  $f$ ;
         $F_i' = f$ ;
    end for
    Evaluate total the interference  $I'$  related to the frequency selection  $F'$ .
    if  $I' \leq I$  then
         $F = F'$ ,  $I = I'$ ,  $it = 0$ ;
    else
         $it = it + 1$ ;
    end if
end while

```

For each selected FON AP, i , we evaluate the interference of each possible frequency setting, I_i^f . On the contrary to the initialization, I_i^f considers both the inner interference and the background interference. The channels with lower interference have a higher probability to be selected. After performing the frequency modifications in the APs, we evaluate the goodness of the solution. If the obtained solution is better than the solution that we had at the beginning of the iteration, the modified solution is saved and the count of iterations without improvement is restarted. Otherwise, the modified solution is excluded the count of iterations without improvement is increased by one. The algorithm stops when a certain number of iterations, $param1$, is reached without improvement or when a solution with null interference is found.

Reinforcement Learning Based Local Search

Frequency assignment problems (FAP) can be presented as a subset of graph coloring problem (GCP). As it was described in [3], the graph coloring problem can be attacked with reinforcement learning based local search (RLS). It is a combination of reinforcement learning techniques with local search. In most cases FAP and GCP are NP-hard and that is why they are computationally challenging. If the problems are attacked with exact numerical methods exponential times are expected. The opposite, heuristic and metaheuristic methods are often referred in finding acceptable sub-optimal solution in satisfactory time limit. The negative side of second approach is that solution optimality is not

guaranteed.

Genetic Algorithm

Genetic Algorithms (GA) lie at the core of Evolutionary Computation, a discipline under the wide umbrella of Computational Intelligence that focuses on the exploitations of principles and processes observed in the evolution of species in Nature to construct self-learning methods for optimization and pattern analysis. GA resort to concepts such as genotype inheritance and controlled mutation so as to efficiently explore a search space encoded as chromosomes (individuals or candidate solutions), which undergo probabilistically drive crossover and mutation operators so as to evolve them towards solutions of enhanced optimality for the problem at hand.

A canonical version of the GA utilized for this challenge is described in Algorithm 2. It should be remarked that the crossover and mutation operators included in the above description can be replaced with tailored alternatives more suited to the problem being tackled, possibly by incorporating heuristic information from the problem statement. Likewise, the initialization of the population of individuals can be also performed in a directed manner, e.g. by assigning more probability of initial assignments to those channel(s) with least background interference levels. This, however, can be worked out further beyond the duration of the challenge.

Algorithm 2 Canonical Genetic Algorithm

```
Initialize a population of  $P$  individuals at random from the set of available frequency channels;  
Evaluate the total sum interference of the network (fitness) as the sum of the interference received  
by every FON AP in its selected frequency;  
for  $it = 0$  to  $I$  do  
  for  $p = 0$  to  $P$  do  
    With probability  $P_c$ , select two parents from the population based on a selection criterion  
    (e.g. Tournament, Roulette-Wheel), and recombine them by means of an uniform crossover  
    operator;  
    With probability  $P_m$ , mutate the offspring by using a random mutation strategy;  
    Evaluate the total sum interference associated to the produced offspring;  
    Add the offspring to the population;  
  end for  
  Sort the population in increasing order of the sum interference associated to each chromosome;  
  Remove the least fit individuals from the extended population, and keep only the best  $P$   
  solutions;  
end for
```

Computational Experiments

In this section, we compare the performance of the GA, RLS and ILS. In order to carry out the comparison, the algorithms have been implemented in Python and run 10 times.

ILS, in all of the runs, was able to find a frequency configuration with null total interference. The mean computational time needed to find such a solution was 285.04 seconds. Results for the GA scheme with $P_c = 0.5$ and $P_m = 0.1$ are depicted in Figure 7 as a boxplot computed over 10 different Monte Carlo realizations. In light of the increased computational complexity of this approach

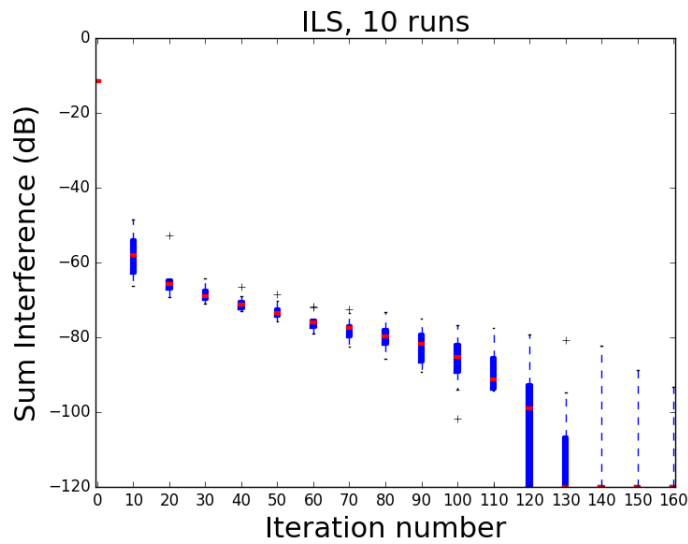


Figure 6: ILS convergence by iterations.

with respect to ILS (330.4 seconds on average) and the lower quality of the produced results our recommendation is to opt for ILS as the practical solution for this problem. It should be noted that GA does not exploit any problem-specific knowledge during its search procedure, hence solutions are provided *blindly* in regards to the particularities of the problem at hand. Incorporation such a knowledge to the definition of GA could be certainly undertaken as future work.

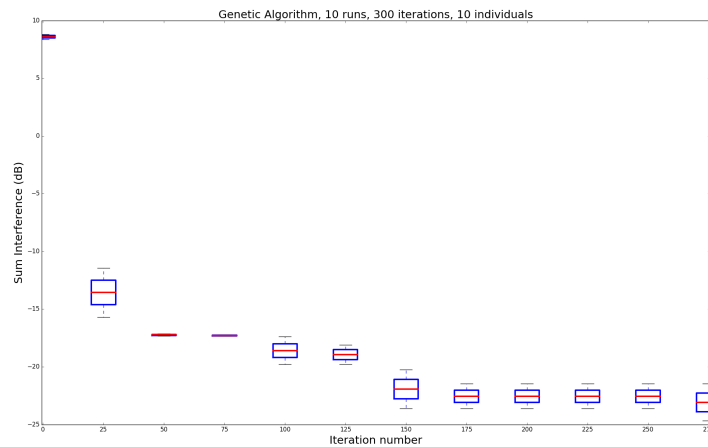


Figure 7: GA convergence by iterations.

At the time of delivery of this report no results were reported by the team for the RLS approach.

Conclusions

As a solution to the problem we proposed Generic algorithm, Iterated Local Search and Reinforcement learning algorithms. By implementing these algorithms in Python and testing them with a given data, we obtained similar results from all three. These three approaches have their advantages and disadvantages but can complement each other. We notice that the heuristic optimization that we used can be very useful for these kind of calculations, but because a large data set was used, these methods can be very time consuming. As further work, we can combine these algorithms or extend the Genetic Algorithm with Iterated Local Search and Reinforcement Learning based Local Search for obtaining better results and more effective solutions. Another further task could be to improve the visualization of the network for better understanding of the obtained results.

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