SP.086. Prove that if a, b, c are the lengths's sides in triangle ABC then:

$$\sin^2 a + \sin^2 b + \sin^2 c \geq 4 \sin s \sin(s-a) \sin(s-b) \sin(s-c)$$

Proposed by Daniel Sitaru - Romania

SP.087. Let z_1, z_2, z_3 be the affixes of A, B respectively C in acute-angled ΔABC.
Prove that:

$$\prod\left(\left|\frac{z_2 - z_3}{z_2 + z_3}\right| + \left|\frac{z_3 - z_1}{z_3 + z_1}\right|\right) \geq \frac{32sr^3}{(s^2 - (2R + r)^2)^2}$$

Proposed by Daniel Sitaru - Romania

SP.088. Let $a, b, c > 0$ such that $ab + bc + ca + abc = 4$.
Prove that

$$(a+1)\sqrt{(b+1)(c+1)} + (b+1)\sqrt{(c+1)(a+1)} + (c+1)\sqrt{(a+1)(b+1)} \geq$$
$$\geq a + b + c + 9$$

Proposed by Nguyen Ngoc Tu – HaGiang –Vietnam

SP.089. Let r_a, r_b, r_c be the exradii of a triangle ABC, h_a, h_b, h_c the altitudes and let R, r, s denote the circumradius, inradius and semiperimeter respectively. Prove that

$$\frac{r_a^2}{h_a} + \frac{r_b^2}{h_b} + \frac{r_c^2}{h_c} \geq \frac{2s^2}{3}\left(\frac{1}{r} - \frac{1}{R}\right)$$

Proposed by Martin Lukarevski – Skopje - Macedonia

SP.090.If $u, v > 0$, with $2u - v > 0$ and α, β, γ are the measures of the angles of triangle ABC, then

$$\sum_{cyc} \frac{\sin\alpha}{u \sin\beta + v\sqrt{\sin\alpha \sin\beta}} \geq \frac{3}{u + v}$$

Proposed by D.M. Bătinețu – Giurgiu; Neculai Stanciu – Romania

UNDERGRADUATE PROBLEMS

UP.076. Evaluate:

$$S = \sum_{n=1}^{\infty}\left(\frac{H_{2n+1}}{n^2}\right)$$

Proposed by Shivam Sharma – New Delhi – India