MATH-LABYRINTH method

INCREASING THE LEVEL OF KNOWLEDGE THROUGH SOLVING MATHEMATICAL PROBLEMS

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2015-1-MK01-KA201-002849

GUIDELINES

Funded by the
Erasmus+ Programme
of the European Union

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for the
MATH-Labyrinth method

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2017

NATIONAL AGENCY for European Educational Programmes and Mobility

Funded by the Erasmus+ Programme of the European Union

PUBLICATION

Guidelines for the Math Labyrinth method

PUBLISHING

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EDITION 200

CIP - Каталогизација во публикација
Национална и универзитетска библиотека "Св. Климент Охридски", Скопје

51:377.02-043.5(036)

GUIDELINES for the math-labyrinth method : Increasing the level of knowledge through solving mathematical problems / Mariche Lazarovska ... [и др.]. - Veles : SOU "Gimnazija Koco Racin", 2017. - 91 стр. : илустр. ; 21 см

Автори: Mariche Lazarovska, Hristina Leova, Zoran Trifunov, Berna Musardo, Mariella D'Adamo, Anna Maria Mele, Tommaso Fama', Carlo Calo', Pavleta Zhelyazkova, Neli Georgieva, Teodora Stavreva, Tsvetinka Oresharova, Galina Hrisandova, Gregory Makrides, Andreas Skotinos, Anargyros Fellouris, Tatjana Atanasova - Pachemska, Marija Miteva, Limonka Lazarova. - Библиографија: стр. 89-90

ISBN 978-608-66136-0-0
1. Lazarovska, Mariche [автор]
а) Математика - Методика - Интерактивен пристап - Прирачници
COBISS.MK-ID 105178378

CONTRIBUTION FOR THE PREPARATION OF THE GUIDELINES

The Guidelines is the outcome of the collaborative work of all the Partners for the development of the European Erasmus+ Math-Labyrinth Project, namely the following:

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Partner Organization:

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6. Goce Delcev State University Stip, Macedonia (Tatjana Atanasova – Pachemska, Marija Miteva, Limonka Lazarova)

The complete output of the project Math Labyrinth consists of the here present Math-Labyrinth Interactive Book and a Guidelines, Collection of Good Practices, a Teacher Training Course and an Evaluation Report. You can download them all from the website www.math-labyrinth.eu

Table of Contents

INTRODUCTION TO THE MATH- LABYRINTH PROJECT

In accordance with the priorities set by the European Commission, and in relation to the needs of the teachers for new approaches in teaching Mathematics, a project **MATH-Labyrinth** was submitted for Erasmus+ funding under Key Action 2 - Strategic Partnerships for innovation and exchange of good practices in 2015.

Students consider mathematics as a difficult discipline and they certainly lack motivation and interest in acquiring skills and competences in this particular subject. The number of low-achieving students in math in high schools around Europe is quite large. On the other hand, they need certain knowledge to pass the examinations at the end of their education. The six organizations in this partnership focused on motivating these students to begin doing math. One of the approaches we suggest is to relate mathematics with problems that students encounter on daily basis.

Many everyday situations and problems require mathematics in order to be solved, thus the aim of the MATH-Labyrinth project is to increase the level of knowledge in Math of low-achieving students through solving mathematical problems.

One of the main objectives of the MATH-Labyrinth project is: **an Interactive book with real-life mathematical problems**. MATH-Labyrinth project intends to develop new methodologies in learning and teaching mathematics to students of age 14-18 which can be used in any school environment.

The aim of this project is the development of a methodology in teaching and learning mathematics with the creation of interactive book that can be used by teachers and students. This book will put the students in the centre of a situation and it will challenge them to begin solving problems and eventually reaching the solution. Through providing help at every stage, the book intends to increase the motivation and the students' understanding of the problem. At different stages students are able to get additional hints in the form of pictures, presentations, videos etc. that will enable them to move forward in the "Labyrinth" and get out of it with a solved problem.

This output is a sample of teaching material and methodology for teaching mathematics mostly to pupils between the age of 14 and 18. The interactive guidebook is comprised of mathematical problems with different degrees of complexity.

The areas in mathematics covered in the contents of the book are relevant to everyday situations and to the syllabuses of the national examinations. It is developed to enhance the brain's ability to visualize and transform knowledge into a solution to a real-life problem.

The teachers are supposed to be students' mentors of creativity. They will help them in making the selection of real-life mathematical problems to be solved by their fellow students. This peer-to-peer approach gives an insight on how pupils develop abilities to think, reason and understand problems, and at the same time teaches those low-achieving students how to understand fractions, geometric puzzles and problems, among other.

The name Labyrinth refers to the complexity of providing solutions; in order to solve a problem of this kind a couple of operations are required and students will have to go back and forth through all the acquired knowledge they have during their education.

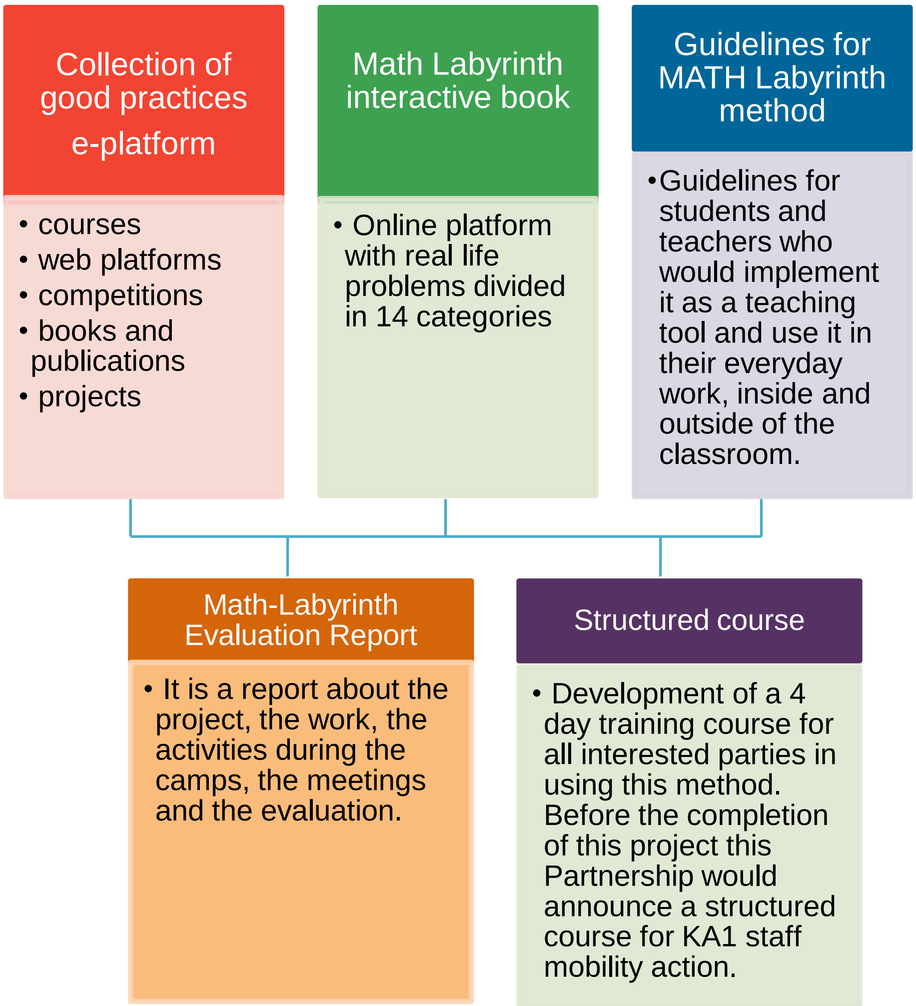
The purpose of the book is not giving them direct answers, but making them think and learn at the same time. The core of it is learning the most common operations and relations and using them in their everyday life. Teachers provide clues and paths towards solving the defined problems and a step by step approach that grabs the attention of students and inspire them to get excited about maths.

Math Labyrinth project gives the following outcomes:

- innovative approach to teaching math and integrated methodology of good practices;
- reduced disparities in learning outcomes affecting all learners, especially underachievers;
- ICT-based methodologies for learning Math and providing more attractive education and training;
- creation of an e-platform for teaching and learning mathematics;

- improved assessment of the key-competences (mathematics and digital skills);
- implementing innovative practices in education and use of open educational resources;
- enhanced professional development of teachers involved in the process of education;
- involvement of students in the process of learning.

STRUCTURE OF THE EUROPEAN ERASMUS+ PROJECT MATHLabyrinth



1. **Collection of good practices** inside and outside Europe is an electronic database. This collection of good practices is a summary of some practices in teaching and learning mathematics that are relevant to the students' age and interest, and it is uploaded on the web platform for future reference.
   The Collection contains web platforms, courses, competitions and contests, books and publications and projects which are considered to be good practices in and outside Europe. Link to the output: http://www.math-labyrinth.eu/o1/collection/

2. **Math Labyrinth book** is an online platform where teachers and students can learn and co-create contents for teaching and learning. The areas in mathematics covered in the contents of the book are relevant to everyday situations and to the syllabuses of the national examinations. It is developed to enhance the brain's ability to visualize and transform knowledge into a solution to a real-life problem. By using this platform and the interactive book, there will be an opportunity for the students to feel more confident to begin doing tasks and solving problems, because the Interactive book gives the students a step by step approach. The method MATH-Labyrinth involves giving clues and hints to students. Link to the output: http://www.math-labyrinth.eu/math-labyrinth-interactive-book/

3. **Guidelines for the MATH Labyrinth method** is intended for teachers who will use this particular method of teaching in their classroom as curricular or extracurricular activity. It provides the aims and objectives of the interactive book, the methodology of getting to the solution of all real-life mathematical problems in it, lesson plans and some useful links, resources and explanations on using different ICT tools.

4. **Math-Labyrinth Evaluation Report** contains forms and analysis of the reports related to the project, the work, the activities during the lessons, the summer camps in schools, and the meetings.

5. **Structured course** is a 4 day-training course for all interested teachers in using this method. This course is announced as KA1 course staff mobility action and can be found on the website of the project or other sites.

METHODOLOGY OF SOLVING PROBLEMS

The goals of mathematics

Mathematics is "the queen of sciences," a definition originating from the power and the brilliance of its results.

Any scientific or technical discipline, from engineering to physics, from economics to information technology, makes an extensive use of tools of analysis, calculation and modeling offered by mathematics.

As an expression of the human mind, mathematics reflects the active will, the contemplative reason, the desire for aesthetic perfection. Its key elements are the logic and the intuition, the analysis and the construction, the entirety and the individuality.

Mathematics is a very challenging subject for the students. The younger generation, who have lived and breathed TV and Internet, do not possess the ability to the intense and prolonged concentration which the study of a complex subject like mathematics and the solving of problems require.

But, if students dislike mathematics, actually they are victims, in some ways. Victims of outdated syllabuses full of endless sequences of jargon, victims of arid and boring teaching methods that should be revitalized and innovated by means of innovations and modernity. The only way to understand and practice mathematics is to make reason and logic become a kind of a “second nature”.

In fact, you can talk about and discuss mathematics without fear: this is the aim of educating the students to "mathematical models." Educating to modeling involves a different way of proposing the study of mathematics, aimed at describing and understanding the real world.

It is very important for us to learn the basics of mathematics no matter how hard it can get. We are facing and we deal with mathematics everyday and everywhere around us. So, mathematics truly is essential in the survival and the progress of each person.

Main goals of mathematics education are to prepare students to:

- solve problems
- communicate and reason mathematically
- make connections between mathematics and its applications
- become mathematically literate
- appreciate and value mathematics
- make informed decisions as contributors to society

So, we must promote Mathematics through problem solving in school education for improving student success and deepening learning.

Problem Solving Techniques

When we say Problem Solving, we don't restrict ourselves to thinking about mathematical problems, but we think about wider goal of education, to motivate students to achieve skills to solve problems.

Mathematical problems can be solved with processes, using the skills creatively in new situations.

So we can say, Problem Solving is a mathematical process. This is the beauty of mathematics that enables us to use the skills in a wide range of situations.

George Polya wrote “How to Solve It” in 1945 as a general guide for solving both mathematical and non-mathematical problems, describing four stages of problem solving.

Polya’s First Principle: Understand the problem

There is no chance a student to be able to solve a problem unless he/she can first understand it. It will almost always be necessary to read a problem several times, both at the start and during working on it. Some of the questions in this phase of problem solving are:

- Do you understand all the words used in the problem?

- Do you know what is given?
- Is there enough information to enable you to find a solution?
- Can you restate the problem in your own words?
- Can you think of a picture or diagram that might help you understand the problem?
- Is this problem similar to another problem you have solved?

Polya's Second Principle: Devise a plan (Find a strategy)

There are many reasonable ways to solve problems. The skill at choosing an appropriate strategy is best learned by solving many problems. You will find choosing a strategy increasingly easy. A partial list of strategies is included:

- Guess and check
- Use a variable
- Make a list
- Eliminate possibilities
- Use symmetry
- Consider special cases
- Use direct reasoning
- Solve an equation
- Do a simulation
- Draw a picture
- Look for a pattern
- Solve a simpler problem
- Draw a diagram
- Use a model
- Work backwards
- Use a formula
- Be ingenious
- Use coordinates

Polya's Third Principle: Carry out the plan

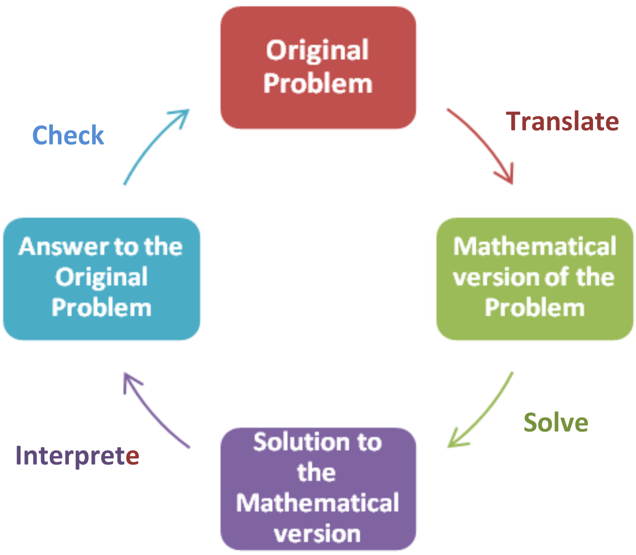
- Implement the strategy or strategies that you have chosen until the problem is solved or until a new course of action is suggested.
- Give yourself a reasonable amount of time in which to solve the problem. If you are not successful, seek hints from others or put the problem aside for a while.

- Do not be afraid to start over. Often, a fresh start and a new strategy will lead to success.

Polya's Fourth Principle: Look back to problem's solutions

If you look back to a problem's solution you will have reflection of what worked, and what didn't. Doing this will enable you to predict what strategy to use to solve future problems.

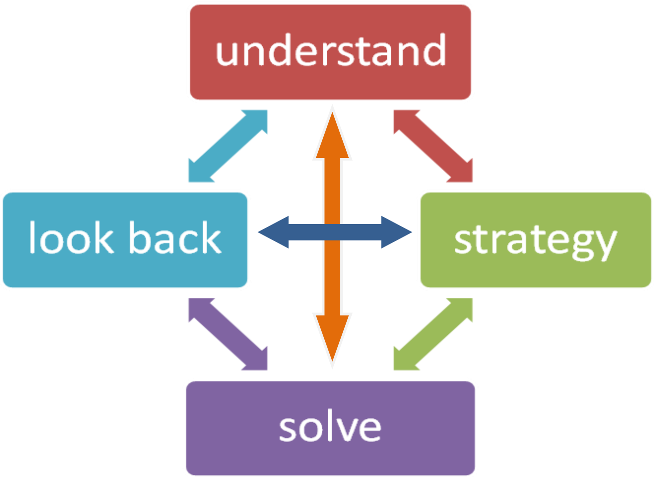
- Is your solution correct? Does your answer satisfy the statement of the problem?
- Can you see an easier solution?
- Can you see how you can extend your solution to a more general case?



This diagram refers to the process of solving a problem through mathematics.

For more difficult problems it may not be possible to easy move through four stages of Problem Solving in chronological order to produce an answer. It is frequently the case that students move backwards and forwards between and across the steps.

What students usually face with in practice is shown on this diagram:



Math Labyrinth methodology

The Math Labyrinth method refers to the strategy of solving real life problems through math, in a step by step approach, by giving hints and clues to students. The Math Labyrinth method is a student-centered approach, because students play an active and participatory role in their own learning.

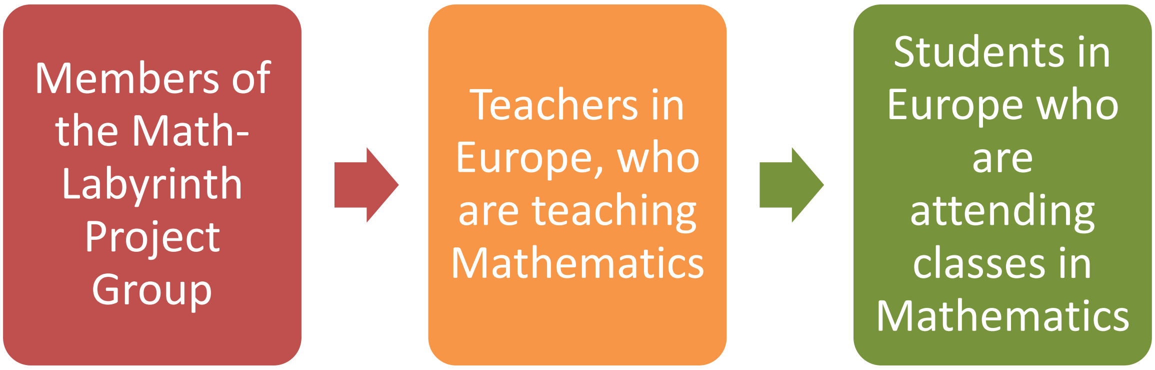
The name Labyrinth refers to the complexity of providing solutions; in order to solve a problem of this kind a couple of operations are required and students will have to go back and forth through all the acquired knowledge they have during their education. The method Labyrinth involves visualization of the problems for better understanding, using free software like GeoGebra.

The purpose of the interactive book is not giving direct answers to students, but making them think and learn at the same time. The core of it is learning the most common operations and relations and using them in their everyday life. Students will recognize that mathematics permeates the world around us.

They will appreciate the usefulness, power and beauty of mathematics and enjoy mathematics and develop patience and persistence when solving problems.

ADVANTAGES FOR TEACHERS AND STUDENTS USING THE MATH-LABYRINTH METHOD

Target groups of the European Erasmus+ Project Math- Labyrinth



- improved competences in an innovative approach to teaching math;
- improved competences of addressing low achievement in basic skills through more effective teaching methods;
- increased level of integrated the teaching of basic skills in maths, promoting problem-based learning;
- increased level of use of ICT-based methodologies for learning math and providing more attractive math education;
- increased level of promoting access to and learning through Open Educational Resources (OER);
- increased motivation and satisfaction in their daily work.

Impact on target groups

A) students

- increased knowledge of mathematics with teaching and learning through the innovative approach;
- opportunity for the student to learn math in an interactive manner with effective methods and techniques of work;

- increased access to ICT, free software and open educational resources;
- increased access to the online platform with mathematical tasks, methods and interactive problem tasks that will facilitate the learning process;
- increased possibilities to communicate with other students from different countries or other participating countries through the online platform in order to get additional information and/or custom programs, tools etc.
- increased level of digital competence, especially regarding OER and online platform;
- students who will love to study math;
- with more positive attitude regarding school education and the role of education in their future career;

B) math teachers (and other teachers in secondary education)

- improved competences in an innovative approach to teaching math;
- improved competences of addressing low achievement in basic skills through more effective teaching methods;
- increased level of integrated the teaching of basic skills in maths, promoting problem-based learning;
- increased level of use of ICT-based methodologies for learning math and providing more attractive math education;
- increased access to OER and IT tools based on scientific information and international cooperation to use in their activities, including a new curricula and methodology;
- increased level of digital competence, especially regarding OER and online platform;

WHAT IS MATH-LABYRINTH

The creation of this interactive e-book has as its primary target to give to upper secondary school students a tool to help them overcome the difficulties they face when studying Mathematics. A tool that tries to bridge the gap between the abstract contents of mathematics and the real life.

All the problems in the e-book are "real-life problems" that accompany the students throughout their studies until their final high school exams.

It is not easy to solve problems, especially those ones that require a combination of knowledge.

Then, the e-book uses a method, the Math-Labyrinth method, which contains suggestions on how these problems should be solved. It is a method that allows students to get to the solution of the problem thanks to these suggestions.

Every real life problem, from the simplest to the most complex one, has been modelled. When the student faces a problem, he will be invited to solve it by himself: if he cannot create the mathematical model that describes the problem, he will receive a series of aids that will lead him find the solution.

The first aid consists of a tip that may be information, a graphic or a link.

After having "asked" for the tip the students should be able to continue, or check the answer to the given hint. After that, he can move on to the following suggestion and then, step by step, until the solution of the problem.

Practically, the Math-Labyrinth method leads the student to recognize in a real-life context an expression, a formula, an equation or a function, which can be treated as a mathematical tool: the solution of a real life problem is therefore connected to the resolution of a mathematical problem.

MATH LABYRINTH IN THE "GEOPEDAGOGICAL" PERSPECTIVE OF LIFELONG LEARNING

European countries share the same strategic objective in education, even in the distinctiveness of their own schooling systems and education policies: to let pupils and students become autonomous and independent learners, who can take an active part in the knowledge society and contribute to the future growth and social cohesion of the European Union. That comes from the idea of the key priority of education in helping a young person develop his or her full potential and become an active participant in society.

What is more, education is thought of as a lifelong process, which never stops. No matter how young or old you are: we need to develop our skills and competences throughout our lives, both for personal fulfillment and for being successful in the constantly changing world of work.

These strategic assumptions were made the "geopedagogical" core of The Recommendation of the European Parliament and of the Council of 18th December 2006 on key competences for lifelong learning.

It set out the minimum knowledge, skills and attitudes which all pupils need to acquire by the end of initial education and training, if they are expected to become autonomous and independent learners. They called them "key competences": as a combination of knowledge, skills and attitudes appropriate to the context, they allow you to open the road towards personal development, active citizenship, employment.

Mathematical competence is one of the eight key competences set out by the 2006 Recommendation. As a key competence, the emphasis is on process and activity, as well as on knowledge; a person who develops mathematical competence is somebody with the ability to apply mathematical thinking to solve a wide range of problems, in everyday life.

Of course, developing that kind of competence implies an approach to teaching that goes beyond traditional subject boundaries and beyond the idea that being mathematically competent only means to know concepts, terms, topics, results and methods.

As in some mathematics educators' assumptions (e.g. Hans Freudenthal, Heinrich Winter among the first) mathematical mastery is largely concerned with the functional use of mathematics, which again goes beyond factual knowledge and procedural skills. It indeed involves cognitive behavior levels, the ability to engage in problem solving, including heuristics, and coming to grips with essential phenomena in nature, society and culture.

Within this context and perspective, Math Labyrinth becomes an innovative and challenging teaching tool and strategy to develop a person's possession of mathematical competence.


DISCOVER THE 8 KEY COMPETENCES FOR LIFELONG LEARNING
COMMUNICATION IN THE MOTHER TONGUE
The ability to express and interpret concepts, thoughts, feelings, facts and opinions in both oral and written form (listening, speaking, reading and writing) and to interact linguistically in an appropriate and creative way in a full range of societal and cultural contexts
01
COMMUNICATION IN FOREIGN LANGUAGES
Involves, in addition to the main skill dimensions of communication in the mother tongue, mediation and intercultural understanding. The level of proficiency depends on several factors and the capacity for listening, speaking, reading and writing
02
MATHEMATICAL COMPETENCE AND BASIC COMPETENCES IN SCIENCE AND TECHNOLOGY
The ability to develop and apply mathematical thinking in order to solve a range of problems in everyday situations, with the emphasis being placed on process, activity and knowledge. The mastery, use and application of knowledge and methodologies that explain the natural world
03
DIGITAL COMPETENCE
Involves the confident and critical use of information society technology (IST) and thus basic skills in information and communication technology (ICT)
04
LEARNING TO LEARN
Is related to learning, the ability to pursue and organise one's own learning, either individually or in groups, in accordance with one's own needs, and awareness of methods and opportunities
05
SOCIAL AND CIVIC COMPETENCES
Refers to personal, interpersonal and intercultural competence and all forms of behaviour that equip individuals to participate in an effective and constructive way in social and working life. Equip individuals to engage in active and democratic participation
06
SENSE OF INITIATIVE AND ENTREPRENEURSHIP
Is the ability to turn ideas into action. It involves creativity, innovation and risk taking, as well as the ability to plan and manage projects in order to achieve objectives. It is the foundation for acquiring more specific skills and knowledge needed by those establishing or contributing to social or commercial activity.
07
CULTURAL AWARENESS AND EXPRESSION
Involves appreciation of the importance of the creative expression of ideas, experiences and emotions in a range of media (music, performing arts, literature and the visual arts)
08

CONTENTS OF THE INTERACTIVE BOOK

The areas in mathematics covered in the contents of the book are relevant to everyday situations and to the syllabuses of the national examinations. There are different levels of difficulty of the exercise from easy to more complex. All the exercises are divided into 14 categories:

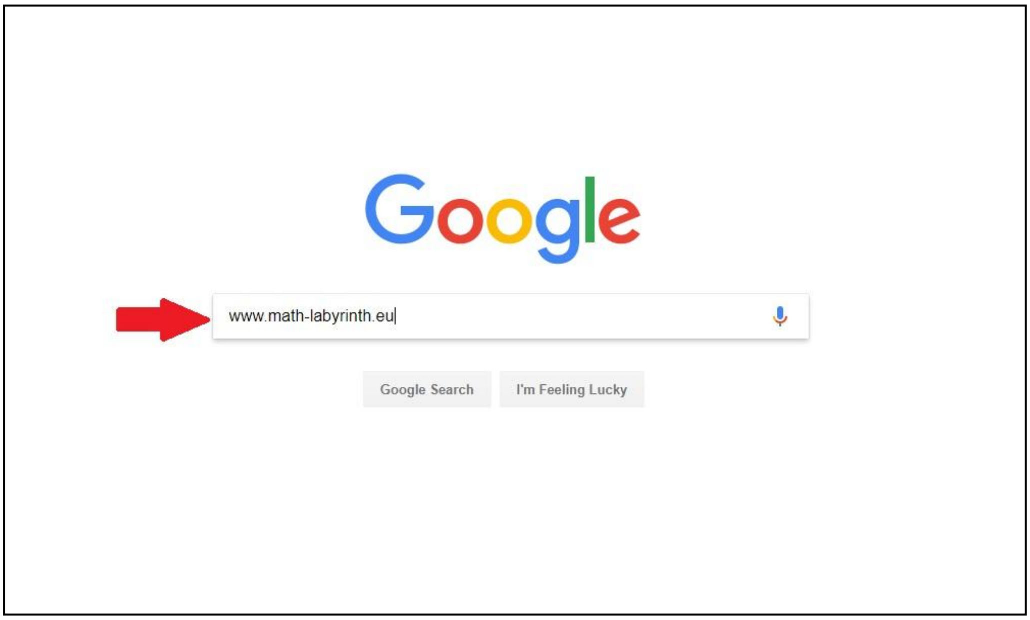
1. Equations, inequalities and systems of equations
2. Construction of triangles and squares
3. Functions and their applications
4. Proportions of quantities
5. Perimeter and area of plane figures
6. Area and volume of solids
7. Differential calculus
8. Definite integral and applications
9. Combinatorics
10. Probability
11. Sequences and progressions
12. Trigonometry
13. Analytical geometry
14. Other

HOW TO USE MATH LABYRINTH

Type

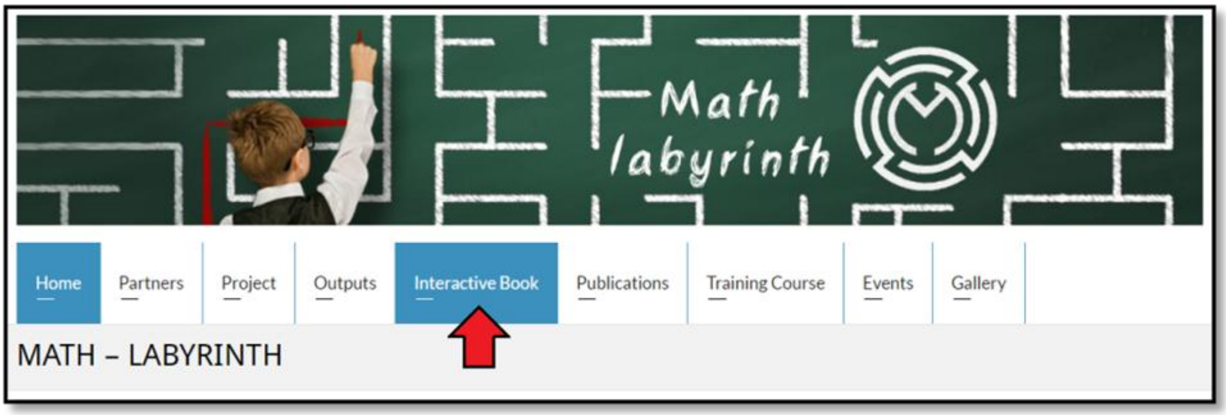
http://www.math-labyrinth.eu/

on your browser



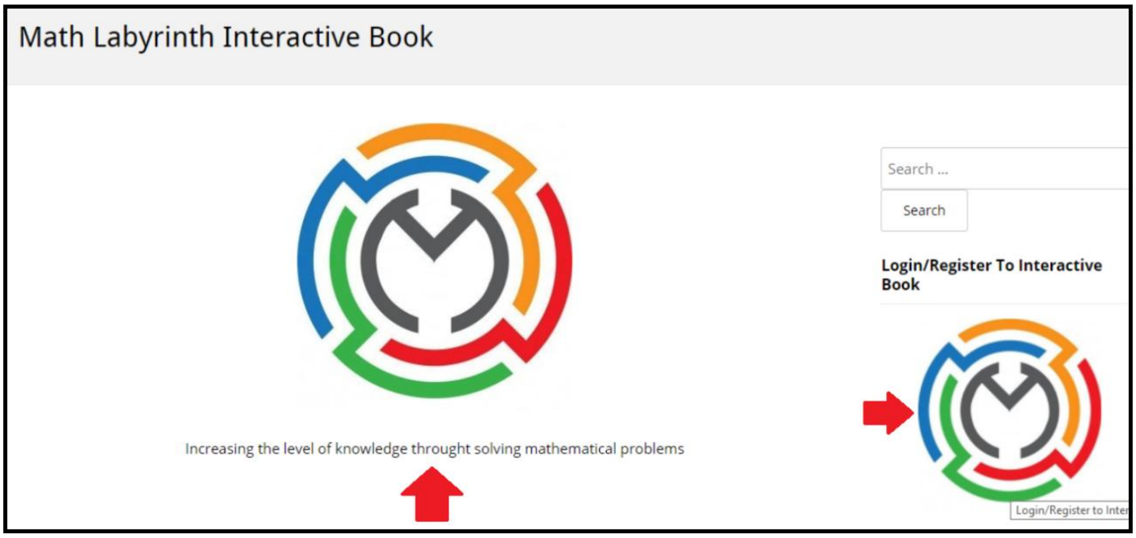
Click on

Interactive Book

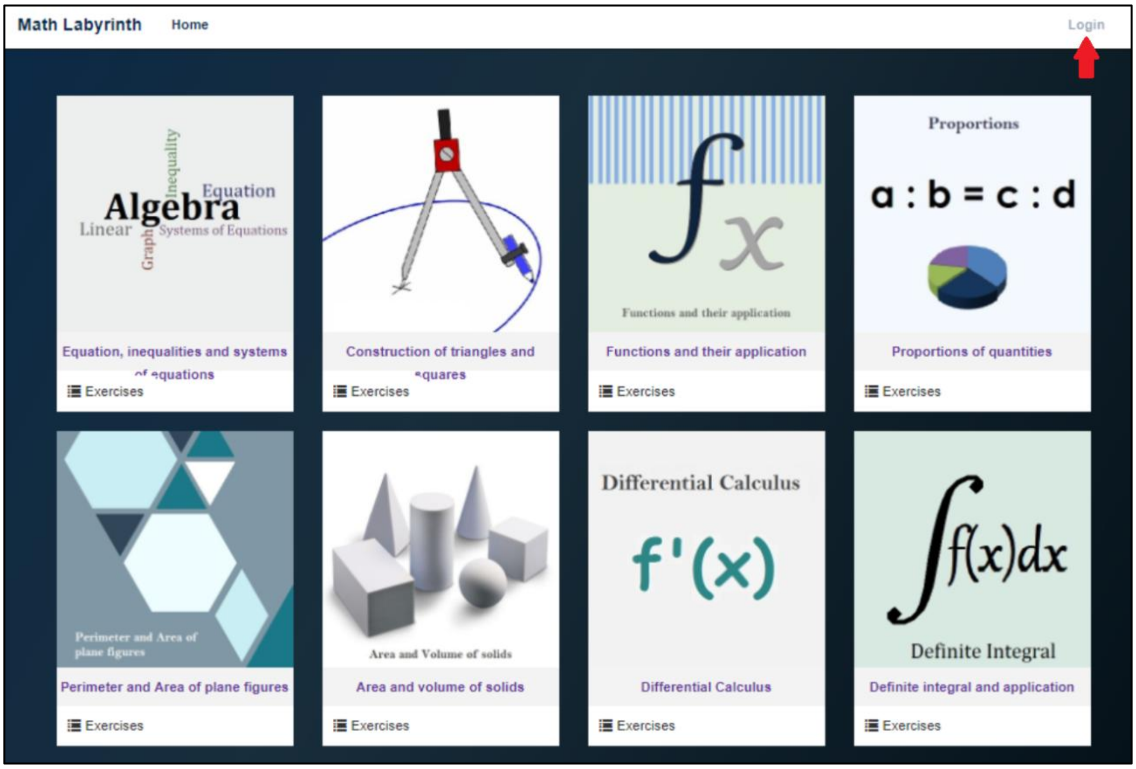


Access to the

Math Labyrinth Book

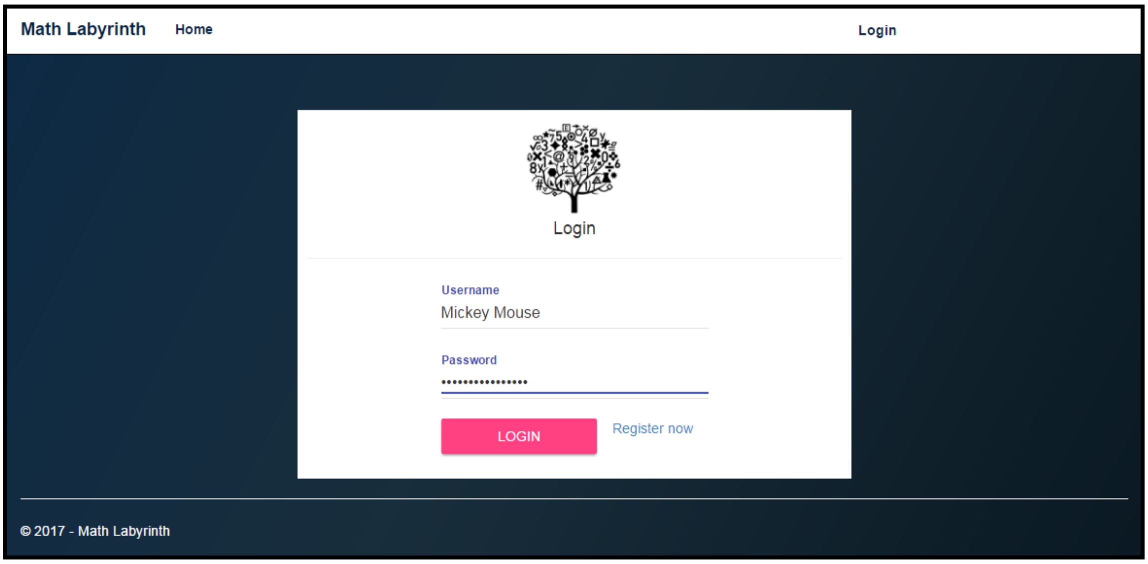


Log in to access the exercises



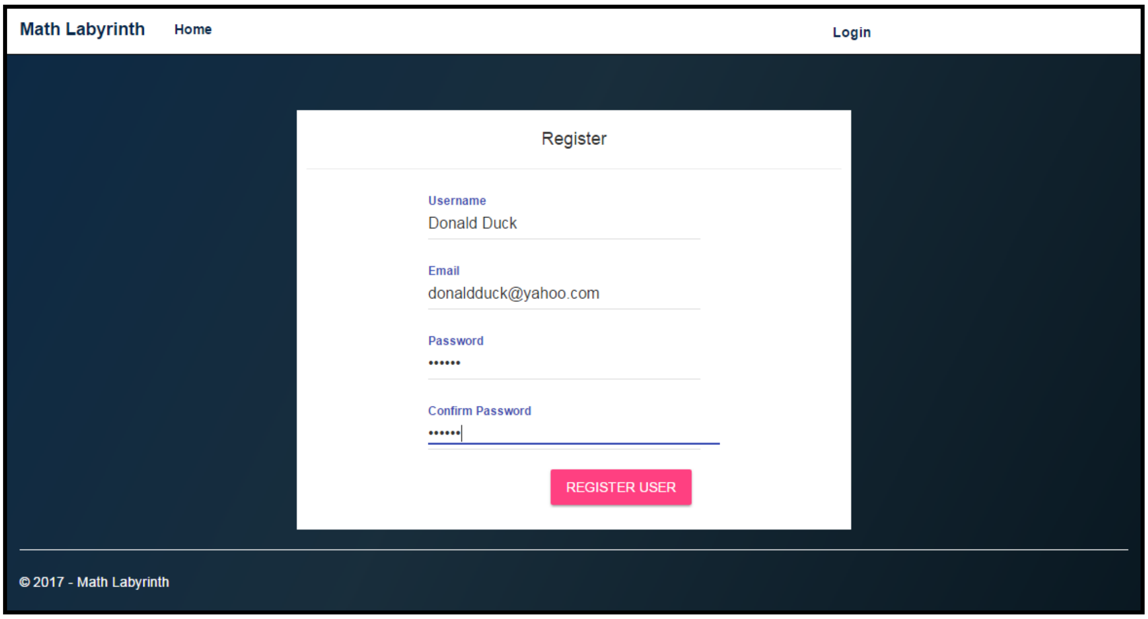
Login with your username and password.

If you don't have an account, register now.

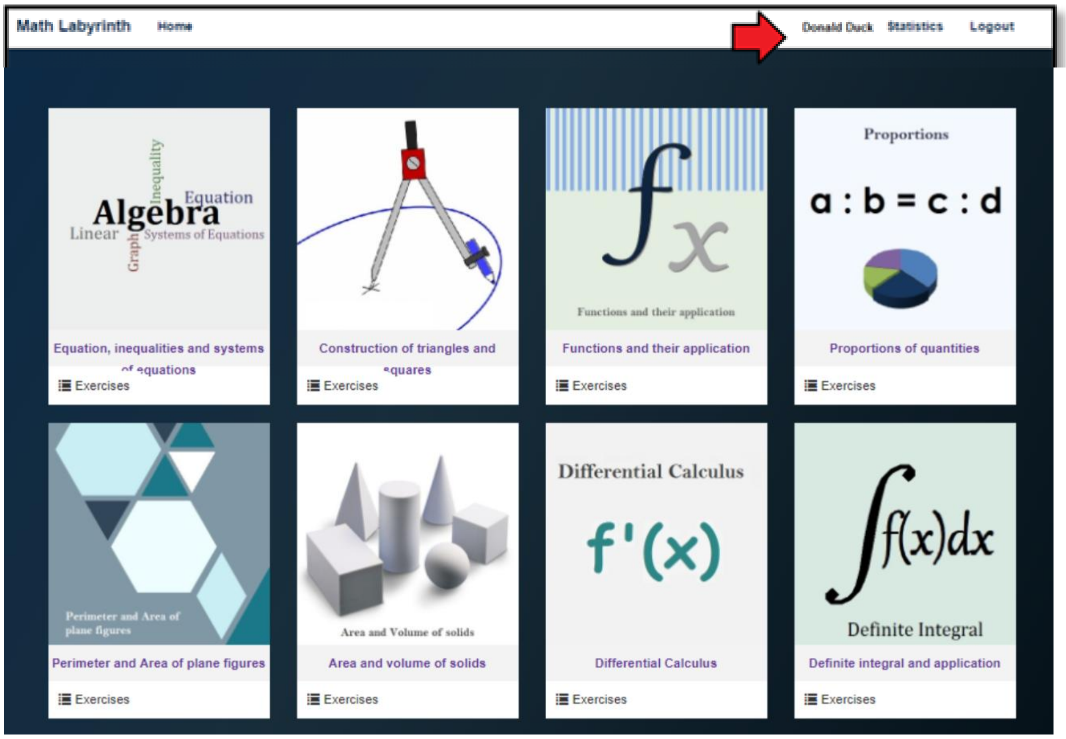


Choose a username and type your e-mail.

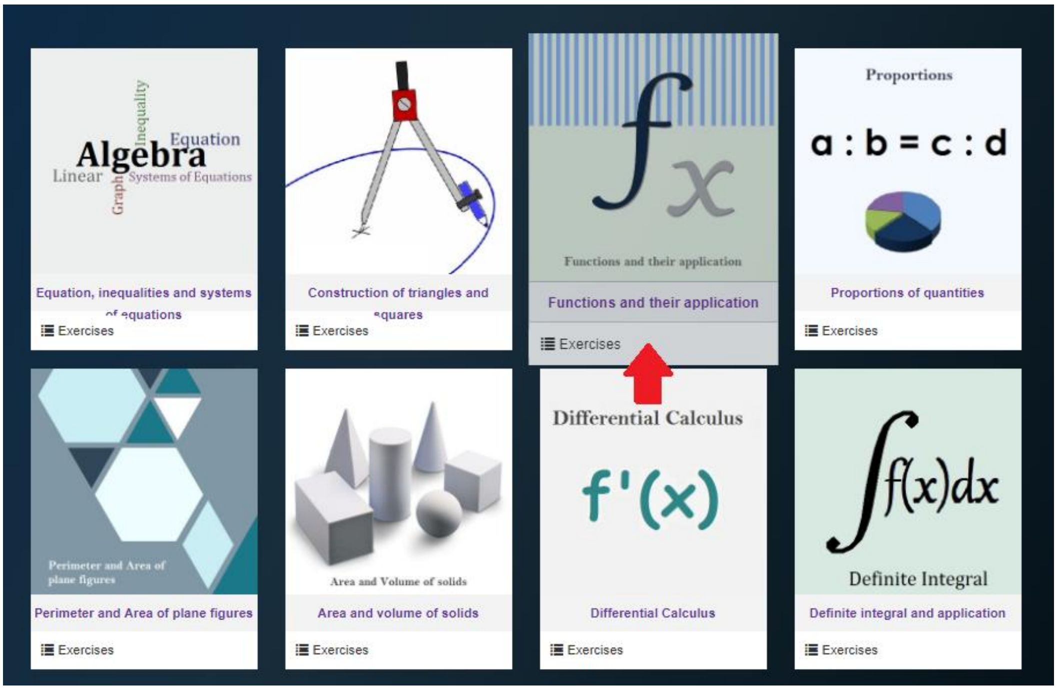
Then choose a password and confirm it.



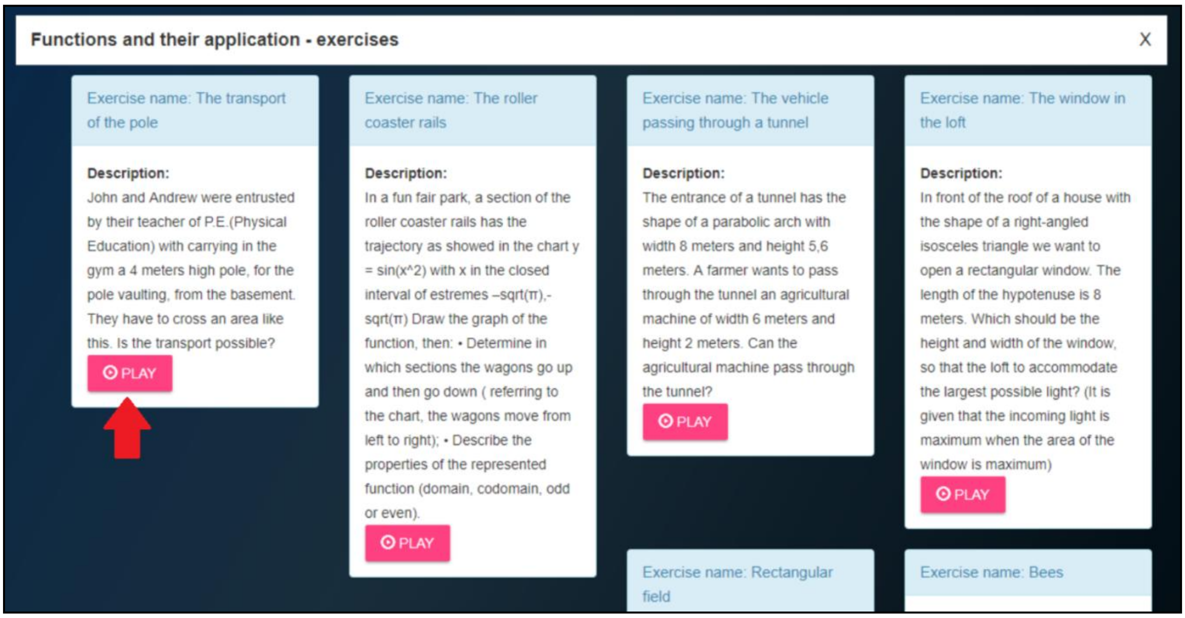
Your username will appear on the top of the window.



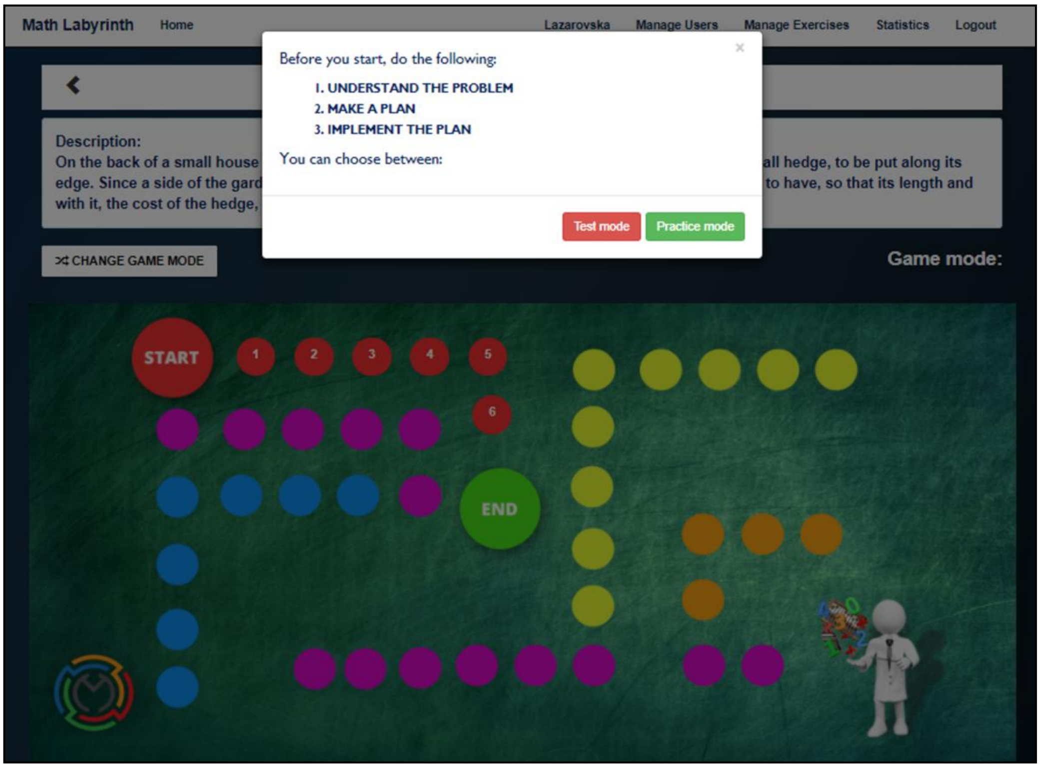
Choose a type of exercise...



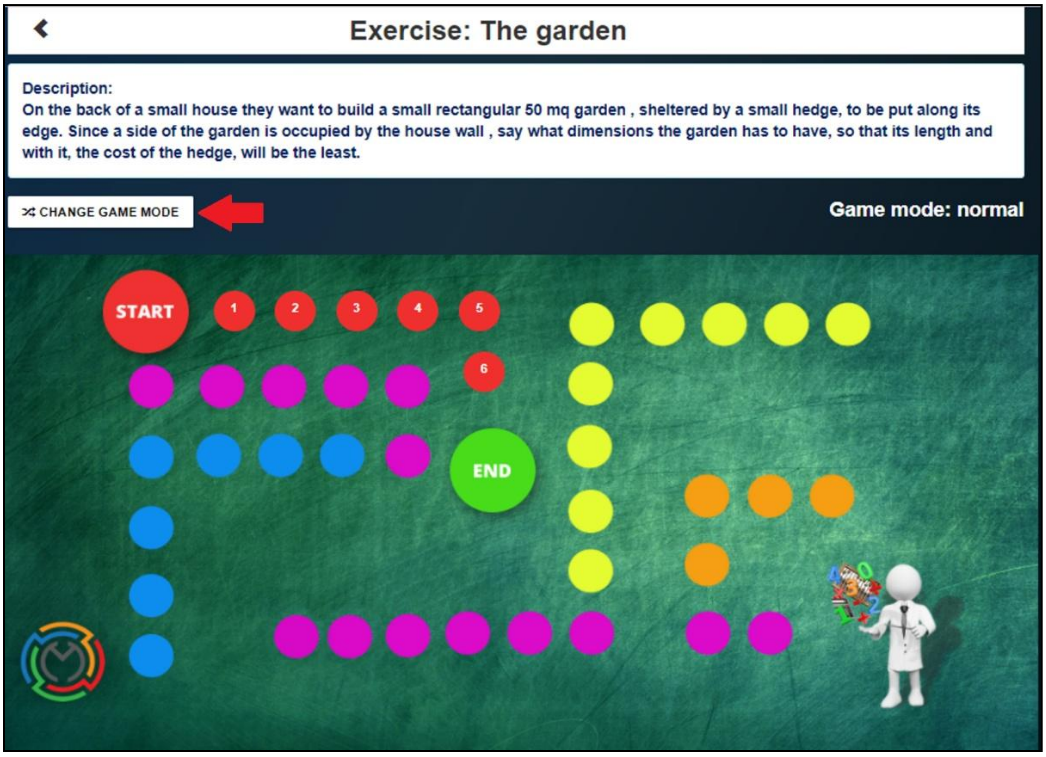
… and then any exercise you want



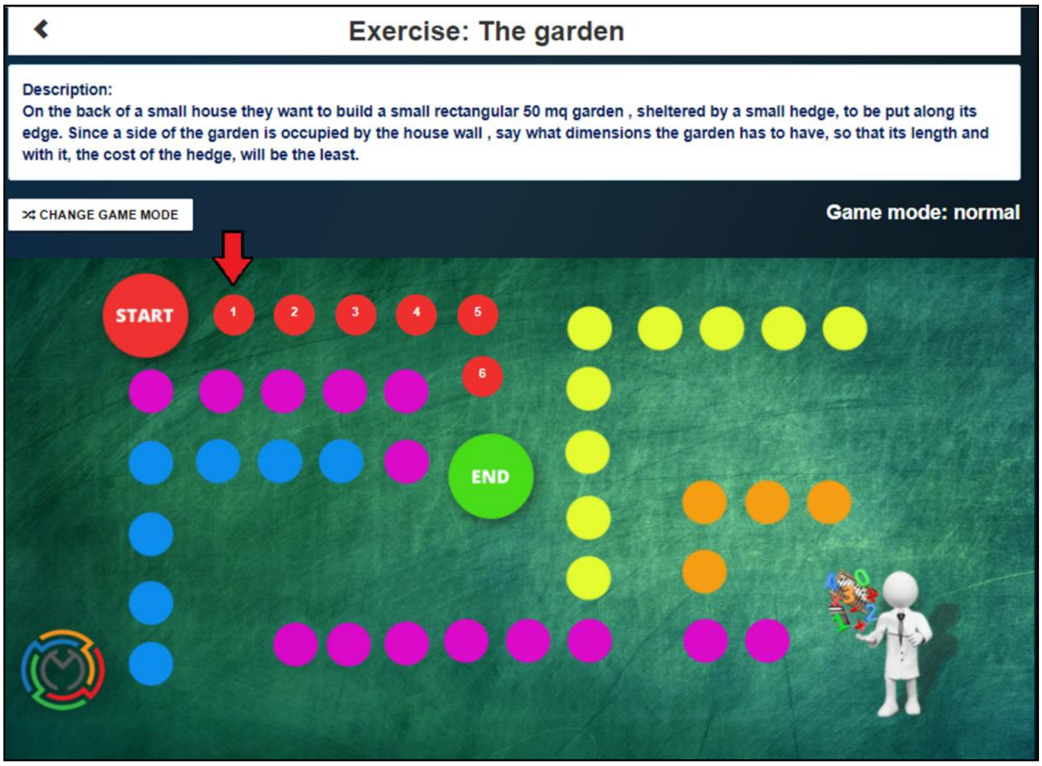
You can choose from two modes: **Test mode** and **Practice mode.**



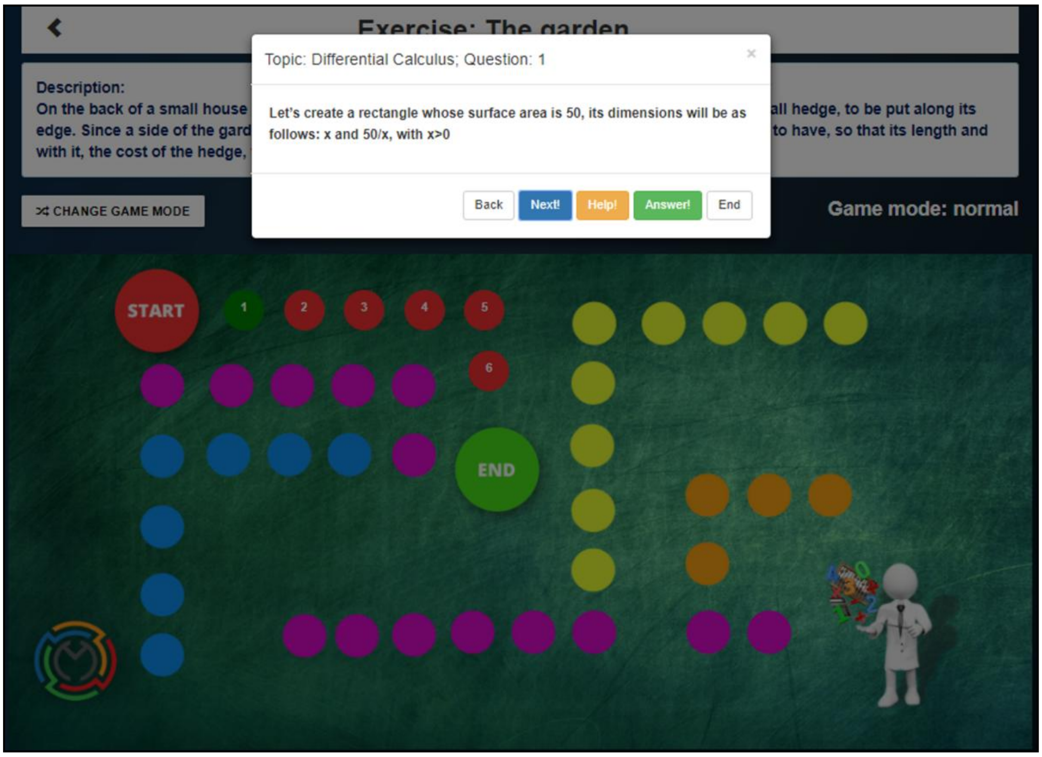
You can also change it afterwards



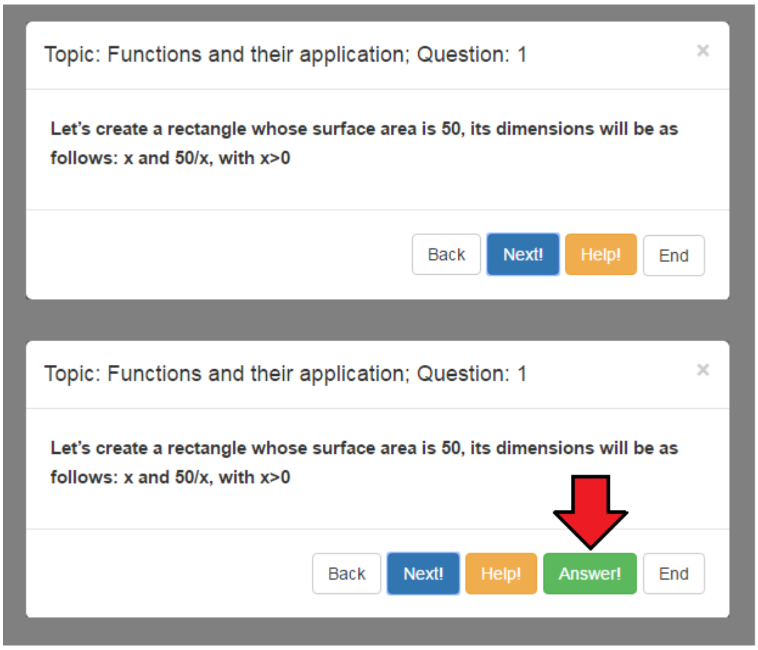
The numbers represent the steps you need to take in order to solve the problem



If you click on any number, a window with suggestions will appear.



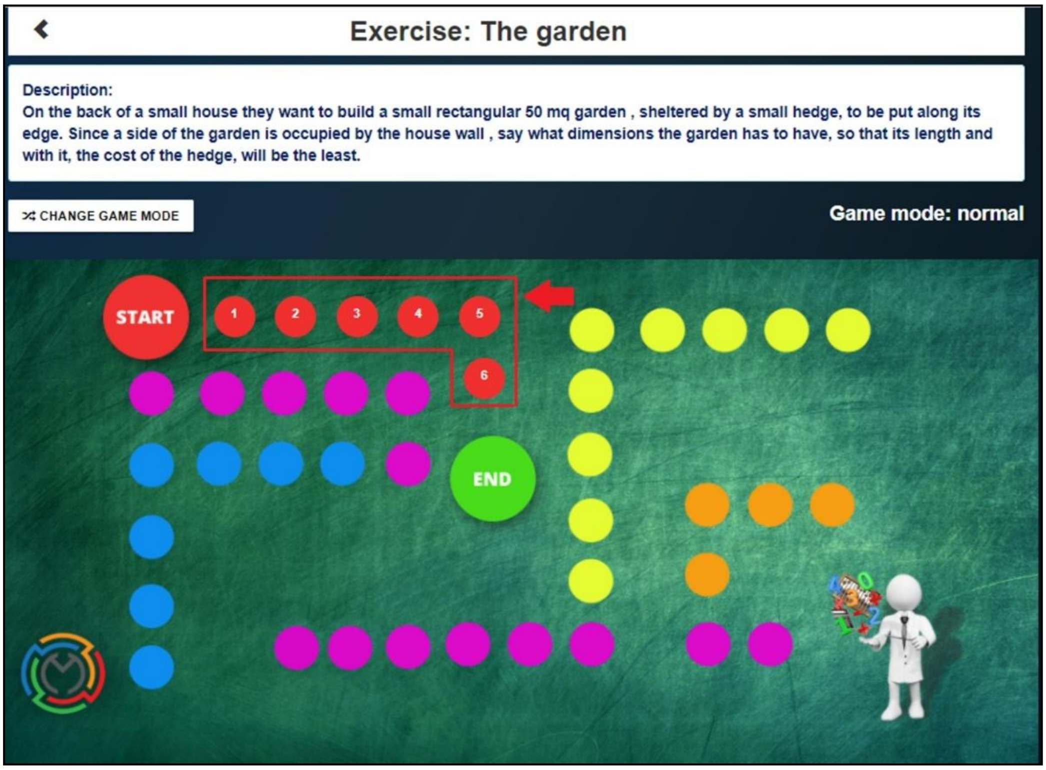
In practice mode you can also get the answer.



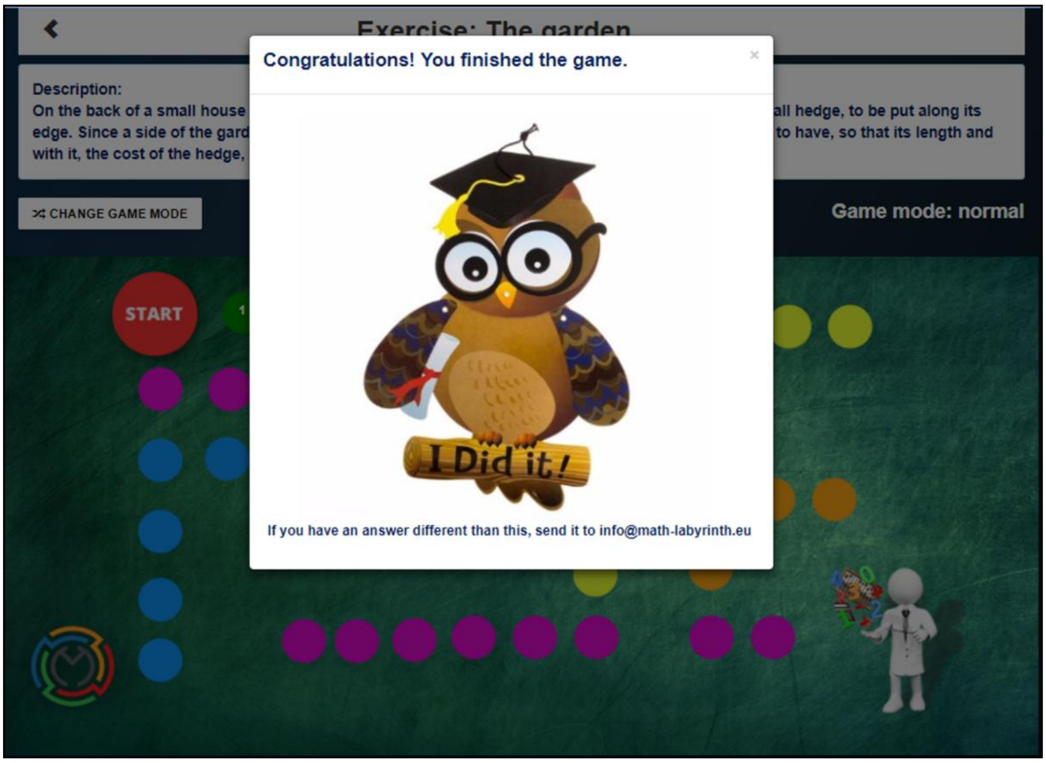
Test Mode

Practice Mode

Keep on the same way for other steps.

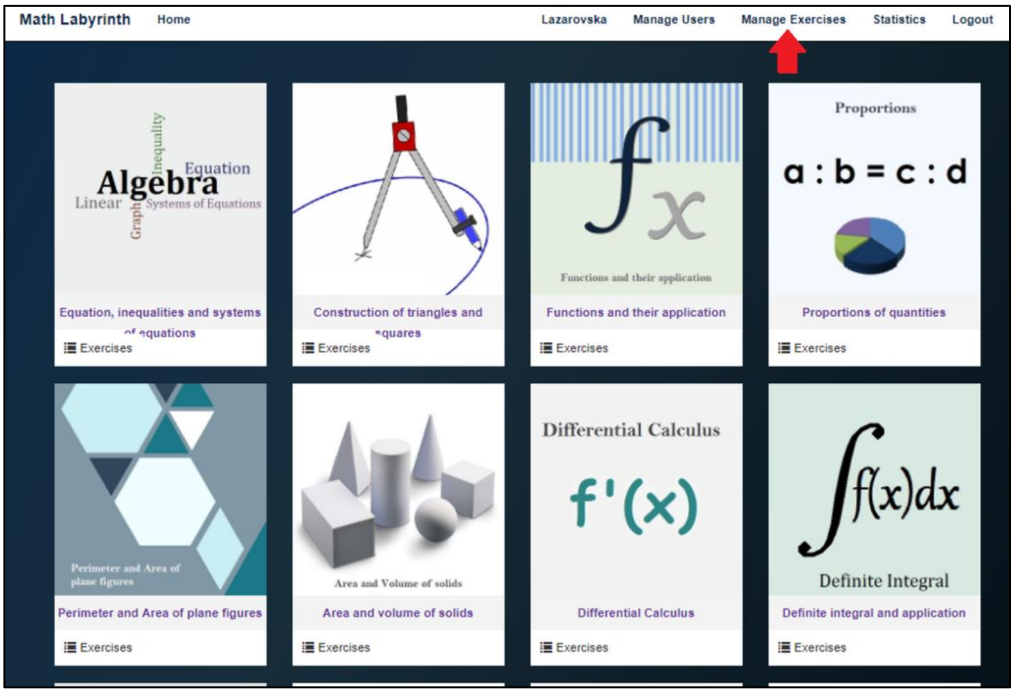


This window indicates that the procedure has been completed

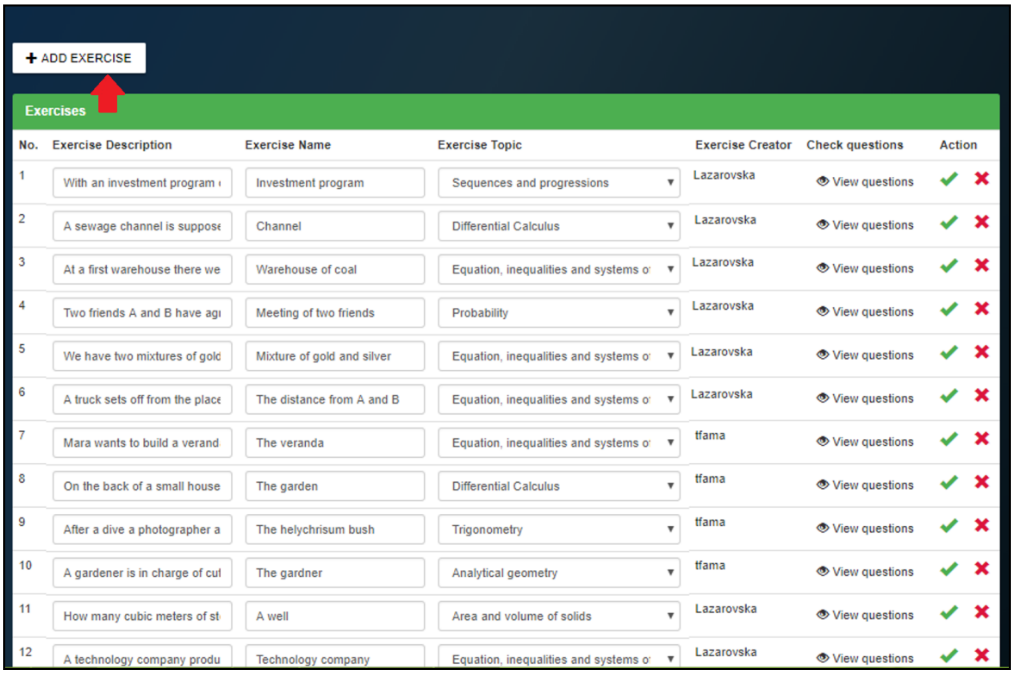


Adding an exercise

To add an exercise to the Interactive book the user needs to have admin role. After registering and given permission as an admin, the user needs to click on **Manage Exercises**.



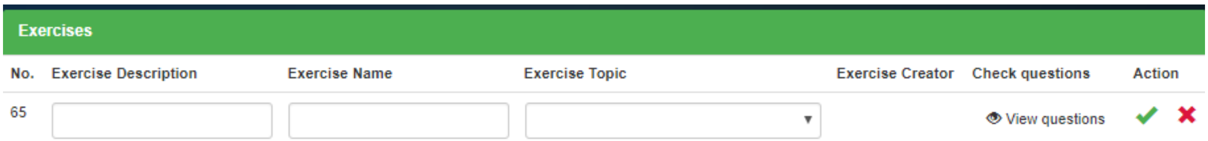
Then click on **ADD EXERCISE**


+ ADD EXERCISE

Exercises

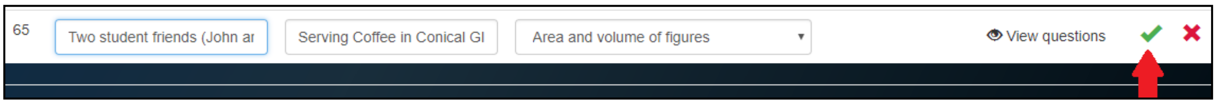
No.	Exercise Description	Exercise Name	Exercise Topic	Exercise Creator	Check questions	Action
1	With an investment program	Investment program	Sequences and progressions	Lazarovska	View questions	✔ ✖
2	A sewage channel is suppose	Channel	Differential Calculus	Lazarovska	View questions	✔ ✖
3	At a first warehouse there we	Warehouse of coal	Equation, inequalities and systems o	Lazarovska	View questions	✔ ✖
4	Two friends A and B have agı	Meeting of two friends	Probability	Lazarovska	View questions	✔ ✖
5	We have two mixtures of golc	Mixture of gold and silver	Equation, inequalities and systems o	Lazarovska	View questions	✔ ✖
6	A truck sets off from the place	The distance from A and B	Equation, inequalities and systems o	Lazarovska	View questions	✔ ✖
7	Mara wants to build a verand	The veranda	Equation, inequalities and systems o	tfama	View questions	✔ ✖
8	On the back of a small house	The garden	Differential Calculus	tfama	View questions	✔ ✖
9	After a dive a photographer a	The helychrisum bush	Trigonometry	tfama	View questions	✔ ✖
10	A gardener is in charge of cul	The gardner	Analytical geometry	tfama	View questions	✔ ✖
11	How many cubic meters of st	A well	Area and volume of solids	Lazarovska	View questions	✔ ✖
12	A technology company produ	Technology company	Equation, inequalities and systems o	Lazarovska	View questions	✔ ✖

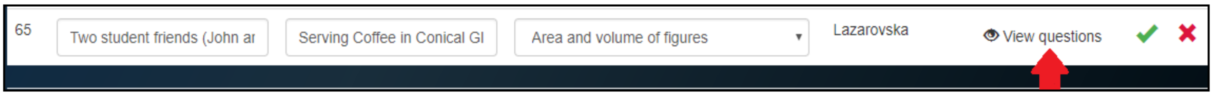
The new row will be added with fields: **Exercise Description, Exercise Name**, and **Exercise Topic**.



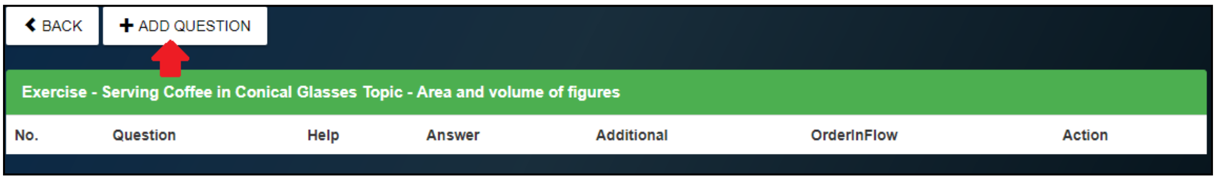
Exercise Description is a full text description of exercise, **Exercise Name** is a short name of the exercise and **Exercise Topic** can be chosen from the fourteen given categories.

After filling all the fields choose the **Action** to save the Exercise and click on **View questions**.

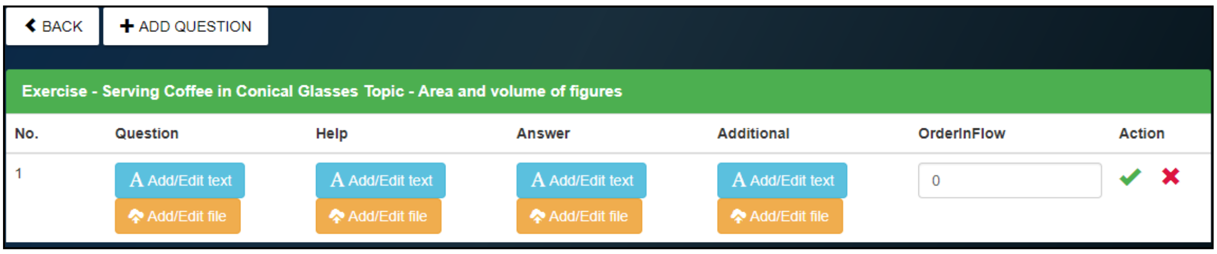




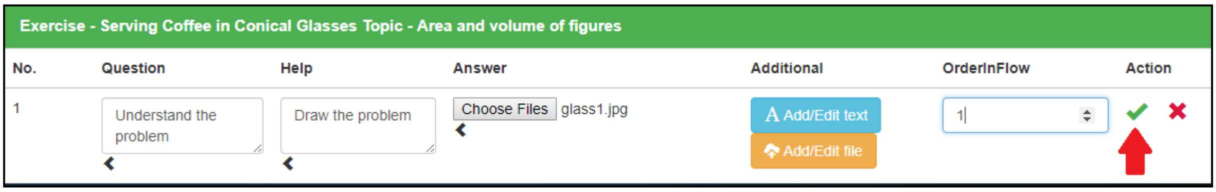
To add a new step/question in the exercise click on **ADD QUESTION**



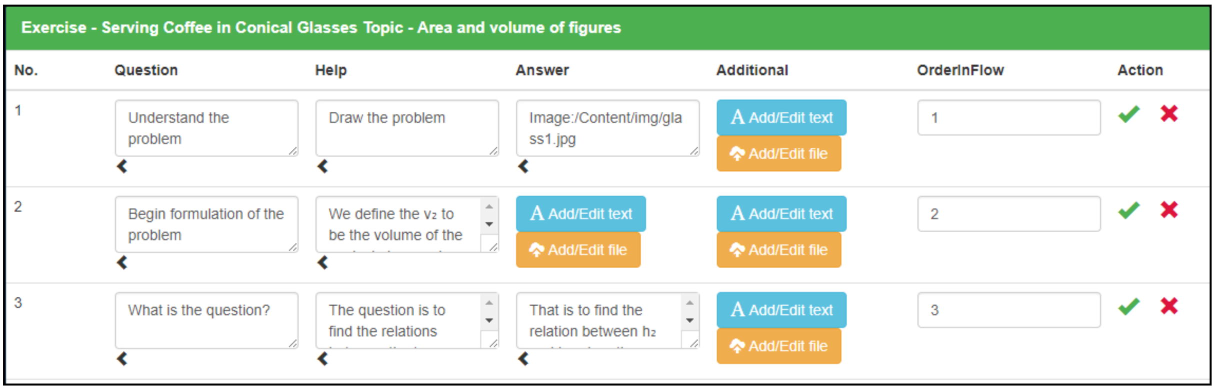
This is the window that will show up



For every question the admin can add a **question/step**, **help**, **answer,** and **additional info**. Text and formulas in latex can be added from the box **Add/Edit text,** picture file can be added from the box **Add/Edit file**. **OrderInFlow** represents the order of a question/step in the exercise and begins with 1.



After filing in the fields click on **Action** to save the question and repeat the same for new questions/steps.



After finishing with all the questions/steps the admin can preview the exercise clicking on **Home** and choosing the category in which the exercise is and then click the button **Play**.

HOW TO RUN THE EXERCISES – EXAMPLES

SUGGESTED LESSON PLAN IN THE AREA OF DIFFERENTIAL CALCULUS

PROBLEM: **A restored house**

TARGET GROUP (17 year-old students)

PREREQUISITES (knowledge and abilities necessary to cope with understanding the problems)

- What is a quadratic function?
- What is parabola?
- What are its basic points and properties?
- Use of calculus to find local minima and maxima of functions
- Be able to solve simultaneous equations

LEARNING OBJECTIVES

- transform a word problem into a mathematical problem
- apply the calculus to real life situations

TOOLS AND MATERIALS

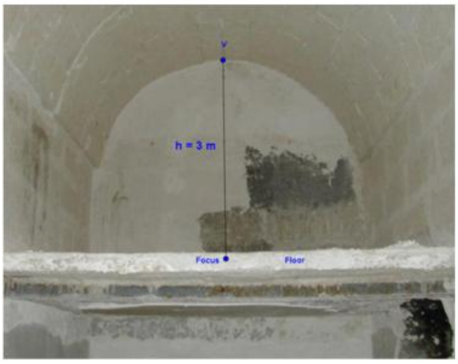
- the Interactive book
- the Guidelines
- GeoGebra applets
- drawings

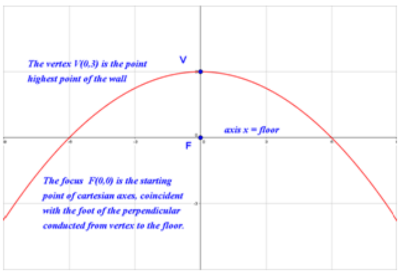
THE PROBLEM

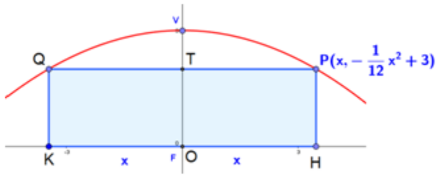
An old house is about to be restored and one more floor has been extracted from a room whose walls are quite high and which has got an arc vault. The arch is a parabolic arc whose maximum height from the floor is 3 meters and it is calculated from the point of the floor which may be considered as the focus of the parabola.

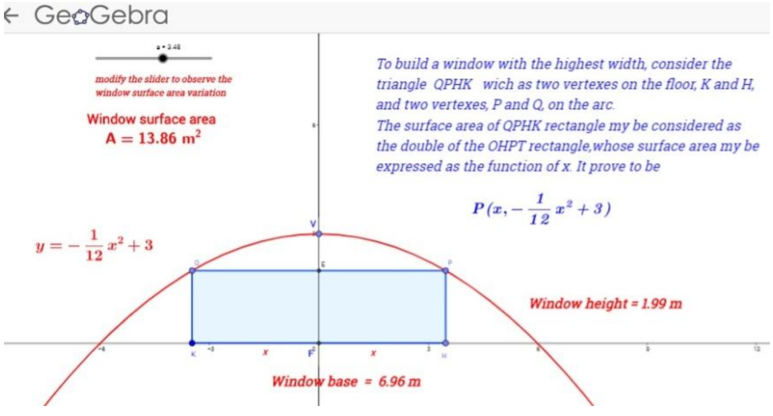
They want to create a window on the new upstairs floor, whose outer wall is indeed the arched one, so that there will be as much more air as possible. What dimensions will the window have?

LESSON

1.The image of the proposed problem is showed to the students so they can see and better understand what you are talking about. To do so, just click on Help 1 ...	 Back Next! Answer! End
2.The students will be guided to modeling the problem	**Consider the house arch as the parabola that has its vertex in the highest point of the wall, V (0,3) and its focus in O (0,0), coincident with the foot of the perpendicular, conducted from the vertex to the floor.** Back Next! End
3.Once they have understood the geometric figure (the parabola) the advice is to calculate the known equation of Vertex and Focus. Help 2 is useful to remember the formulas to use.	Calculate the equation of the parabola, $y = ax^2 + bx + c$ Back Next! Help! Answer! End We know vertex and focus as well as the following relations: $V\left(-\frac{b}{2a}; \frac{\Delta}{4a}\right)$ $F\left(-\frac{b}{2a}; \frac{1-\Delta}{4a}\right)$ where $\Delta = b^2 - 4ac$ Back Next! Answer! End

4.You set the system that	Set up the system $\begin{cases} -\frac{b}{2a}=0 \\ -\frac{\Delta}{4a}=3 \\ \frac{1-\Delta}{4a}=0 \end{cases}$ Back Next! End
5.…when resolved…	$\begin{cases} b=0 \\ \Delta=-12a \\ \Delta=1 \end{cases} \Rightarrow$ $\begin{cases} b=0 \\ a=-\frac{1}{12} \\ b^2-4ac=1 \end{cases} \Rightarrow$ $\begin{cases} b=0 \\ a=-\frac{1}{12} \\ b^2-4ac=-12a \end{cases}$ Back Next! Answer! End and substituting into the second equation the values of a and b, it appears that $a=-\frac{1}{12}; b=0; c=3$ Back Next! End
6....will allow to write the equation of the parabola	 Back Next! Answer! End $y=-\frac{1}{12}x^2+3$ it is the arch equation Back Next! End

7. Now the students are invited to build in the parabolic arc the window of maximum area...	 Back Next! End
8.…and calculate the area of the rectangle inscribed as a function of *x* (the coordinates of the variable P point on the arc of the parabola)	Topic: Differential Calculus; Help: 6 **The surface area of the rectangle PQKH is the double of the OHPT rectangle** Back Next! Answer! End $A(x) = KH \cdot PH = 2x(-\frac{1}{12}x^2+3)$ i.e. $A(x) = -\frac{1}{6}x^3 + 6x$ Back Next! End
9. It is now suggested how to calculate the *x* value for which the area found is the maximum one. At this point, the derivation of a function is calculated and used.	**Do the first derivative of A(x) and find out the value of the x by which it is made null.** Back Next! Answer! End $A'(x) = -\frac{1}{2}x^2 + 6 = 0$ $x = \pm 2\sqrt{3}$ Back Next! End

10. The use of GeoGebra software is very effective to visualize the result obtained. Students are invited to realize the graph of the solution of the problem. However, they may still get a visualisation at the suggested link	https://www.geogebra.org/m/n8kCSKEX  **For** $x = 2\sqrt{3} = 3,46m$ **the window will have the maximum area of** $A_{max} = 8\sqrt{3} = 13,86m^2$ Back Next! End
11. The student can easily infer from the result obtained the dimensions that the window will have.	**This means that, in order to build the window, the bricklayer will have to do a break in the outer wall 2 m high and about 2,5 m far from the lateral walls.** Back Next!

WORKSHEET FOR STUDENTS

1. What is the equation for quadratic function?
2. What is a parabola?
3. What is the equation of a parabola with axis parallel to the y axis?
4. How do you determine the Focus, Vertex, and Directrix of a parabola with an axis parallel to the y axis?
5. How do you determine the equation of the parabola with an axis parallel to the axis of the y knowing the Focus and the Directrix?
6. Calculate the derivative of the following function: $y = x^4 - 5x^2 + 3$
7. Calculate the local maximum and the local minimum points of the following function $y = \frac{x^3}{3} + x^2 - 3x$
8. A manufacturer of tennis balls has a daily cost of $C(x) = 200 - 10x + 0.01x^2$ where C(x) is the total cost in euro and x is the number of tennis balls produced. What number of produced tennis balls will minimize the daily cost?
9. While playing basketball Frank shoots an air-ball. The height in meters reached by the ball after *t* seconds is given by function $h(t) = -16t^2 + 32t + 8$. How long will it take for the ball to hit the ground? What is the maximum height that the ball reaches?
10. The perimeter of a rectangle is 120 cm. What is the maximum possible area of such a rectangle?

SUGGESTED LESSON PLAN IN THE AREA OF FUNCTIONS AND THEIR APPLICATION

PROBLEM: **The veranda**

TARGET GROUP (17 year-old students)

PREREQUISITES (knowledge and abilities necessary to cope with understanding the problems)

- What is an equation?
- Be able to solve second degree equations
- What are abscissas and ordinates?
- Be able to calculate the area of a polygon.

LEARNING OBJECTIVES

- transform a word problem into a mathematical problem
- apply the calculus to real life situations

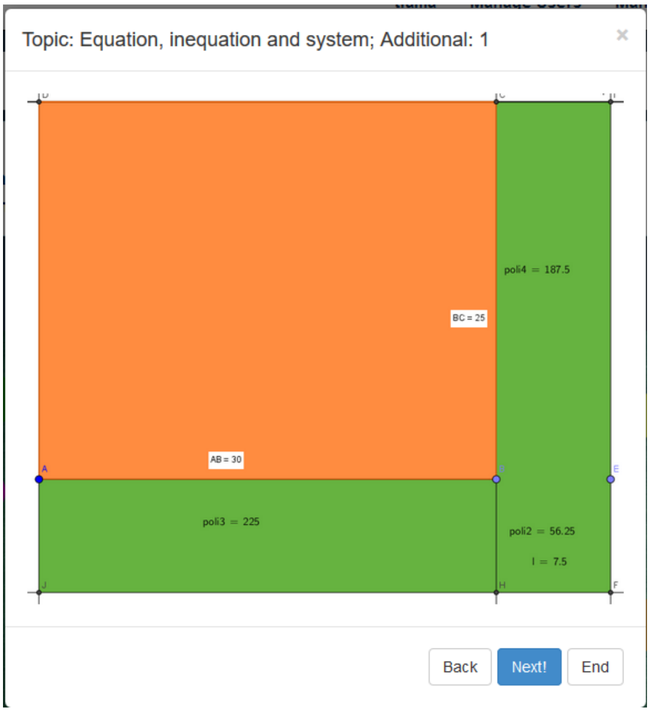
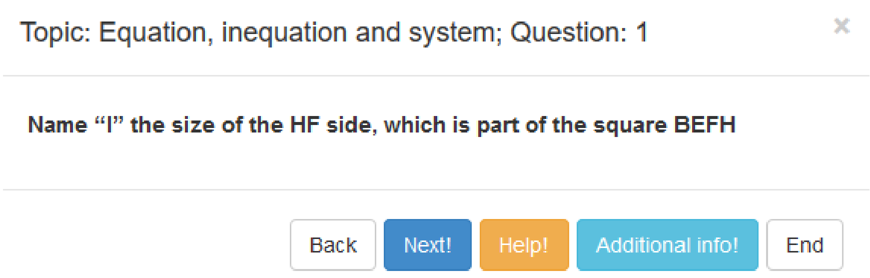
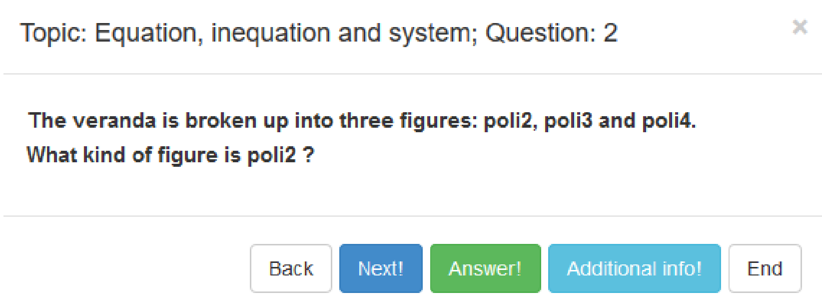
TOOLS AND MATERIALS

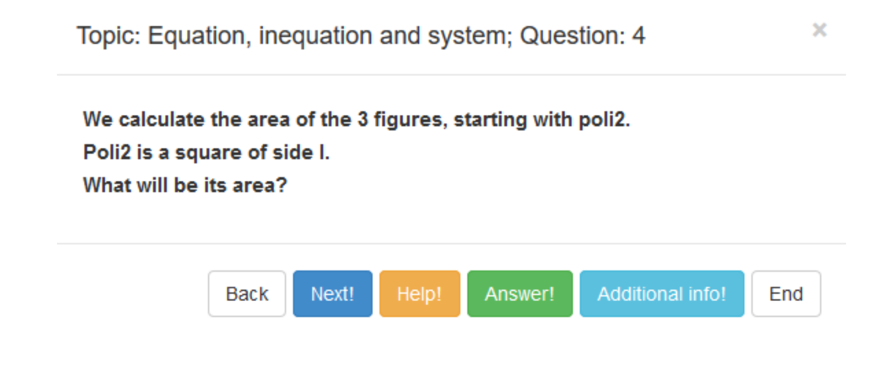
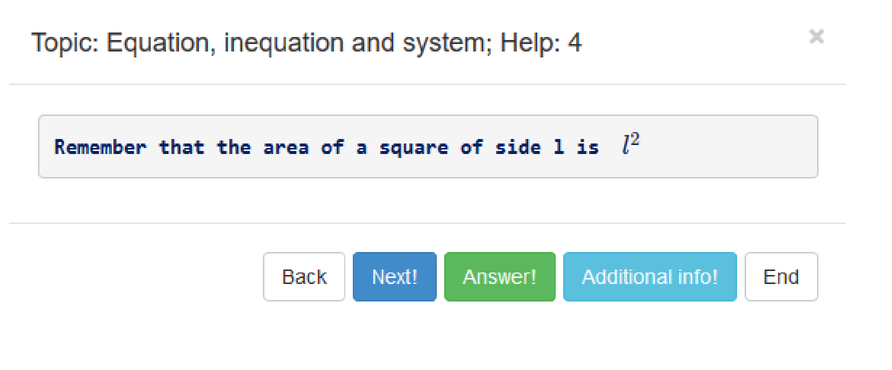
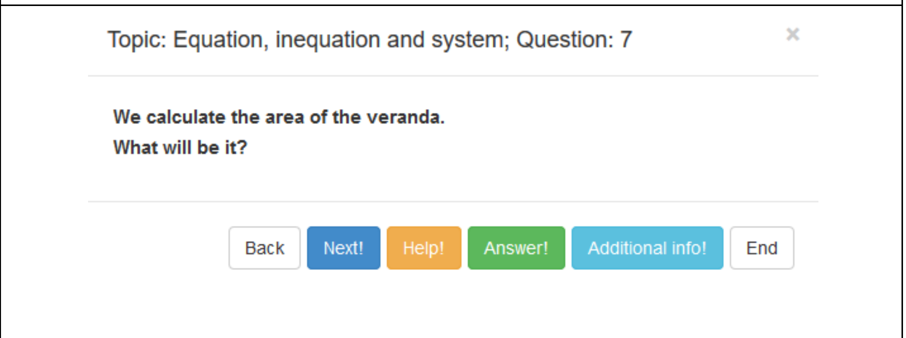
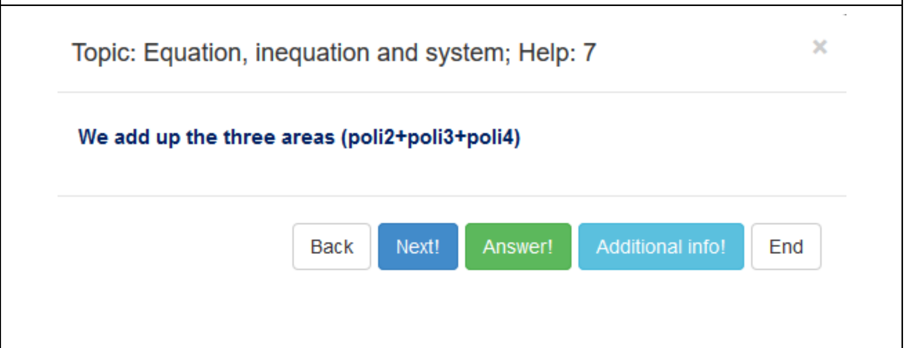
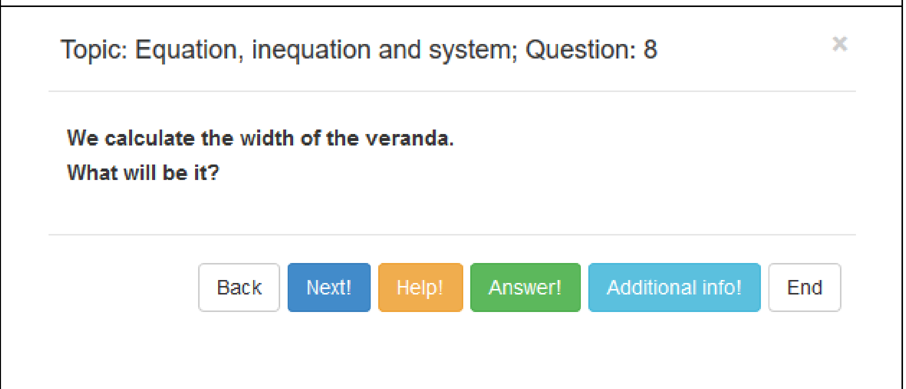
- the Interactive book
- the Guidelines
- GeoGebra applets
- Drawings

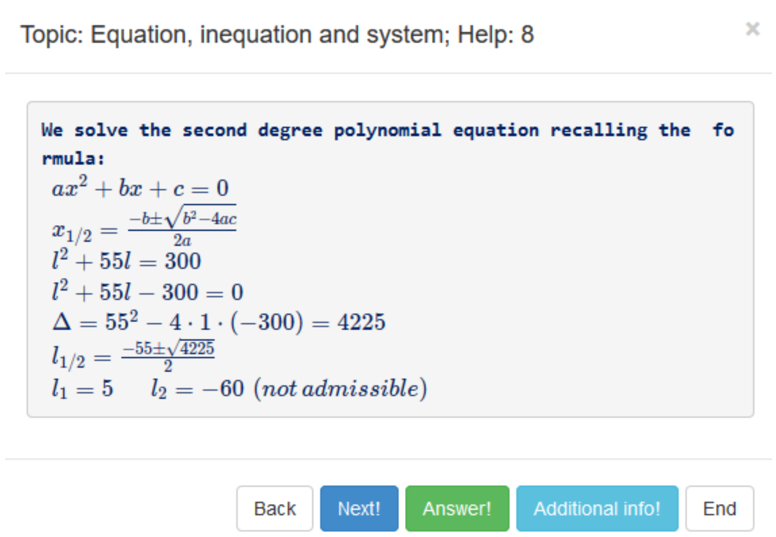
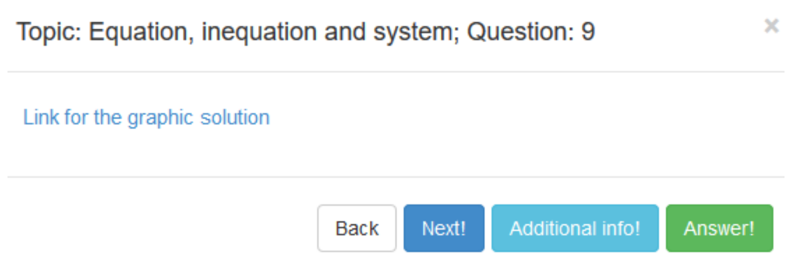
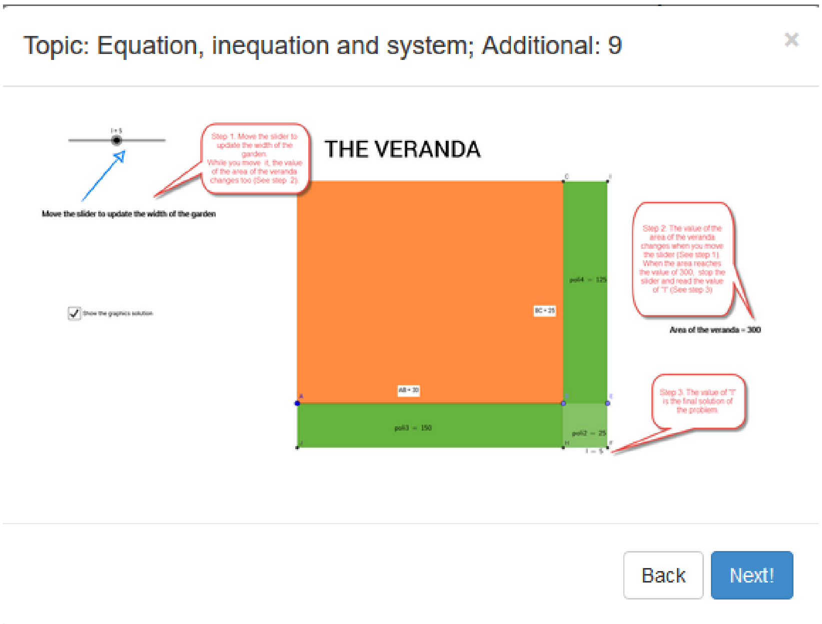
THE PROBLEM

Mara wants to build a veranda along the southern and eastern sides of her beach house. If Mara has got building material to cover a surface of 300 m^2, how wide should the veranda be?

LESSON

<table>
<tr><td>1. Ask students to draw a picture/figure related to the problem, so that they can visualize and understand what it is about.</td><td>
</td></tr>
<tr><td>2. The student is guided to find an algebraic solution, setting the unknown "l" and thus modeling the problem.</td><td>
</td></tr>
<tr><td>3. The student should reflect on the type of figures resulting after “breaking” the porch....</td><td>
</td></tr>
</table>

4 …and calculate the area of each of the above mentioned parts.	
5. HELP By activating Help, the student will recall the formula for calculating the area of a square.	
6. Once the surface of each of the three parts is calculated, the student is asked to calculate the entire veranda area.	
7. HELP. By activating Help, the student is helped to calculate the area of the entire veranda	
8. To calculate the depth of the veranda, it is necessary to set and solve a 2nd degree equation, equaling the area of the veranda found	

with the known figure of the problem (300).	
9. HELP. By activating the Help, the student will get the resolving equation displayed . He will also be able to revise the formula to resolve a 2nd degree equation.	
10. You can also find the graphic solution of the problem. By clicking on a link, in fact, a Geogebra sheet is opened, with the already modeled problem	
11...and the student has to just move the slider to find the graphic solution	

WORKSHEET FOR STUDENTS

1. How do you calculate the area of a rectangle? And the area of the square?
2. What is the formula for the calculation of the discriminant of a second degree equation?
3. What is the formula giving the solutions of a second degree equation?
4. Solve the following second degree equation: $16x^2 - 8x + 1 = 0$
5. Solve the following second degree equation: $x^2 + x + 1 = 0$
6. Two cars start out at the same point. One car starts out driving north at 25 km/h. Two hours later the second car starts driving east at 20 km/h. How long after the first car starts traveling does it take for the two cars to be 300 kilometers apart?
7. An office has two envelope stuffing machines. Working together they can stuff a batch of envelopes in 2 hours. Working separately it will take the second machine 1 hour longer than the first machine to stuff a batch of envelopes. How long would it take each machine to stuff a batch of envelopes by themselves?

SUGGESTED LESSON PLAN IN THE AREA OF SQUARE FUNCTIONS

PROBLEM: **The window in the loft**

TARGET GROUP (16 year old students)

PREREQUISITES (knowledge and abilities necessary to cope with understanding the problems)

- What is a right-angled isosceles triangle?
- To know the formula for the area of the rectangle
- What is a quadratic function?
- What is parabola?
- When does a quadratic function have a maximum?
- Be able to solve simultaneous equations

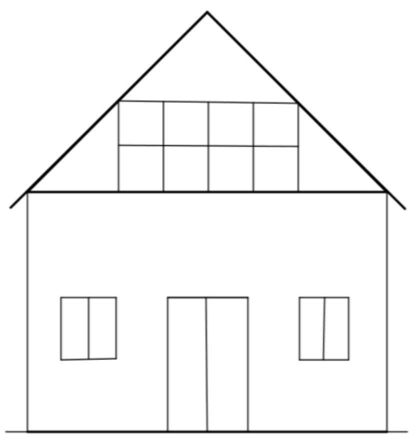
LEARNING OBJECTIVES

- transform a word problem into a mathematical problem
- calculate the maximum of a quadratic function
- use the maximum of a quadratic function into a real life problem

TOOLS AND MATERIALS

- the Interactive book
- the Guidelines
- GeoGebra applets
- drawings

THE PROBLEM

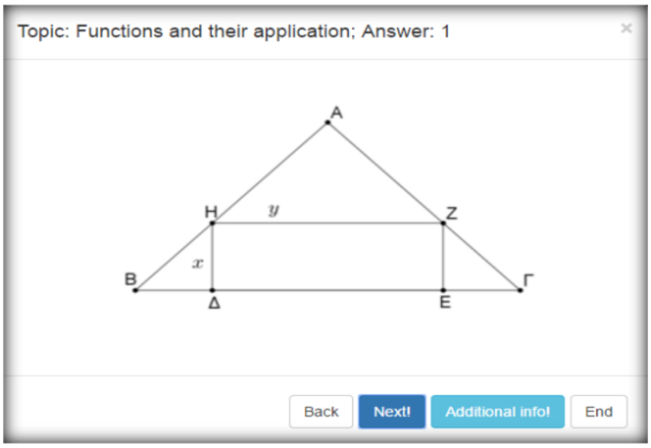


In front of the roof of a house with the shape of a right-angled isosceles triangle we want to open a rectangular window, as shown in the figure below. The length of the hypotenuse is 8 meters. Which should be the height and width of the window, so that the loft to accommodate the largest possible light?

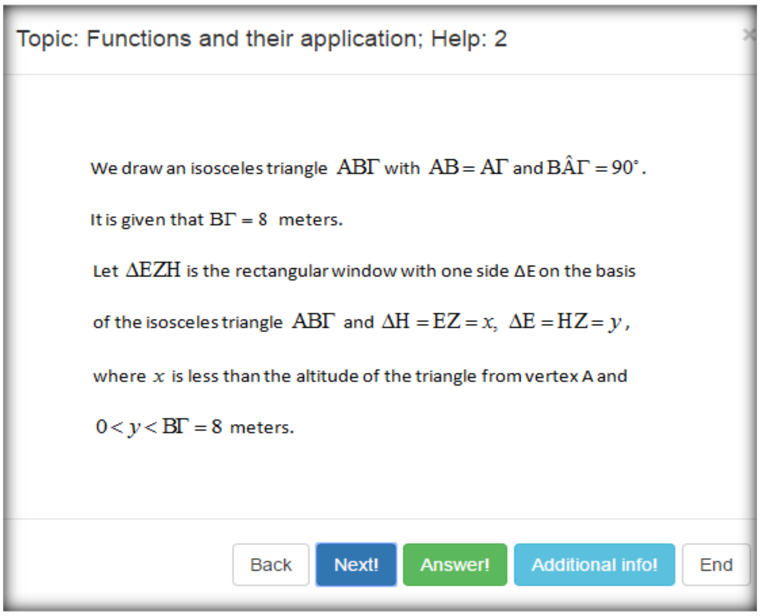
(*It is given that the incoming light is maximum when the area of the window is maximum*)

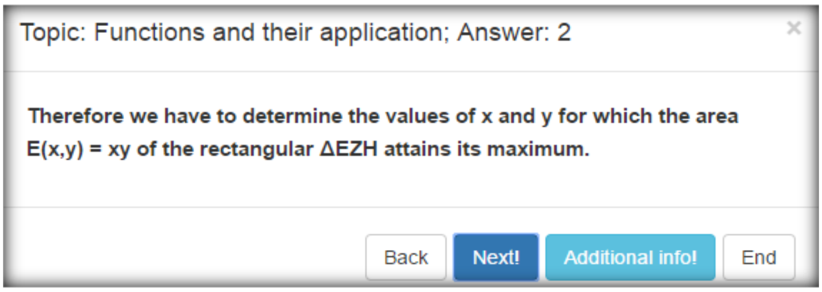
LESSON

1. Ask students to draw a picture/figure related to the problem, so that they can visualize and understand what it is about.



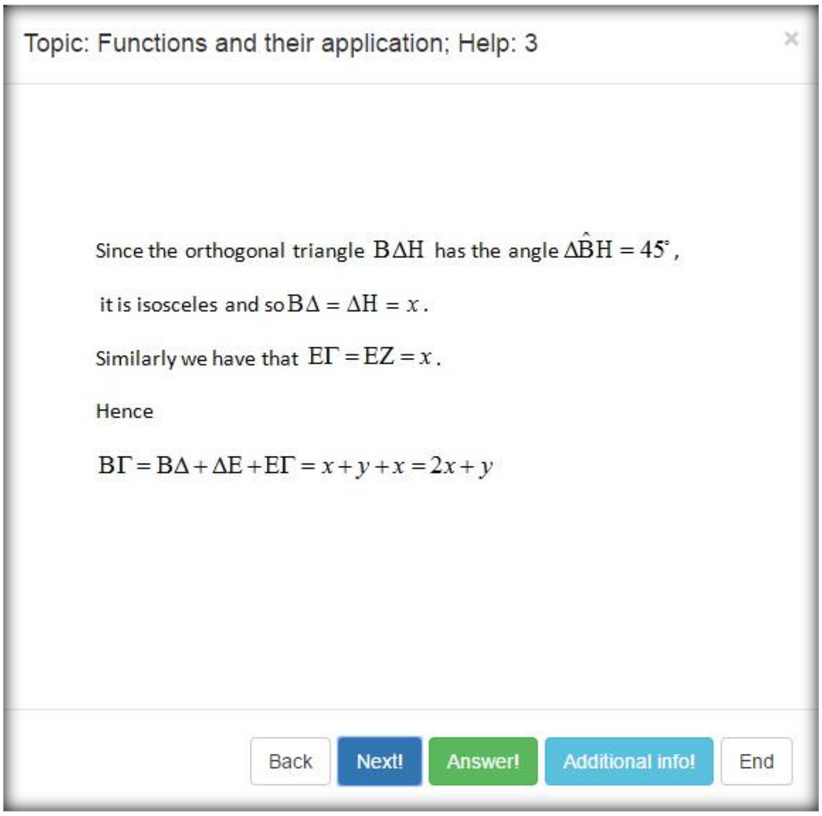
2. Students need to formulate the problem. Student needs to transform a word problem into a mathematical problem. To do that, they should remind about right-angled isosceles triangle and formula for the area of rectangle.





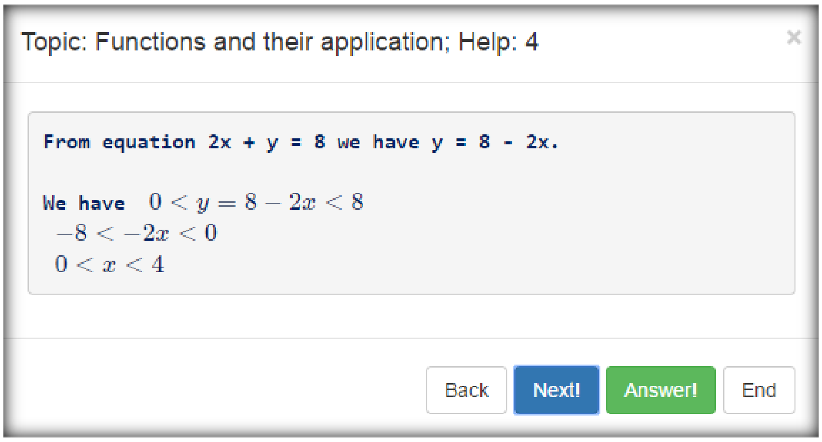
3. In the exercise some conditions are given, so the student will be ask to apply them in solving the problem.

   What relation is satisfied by x and y?



   Using a sketch and knowledge about right-angled isosceles triangle, student came to conclusion for the relation between x and y: 2x + y = 8.

4. To continue with work the student needs to express the area of the rectangle as a function of one variable. He will use the method of substitution and solving simple inequalities.

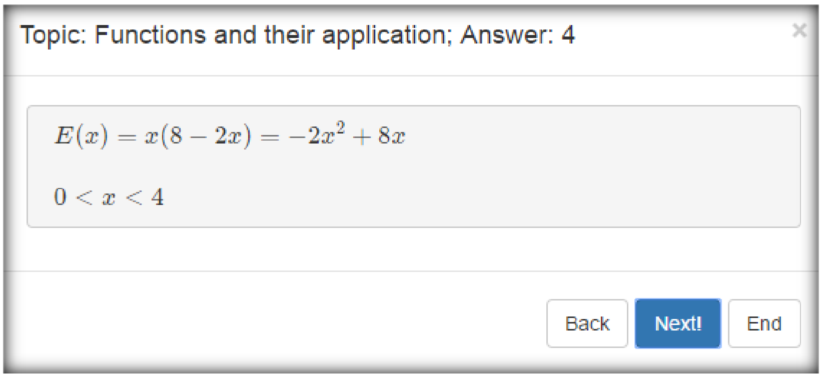

Topic: Functions and their application; Help: 4

From equation 2x + y = 8 we have y = 8 - 2x.

We have $0 < y = 8 - 2x < 8$

$-8 < -2x < 0$

$0 < x < 4$


Topic: Functions and their application; Answer: 4

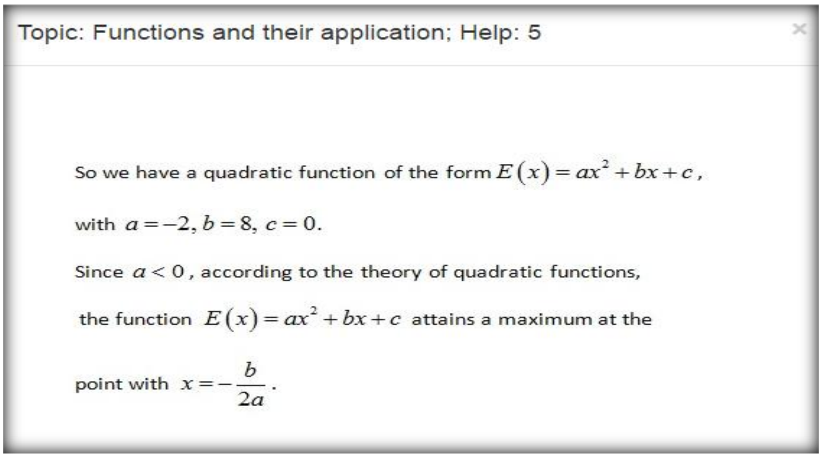
$E(x) = x(8 - 2x) = -2x^2 + 8x$

$0 < x < 4$

5. Student following the previous steps came to the conclusion that the area of a rectangle is a quadratic function. But at the beginning of solving this problem, the student determined that he/she will need the value of height and width of a rectangle with a maximum area.

   So at this step the student will need to find where the function E(x) has a maximum?

   The students will recall of their knowledge of what quadratic function is and when the quadratic function has a maximum?


Topic: Functions and their application; Help: 5

So we have a quadratic function of the form $E(x) = ax^2 + bx + c$,

with $a = -2, b = 8, c = 0$.

Since $a < 0$, according to the theory of quadratic functions,

the function $E(x) = ax^2 + bx + c$ attains a maximum at the

point with $x = -\frac{b}{2a}$.

Topic: Functions and their application; Answer: 5

Hence for our function we have
$x = -\frac{b}{2a} = -\frac{8}{-4} = 2$

Than we find
y = 8 - 2x = 4

6. In this exercise the student uses the maximum of a quadratic function into a real life problem. So to the question "Which should be the height and width of the window, so that the loft to accommodate the largest possible light?" the answer is:

Topic: Functions and their application; Answer: 6

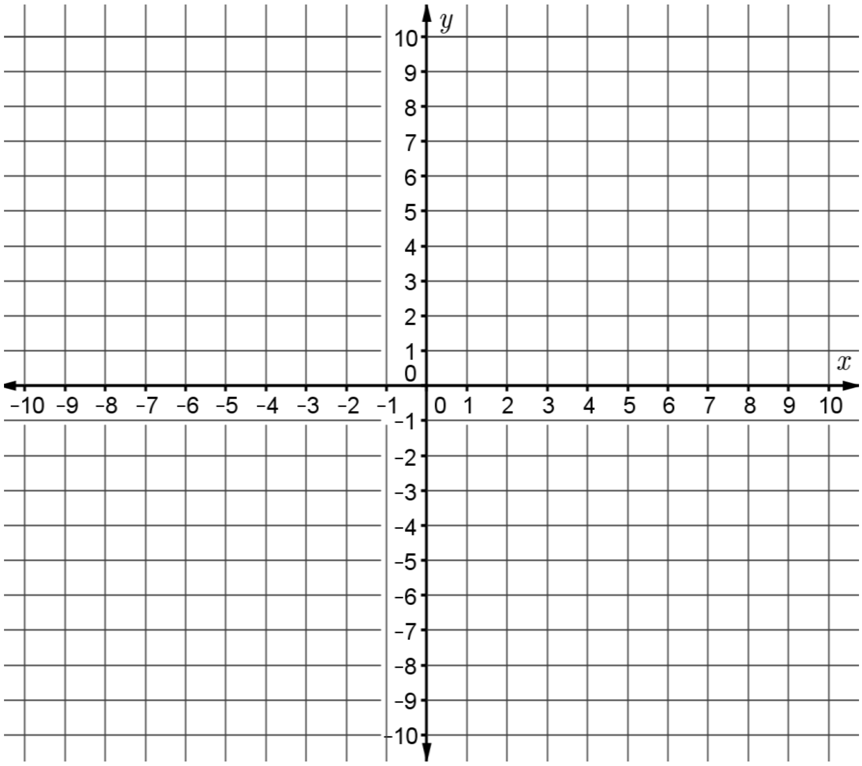
Hence the dimensions of the window must be x=2 meters (height) and y = 4 meters (width) and its maximum area is
E(x,y) = xy = 8 m^2

Back Next!

WORKSHEET FOR STUDENTS

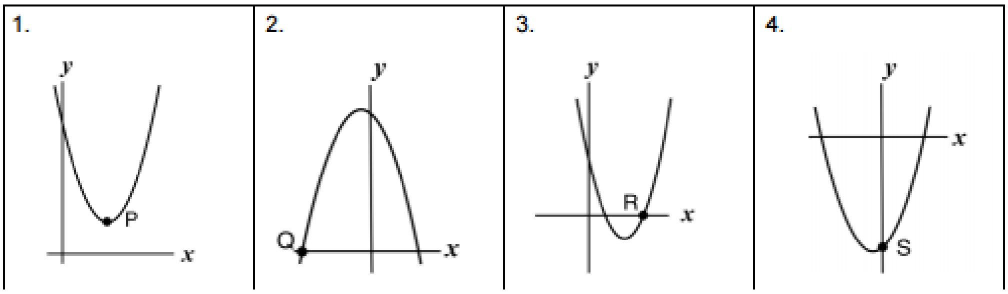
1. What is the equation for a quadratic function?

2. Graph the function $f(x) = x^2 + 2x - 3$.



3. Does this function have a minimum? If so, where is it?
4. Does this function have a maximum? If so, where is it?
5. Here are 4 equations of quadratic functions and 4 sketches of the graphs of quadratic functions.

A. $y = x^2 - 6x + 8$	B. $y = (x-6)(x+8)$	C. $y = (x-6)^2 + 8$	D. $y = -(x+8)(x-6)$



a) Match the equation to its graph and explain your decision.

Equation A matches Graph ________ because

____

Equation B matches Graph ________ because

____

Equation C matches Graph ________ because

____

Equation D matches Graph ________ because

____

b) Write the coordinates of the points P(,) Q(,) R(,) S(,)

6. The graph of a quadratic function has an y intercept at (0,5) and a minimum at (3, -4).

a) Write the equation of its curve

____

b) Write the coordinates of the root(s) of the quadratic function.

7. Joseph threw a waffle ball out of a window that is four meters high. The position of the waffle ball in Cartersian coordinates Oxy is determined by the parabola $y = -x^2 + 4$. At how many meters far from the building does the ball hit the ground?

8. A ball is dropped from a height of 60 meters. The quadratic equation $d = -5t^2 + 60$ provides the distance d, of the ball, after t seconds. After how many seconds, does the ball hit the ground?

9. The height in meters of a projectile at t seconds can be found by the function $h(t) = -5t^2 + 40t + 1.2$ Find the height of the projectile 4 seconds after it is launched.

SUGGESTED LESSON PLAN IN THE AREA OF PROBABILITY

PROBLEM: **Meeting of two friends**

TARGET GROUP (17-18 year old students)

PREREQUISITES (knowledge and abilities necessary to cope with understanding the problems)

- What is probability?
- How to determine the probability of the event A?
- What is the number of the elementary events Ω, and the number of the favorable outcomes of event A?
- Be able to solve the inequalities

LEARNING OBJECTIVES

The student will be able to:

- transform a word problem into a mathematical problem
- formulate a strategy for solving problem
- calculate the probability of the event A
- determine the probability of the event A, using the geometric probability
- use the probability into a real life problem
- connect probability to the real world
- develop problem-solving skills.

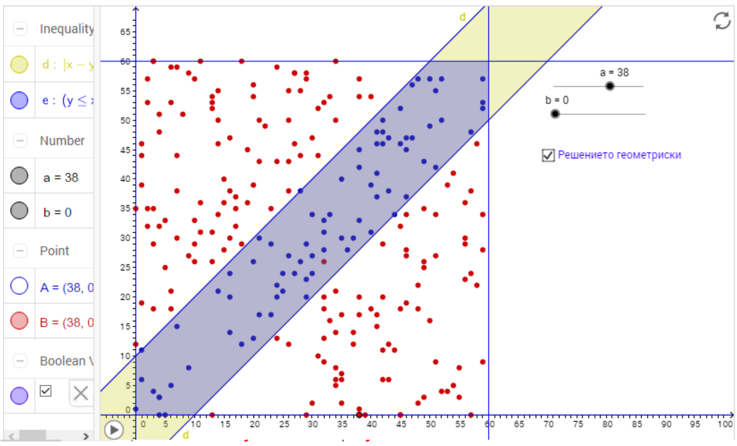
TOOLS AND MATERIALS

- the Interactive book
- the Guidelines
- GeoGebra applets
- drawings

THE PROBLEM

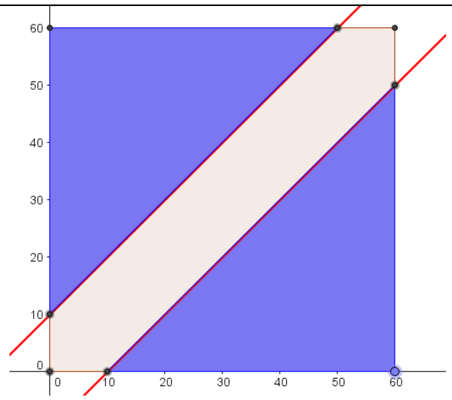
Two friends A and B have agreed to meet in the city center of Skopje from 12.00 to 13.00 hour. Determine the probability that the two friends will meet each other if each of them after their arrival at the meeting point, waits for 10 minutes for the other friend to come, and if he does not come to the meeting – he leaves.

LESSON

1. Students need to mark the time when the friends will arrive in the square using variables.	Topic: Probability; Answer: 1 × **Let's mark the time of arrival in minutes of the friend A with "x", and the time of arrival of the friend B with "y".** Back Next! End
2. Students need to understand the variables and will be asked in which limits the variables are ranging?	Topic: Probability; Answer: 2 × $0 \leq x \leq 60$ **and** $0 \leq y \leq 60$ Back Next! End
3. The students will be suggested to use a point S with coordinates x and y. Or: *S*(*x*,*y*) to mark together the arrival time of the both friends. The student will be given visualization of the problem in the GeoGebra and instructions how to make it by themselves.	 http://tube.geogebra.org/m/UhentK8w?doneurl=%2Fsearch%2Fperform%2Fsearch%2Fzoran%2Fpage%2F2%2Fr%2F0

4. The student is guided to find the probability of the event A, by using the geometric probability.	Topic: Probability; Answer: 6 × $\|\Omega\|$ - the area of the figure of elementary events, $\|A\|$ - the area of the figure of favorable events, $P(A) = \frac{\|A\|}{\|\Omega\|}$ Back Next! End
5. The student is helped to determine the set of the elementary events Ω.	Topic: Probability; Answer: 7 × A square with a side of 60 units. $\Omega = \{(x,y)\ \|0 \le x \le 60; 0 \le y \le 60; x,y \in Z\}$ Back Next! End
6. The student is helped to calculate the area of Ω	Topic: Probability; Help: 8 × The area of a square with a side a, $P = a^2$. Make a sketch (drawing). Back Next! Answer! Additional info! End $\|\Omega\| = 60^2 = 3600$
7. The student is guided by the questions: What should be fulfilled for the friends to meet? What is the difference in minutes when they had arrived at the square?	Topic: Probability; Answer: 9 × The difference in an absolute value has to be less than or equ al to 10 minutes. $\|x-y\| \le 10$ Back Next! End
8. Students need to mark the event to happen. Event A - the friends met at the Square. What is the set of favorable events?	Topic: Probability; Answer: 11 × $A = \{(x,y)\ \|x,y \in Z; \|x-y\| \le 10\}$ Back Next! End

9.Students are asked to determine the dimension of A by solving inequalities	Topic: Probability; Answer: 12 × To determine the area of the figure obtained for the favorable events, by solving the inequality. $\lvert x-y\rvert \leq 10$. Back Next! End
10. Which two inequalities are obtained? The cross section of the two solutions is the area of A.	Topic: Probability; Answer: 15 × $y \leq x+10$ and $y \geq x-10$ Back Next! End
11.The students need to define/determine the graph of the straight lines. Find the sections of the straight lines and the sides of the square. Make a sketch of the graph.	$y = x+10$ (Intersection with the straight lines y=60 and x=0) $y = x-10$ (Intersection with the straight lines y=0 и x=60)
12. The students need to calculate the area of the figure.	Mark the straight lines on the graph with red colour and define where the point S(x,y) would lie if the two friends meet, and where if they do not. If the point S(x,y) lies in the blue part of the square then the two friends will not meet. If it lies in the inner part marked with the straight lines and the sides of the square (marked with red colour in the image), then the friends would meet.

<table>
<tr><td></td><td>
</td></tr>
<tr><td>13. Students are helped by the questions: What is the area of A?
How to determine it?
What the two blue triangles form?</td><td>Topic: Probability; Answer: 18

The area of A is a difference from the area of the whole square and the square made up of two isosceles right triangles with a cathetus of 50 units.
$|A| = 60^2 - (60-10)^2 = 60^2 - 50^2$
and
$|A| = 3600 - 2500 = 1100$

Back Next! End</td></tr>
<tr><td>14. Students need to calculate the probability of the event?</td><td>Topic: Probability; Answer: 19

$P(A) = \frac{|A|}{|\Omega|} = \frac{1100}{3600} = \frac{11}{36}$
$P(A) = 0,31$

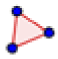
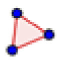
Back Next! End</td></tr>
<tr><td>15. And expressed as a percentage. Conclusion</td><td>Topic: Probability; Answer: 20

31%
The probability that the both friends will meet each other is 31%.

Back Next!</td></tr>
</table>

GeoGebra-aplet

Construction Protocol in GeoGebra

No.	Toolbar Icon	Command/ Value
1	a = 2	Slip a, from 0 to 60, step 1, check „ Accidentally “, Turn animation on
2	a = 2	Slip b, from 0 to 60, step 1, check „ Accidentally “, Turn animation on
3		In the window for entry, point S=(a,b)
4	a = 2	Right click on the point S and in the characteristics in the tab - Basic to turn on – trace, in the color tab to choose red, and in the tab – Advanced > Term for indication, to enter: abs(a-b)<10
5		In the window for entry, point S_1=(a,b)
6		Right click on the point S_1 and in the characteristics in the tab - Basic to turn on – trace, in the color tab to choose blue, and in the tab – Advanced > Term for indication, to enter: abs(a-b)>10
7		Points A=(0,0), B=(60,0), C=(60,60), D=(0,60)
8		Polygon ABCD
9		In the window for entry straight lines: y=x+10, y=x-10
10	ABC	Insert text: Show the solution
11		Checkbox - show / hide objects: Select the straight lines y=x+10, y=x-10.
12		Turn on the animation

WORKSHEET FOR STUDENTS

1. A card is drawn at random from a deck of cards. Find the probability of getting a queen.
2. A jar contains 3 red marbles, 7 green marbles and 10 white marbles. If a marble is drawn from the jar at random, what is the probability that this marble is white?
3. The blood group of 200 people is distributed as follows: 50 have type A blood, 65 have B blood type, 70 have O blood type and 15 have type AB blood. If a person from this group is selected at random, what is the probability that this person has O blood type?

4. What is the probability of getting an odd number when rolling a single 6-sided die?
5. What is the probability of getting a 7 after rolling a single die numbered 1 to 6?
6. What is the probability of choosing the letter i from the word probability?
7. A jury of 12 is to be selected from a list of 35 males and 15 females. What is the probability that there is no more than one female on the jury?
8. There are 3 defective calculators in a box of 100. Out of five selected, what is the probability that:
   a) four are good and one is defective
   b) there are at most two defective
   c) all five are good
9. A committee of six is randomly selected from a club with 18 male and 12 female members
   a) What is the probability that there is at least one female on the committee?
   b) Find the probability that there are three males and three females on the committee.

SUGGESTED LESSON PLAN

IN THE AREA OF MOTION PROBLEMS

MODELING WITH LINEAR EQUATIONS

PROBLEM: **Motion problem on trains**

TARGET GROUP (14-15 year old students)

PREREQUISITES:

- Which are the components in the uniform motion?
- What relation is there between them?
- In which situations may we encounter when two trains are running?
- What does a vehicle with definite length mean to cross a stationary object?

LEARNING OBJECTIVES

- to model situations in motion through equations
- to calculate the result according to the modeled situation

TOOLS AND MATERIALS

- the Interactive book
- the Guidelines
- GeoGebra applets
- drawings

THE PROBLEM

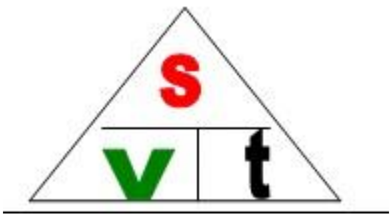
A train crosses a pole in 6 seconds and runs over a 160-metre-long bridge in 14 seconds. Another train has the same length and it runs at a speed of 36km/h. In what time (in seconds) will they cross each other travelling in opposite direction?

LESSON:

- So, let’s try to make sense of the text! We can not perceive the whole text at once, so we'll divide it into parts.

- We read "Train crosses a pole". Therefore, the problem is motion. Therefore, we need to remind (the theory) the main characteristics of motion and the basic dependencies between them.

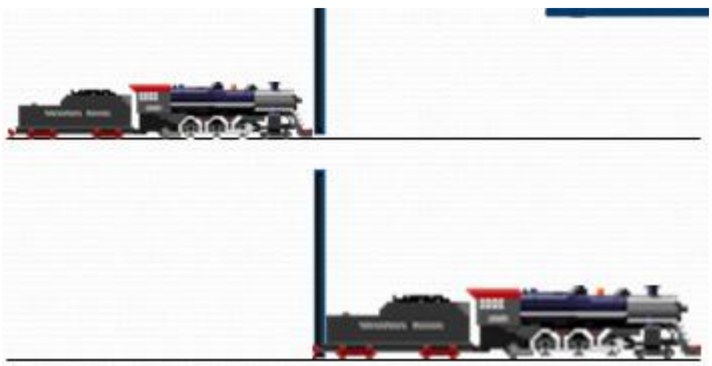
  - T - time
  - S – distance
  - V – speed



1. <u>We read "Train crosses a pole". Is the situation clear?</u>

   - To illustrate this situation and analyze it. What reasonings, conclusions will we made? To clarify it as a vibrant, practical situation!

   To illustrate the situation!



   - When has the train crossed the pole? (When even the last carriage has crossed.)
   - Consider the sketch! Compare the text to the sketch!
   - As a practical situation, we have clarified the text "Train crossed a pole", and now "Train crosses a pole in 6 seconds".

- We have the time when the train crossed the pole, what reflections should be made in this time of discrepancy? (In 6 seconds the train crossed the pole, and at what speed?) What distance has it travelled?
- Answer these questions! Look at the sketch!

CONCLUSIONS:

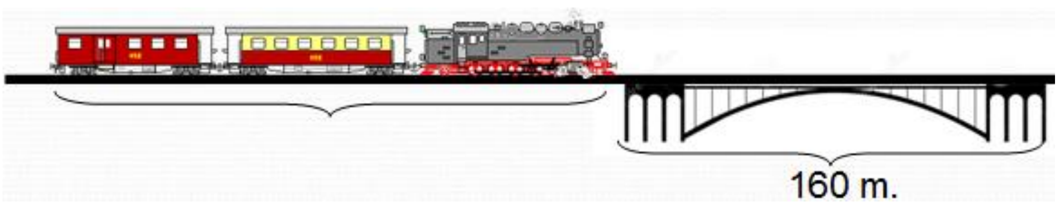
- The distance travelled is equal to the length of the train $S = D_1$
- The relative speed is equal to the speed of the train $V = V_1$.

You may encounter a similar situation in another problem. Summarize!

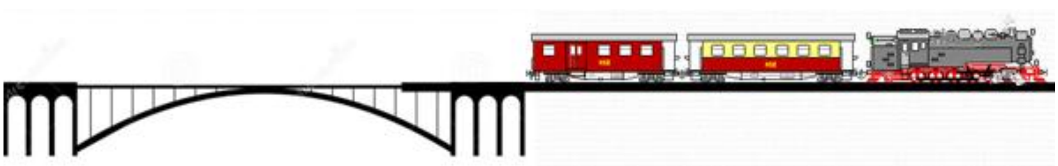
- A body of a definite length (train) crosses a stationary object without length (without stretching).
- The relative speed is the speed of the moving object.
- The distance travelled is the length of the stretch object.

2. Let’s interpret the text "over a bridge, ...". The train crosses the bridge.

- Compare the two situations. The train is crossing a pole, and now it travels over a bridge. (The bridge has a length)
- Let's clarify this situation:



- "When will we think the train has crossed the bridge?" (The train crossed the bridge when and its last carriage crossed the bridge.)



CONCLUSIONS:

- The distance travelled is equal to the sum of the length of the train and the length of the bridge ($S = D_1 + M$)
- The relative speed is equal to the speed of the train $V = V_1$.

We read "Train crosses the pole in 6 seconds," - think! (Pause for reflection). We read "over a bridge that is 160 meters long - in 14 seconds" - think! (Pause for reflection).

We read "Another train has the same length and runs at a speed of 36km / h." Are there any unclear situations in the last sentence? (No / Yes)

3. <u>In what time (in seconds) will they cross each other travelling in opposite direction?</u>

- To clarify the situation ”the trains run in opposite directions and they cross each other”!
- To illustrate the situation!

- Draw a sketch!
- When will we think the trains have crossed each other? (We think the trains have crossed each other when and their last carriages crossed.)
- Determine the dependencies between quantitative characteristics, the relative speed, travelled distance, time taken to cross each other!

CONCLUSIONS:

- The travelled distance is equal to the sum of the length of the first train and the length of the second ($S = D_1 + D_2$);
- The relative speed is equal to the sum of the speed of the first train and the speed of the second one ($V = V_1 + V_2$).

To summarize the three situations:

1. A train crosses a stationary object (we emphasize that the pole has only a height):

A) the travelled distance is equal to the length of the train (D);

B) the relative speed is the speed of the train.

2. Train (moving object with stretch) passes over a bridge (object with stretch but stationary):

A) the distance travelled is equal to sum of the length of the train and the length of the bridge ($S = D1 + M$);

B) The relative speed over the bridge is equal to the speed of the train.

3. At crossing two moving objects with respective length and speed;

A) the distance travelled is equal to the sum of the lengths of the two trains ($S = D_1 + D_2$);

B) the relative speed is equal to the sum of the speeds of the two trains ($V = V_1 + V_2$).

Let's read the whole problem: "A train crosses a pole in 6 seconds and it travels over a bridge that is 160 meters long - in 14 seconds. Another train has the same length and travels at a speed of 36km / h.

In what time (in seconds) will they cross each other travelling in opposite direction?"

- Is the condition understood? (Yes/No)
- Solve the problem!

1. The train crosses a pole

Let the length of the train is x m.

D: x > 0 t = 6 s. $V = \frac{x}{6} m/s$

2. The train travels over the bridge

S = (160 + x) m t = 14 s => $V = \frac{160+x}{14} m/s$

3. Create equation using the conclusions of the speed in 1st and 2nd situation!

$$\frac{x}{6} = \frac{160+x}{14} \Big| \cdot 42,\ D: x > 0$$
$$7x = 3(160 + x)$$
$$7x = 480 + 3x$$
$$7x - 3x = 480$$
$$4x = 480 \Big| :4$$
$$x = 120 \in D$$
$$V = \frac{120}{6} = 20 m/s$$

Answer: The length of the train is 120m; The speed of the train is 20 m/s.

1st train **2nd train**

V_1 = 20m/s

D_1 = 120m

V_2 = 36km/h = 10 m/s

D_2 = 120m

Relative speed: $V = V_1 + V_2$ V = 20m/s + 10m/s = 30m/s

Travelled distance of crossing each other : $S = D_1 + D_2$ S = 120 +120 = 240m

Time taken to cross each other: $t = \frac{S}{V} = \frac{240}{30} = 8s$

Answer: The time (in seconds) which they take to cross each other, is 8s.

Let's describe the approach to the problem!

- We consider the condition, each situation individually:

1. A train crosses a pole;
2. A train travels over a bridge;
3. A train crosses another train.

- What is the approach to each of these situations?

1. We illustrate the situation;
2. We divide its quantitative characteristics - distance, speed, time.
3. We emphasize the speed of the motion and the travelled distance.

Conclusion:

In order to solve a practical problem, we must first understand the condition in detail. At first it seems to us that we do not understand anything, so in parts, we have to read ourselves and conceive of every finished thought from the text. If we have not encountered such a situation, we will try to imagine it separately from the text, to clarify it for ourselves (as a sub problem).

It is essential, after we have practically clarified the situation set out by the text, to determine the main dependencies between the characteristics in the situation.

WORKSHEET FOR STUDENTS

1. Two cars are 500 kilometers apart and moving directly towards each other. One car is moving at a speed of 100 km/h and the other is moving at 70 km/h. Assuming that the cars start moving at the same time how long does it take for the two cars to meet?
2. Two boats start out 100 kilometers apart and start moving to the right at the same time. The boat on the left is moving at twice the speed as the boat on the right. Five hours after starting the boat on the left catches up with the boat on the right. How fast was each boat moving?
3. Two planes leave the same point at 8 AM. Plane 1 heads East at 600 km/h and Plane 2 heads West at 450 km/h. How long will they be 1400 km apart? At what time will they be 1400 km apart? How far has each plane traveled?
4. In still water, Peter's boat goes 4 times as fast as the current in the river. He takes a 15-kilometers trip up the river and returns in 4 hours. Find the rate of the current.
5. A shirt is on sale for 15.00 euro and has been marked down 35%. How much was the shirt being sold for before the sale?
6. An office has two envelope stuffing machines. Machine A can stuff a batch of envelopes in 5 hours, while Machine B can stuff a batch of envelopes in 3 hours. How long would it take the two machines working together to stuff a batch of envelopes?
7. Mary can clean an office complex in 5 hours. Working together John and Mary can clean the office complex in 3.5 hours. How long would it take John to clean the office complex by himself?

SUGGESTED LESSON PLAN

MODELING WITH A SECOND-DEGREE SYSTEM WITH TWO UNKNOWNS

PROBLEM: **Sheet metal roof**

TARGET GROUP (16 year - old students)

PREREQUISITES:

- What does square function mean?
- What does hyperbola mean?
- How do we solve a second-degree system with two unknowns?
- What do we know about the theorems of Vieta?

LEARNING OBJECTIVES

- To model situations with a second-degree system with two unknowns
- To apply the Vieta's formulas, Pythagorean theorem, function graph;
- To illustrate a graphic solution
- To give more than one solution of one problem

TOOLS AND MATERIALS

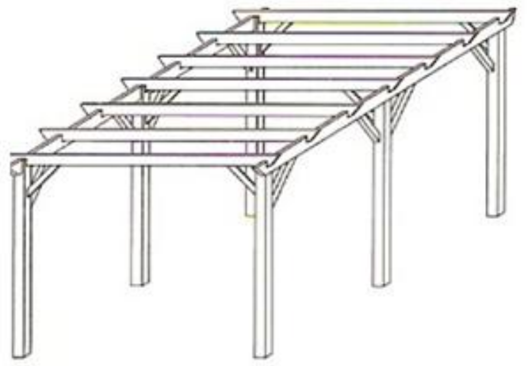
- the Interactive book
- the Guidelines
- GeoGebra applets
- drawings

<u>Problem:</u>

To make the ceiling of a shed a metal sheet must be cut in the shape of a rectangle with area of 12 square metres and 5 metres diagonal. Find the sides of the metal sheet.

LESSON:

To illustrate the situation



The shape of the metal sheet is a rectangle. We look for its dimensions, i.e. the sides a and b. Let's apply the formula for the area of e rectangle-

$$S = a \cdot b$$

As soon as we look for the length and width of the metal sheet, we have two unknowns: x - the length and y - the width.

I way: We have to create two equations with these quantities and solve them as a system. What dependencies can you see between the two quantities?

1. From the condition of the problem, the area of the metal sheet is 12 m^2, consequently xy = 12 will be one of the two equations.
2. Do you see another dependence between x and y?

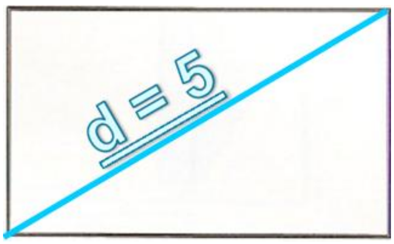
   They are catheters in the rectangular ΔABC whose hypotenuse is 5m.

   Let us remember the Pythagorean theorem: $c^2 = a^2 + b^2$

   consequently $5^2 = x^2 + y^2$. Here is the second equation of the system.

   Set of all allowable values!

   - As x and y are sides of a rectangle, they must be positive numbers, i.e. D: x> 0, y> 0.



3. To create a system and to solve it!

$$\begin{cases} x^2 + y^2 = 25 \\ x \cdot y = 12 \end{cases}$$

In how many ways can we solve the system?

- By substitution and addition.

<u>Let's solve it by substitution:</u>

$$\begin{cases} x^2 + y^2 = 25 \\ x \cdot y = 12 \end{cases} \Leftrightarrow \begin{cases} x^2 + y^2 = 25 \\ y = \dfrac{12}{x} \end{cases} \Leftrightarrow \begin{cases} x^4 - 25x^2 + 144 = 0 \\ y = \dfrac{12}{x} \end{cases}$$

$$\begin{cases} x_1 = 4 \\ y_1 = 3 \end{cases}; \begin{cases} x_2 = -4 \\ y_2 = -3 \end{cases}; \begin{cases} x_3 = 3 \\ y_3 = 4 \end{cases}; \begin{cases} x_4 = -3 \\ y_4 = -4 \end{cases}$$

<u>CONCLUSIONS</u>: The solutions (-4, -3) and (-3, -4) are unacceptable values and consequently they are not solutions of the problems, i.e. the metal sheet must have dimensions of 4 m and 3 m.

<u>Let's solve the system and by addition:</u>

$$\begin{cases} x^2 + y^2 = 25 \\ x \cdot y = 12 / \cdot 2 \end{cases}$$

$$\begin{cases} x^2 + y^2 = 25 \\ 2xy = 24 \end{cases} (+)$$

$$(x + y)^2 = 49$$

$$\swarrow \searrow$$

$$\begin{cases} x + y = 7 \\ x \cdot y = 12 \end{cases} \qquad \begin{cases} x + y = -7 \\ x \cdot y = 12 \end{cases}$$

$$\begin{cases} x_1 = 4 \\ y_1 = 3 \end{cases}; \begin{cases} x_2 = 3 \\ y_2 = 4 \end{cases}; \begin{cases} x_3 = -4 \\ y_3 = -3 \end{cases}; \begin{cases} x_4 = -3 \\ y_4 = -4 \end{cases}$$

<u>SOLUTIONS</u>: The solutions (-4, -3) and (-3, -4) are unacceptable values and consequently they are not the solutions of the problems, i.e. the metal sheet must have dimensions of 4 m and 3 m.

<u>II way</u>: Can you solve the problem in another way?

Let's use our knowledge about the Vieta's formulas!

Which formula of abbreviated multiplication could we use in the problem?

$x^2 + y^2 = (x+y)^2 - 2xy$

Vieta's Formulas
If x_1 and x_2 are roots of $x^2 + px + q = 0$ so $x_1 + x_2 = -p$, $x_1 x_2 = q$
If x_1 and x_2 are roots of $ax^2 + bx + c = 0$ so $x_1 + x_2 = -\frac{b}{a}$, $x_1 x_2 = \frac{c}{a}$

$\begin{cases} x^2 + y^2 = 5^2 \\ x \cdot y = 12 \end{cases}$ $\quad x^2 + y^2 = (x+y)^2 - 2xy \quad \Rightarrow \quad (x+y)^2 - 24 = 25 \Rightarrow (x+y)^2 = 49$

$\Rightarrow x + y = 7 \vee x + y = -7$

$\begin{cases} x + y = 7 \\ xy = 12 \end{cases}$ x and y are solutions of $p^2 - 7p + 12 = 0 \quad \Rightarrow p_1 = 4, p_2 = 3$

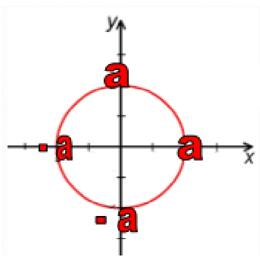
$\begin{cases} x + y = -7 \\ xy = 12 \end{cases}$ x and y are solutions of $q^2 + 7q + 12 = 0 \quad \Rightarrow q_1 = -4, q_2 = -3$

The solutions are: (4,3), (-4,-3), (3,4), (-3,-4)

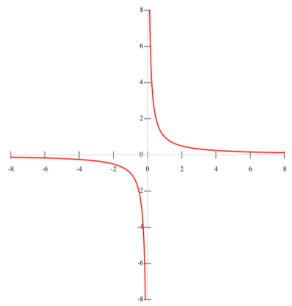
CONCLUSIONS: Solutions of x + y = -7 are unacceptable values (D: x> 0, y> 0) and consequently they are not solutions of the problems, i.e. the metal sheet must have dimensions of 4 m and 3 m.

<u>III way</u>: Let us remember our knowledge about the function graph and to see if we can solve the problem and graphically.

- What is the graph of the equation $x^2 + y^2 = a^2$?
- Circle with radius a**.**
- Draw it!



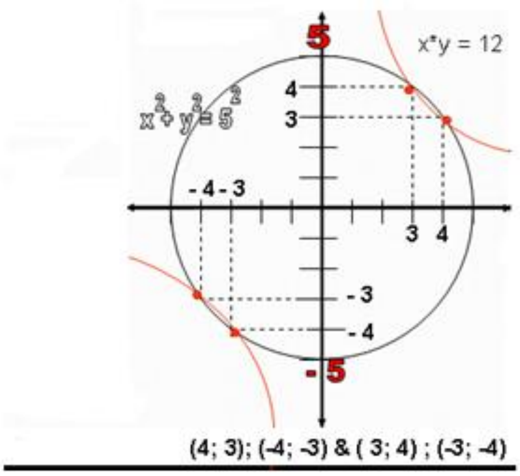
- What is the graph of the equation $x \cdot y = a$?
- Hyperbola in the I and III quadrant.
- Draw it!



To summarize and conclude!

CONCLUSIONS: We are looking for the possible solution of the two equations. Consequently these are the points in which the two graphs intersect.

Make a drawing



Conclusion: The solutions of the problems are 4 and 3 because the negative values are not solutions (D: x> 0, y> 0) (D: x> 0, y> 0)

Answer: The metal sheet must have dimensions of 4 m and 3 m.

Conclusion: In order to solve a practical problem we need the following: first to the condition understand in details. If we have not encountered such a situation, we will try to imagine it and make it clear. We need to determine how to approach and how (what method to use) to solve it.

It is essential that after we have practically clarified the situation set out by the text, we determine the basic dependencies between the quantities in the situation.

WORKSHEET FOR STUDENTS

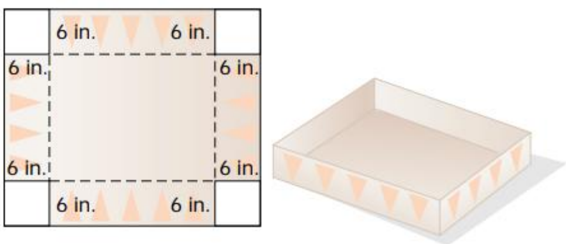
1. Solve the system $\begin{cases} x^2 + y^2 = 10 \\ 16x^2 + y^2 = 25 \end{cases}$

2. An engineer is to design a rectangular computer screen with a 19-inch diagonal and a 175-square-inch area. Find the dimensions of the screen to the nearest tenth of an inch.

3. Numbers. Find two real numbers such that their sum is 3 and the sum of their squares is 5.

4. Geometry. Find the dimensions of a rectangle with an area of 32 square meters if its perimeter is 36 meters long.

5. Construction. An open-topped rectangular box is formed by cutting a 6-inch square from each corner of a rectangular piece of cardboard and bending up the ends and sides. The area of the cardboard before the corners are removed is 768 square inches, and the volume of the box is 1,440 cubic inches. Find the dimensions of the original piece of cardboard.



WORKSHEET FOR STUDENTS

1. For the patronage of a school, a 1000 meters race was organized. The school had provided prizes for every participant in the race and it was sport equipment. The total amount of the prize given as sport equipment is 5000 euro. The student who won the race could choose sport equipment for 800 euro, and each student who arrived next at the end, got a prize of 50 euro cheaper than the previous one. How many students have taken part in this race?

2. A grandfather has given to his grandson a bucket and asked him to bring 8 l water from the well. The bucket is 24 cm deep, and its diameters at the bottom and the top are 16 and 24 cm respectively. Does the grandson can bring such quantity of water by going only once to the well?

3. The menu of a restaurant offers a salad of tomatoes, cucumbers and cheese. The portion is 750 grams. The restaurant buys tomatoes at a price of 3 euro per kilogram, cucumbers - 2 euro per kg and cheese - 7 euro per kg. How many grams of cheese are there in one portion of salad, if the cucumbers weight is 250 grams and the whole portion costs the restaurant 2.40 euros?

4. Peter gets from his grandmother 4 euro twice a week and he buys his favorite juice and chocolate with that money. The chocolate costs $\frac{3}{5}$ of the money, and the rest Peter spends for the juice. However, in that store the price for the chocolate becomes higher for 12%, and the juice gets 15% discount. Can Peter buy his favorite juice and chocolate with 4 euro twice a week?

5. Maria and her brother have joint room. Maria can clean it for 10 minutes and her brother can clean it for 25 minutes. How long will take cleaning the room if they work together?

6. A cylindrical cup has radius of the base that equals 25% of its height. How much of the height of the cup you can pour natural juice in, so when you put one ball of ice cream with a radius equals to the radius of the base of the cup, the juice would not overflow?

EVALUATION

Evaluation is a process that critically examines a program. It involves collecting and analyzing information about a program's activities, characteristics, and outcomes. Its purpose is to make judgments about a program, to improve its effectiveness, and/or to inform programming decisions (Patton, 1987).

The project Mathematical Labyrinth relates to creation of a web platform with an interactive book that contains real-life mathematical problems. Our basic intention and purpose of the project was to increase the knowledge of students in acquiring skills and competences in mathematics. In order to evaluate to what extent we have achieved our goals there were several forms created and prepared that can be used a sample material for assessment and evaluation.

It was important to assess and adapt the activities to ensure they are as effective as they can be. Evaluation helped us identify areas for improvement and ultimately helped us realize our goals more efficiently. Additionally, when we shared our results about what was more and less effective, we helped advance environmental education.

Evaluations fall into one of two broad categories: formative and summative. Formative evaluations are conducted during program development and implementation and are useful if you want direction on how to best achieve your goals or improve your program. Summative evaluations should be completed once your programs are well established and will tell you to what extent the program is achieving its goals.

Within the categories of formative and summative, there are different types of evaluation.

TYPE OF EVALUATION	PURPOSE
Formative	
1. Needs Assessment	Determines who needs the program, how great the need is, and what can be done to best meet the need. An EE needs assessment can help determine what audiences are not currently served by programs and provide insight into what characteristics new programs should have to meet these audiences' needs.
2. Process or Implementation Evaluation	Examines the process of implementing the program and determines whether the program is operating as planned. Can be done continuously or as a one-time assessment. Results are used to improve the program. A process evaluation of an EE program may focus on the number and type of participants reached and/or determining how satisfied these individuals are with the program.
Summative	
1. Outcome Evaluation	Investigates to what extent the program is achieving its outcomes. These outcomes are the short-term and medium-term changes in program participants that result directly from the program. For example, EE outcome evaluations may examine improvements in participants' knowledge, skills, attitudes, intentions, or behaviors.
2. Impact Evaluation	Determines any broader, longer-term changes that have occurred as a result of the program. These impacts are the net effects, typically on the entire school, community, organization, society, or environment. Impact evaluations may focus on the educational, environmental quality, or human health impacts of EE programs.

Adapted and used from: MEERA

During the process of evaluation we kept to the following principles and standards to make it relevant and eligible for further use of teachers and relevant stakeholders.

- **Good evaluation is tailored to your program and builds on existing evaluation knowledge and resources.**

The evaluation should be crafted to address the specific goals and objectives of the program. However, it is likely that other environmental educators have created and field-tested similar evaluation designs and instruments. Rather than starting from scratch, looking at what others have done can help you conduct a better evaluation.

- **Good evaluation is inclusive.**

It ensures that diverse viewpoints are taken into account and that results are as complete and unbiased as possible. Input should be sought from all of those involved and affected by the evaluation such as students, parents, teachers, program staff, or community members. One way to ensure your evaluation is inclusive is by following the practice of participatory evaluation.

- **Good evaluation is honest.**

Evaluation results are likely to suggest that your program has strengths as well as limitations. The evaluation should not be a simple declaration of program success or failure. Evidence that the program is not achieving all of its ambitious objectives can be hard to swallow, but it can also help you learn where to best put your limited resources.

- **Good evaluation is replicable and its methods are as rigorous as circumstances allow.**

A good evaluation is one that is likely to be replicable, meaning that someone else should be able to conduct the same evaluation and get the same results. The higher the quality of your evaluation design, its data collection methods and its data analysis, the more accurate its conclusions and the more confident others will be in its findings.

MATHEMATICAL CAMPS

In the proposal we planned and organized activities to assess the effectiveness of the interactive book. In this relation, there were three summer mathematical camps organized by the three secondary schools involved in the project as partners.

The mathematical camps were organized with the total number of 90 students who were selected by criteria set up by the math teachers. The camps lasted for 5 days in an educational environment with agreed agenda and previously arranged activities. The intention was logging in and using the interactive book to make the students use the platform and increase their motivation, skills and competences in math, as well as improving their attention and logics.

To make an adequate analysis of the students' skills and motivation there was an entry and a final test. The students were supposed to solve real-life problems created by their teachers according to their age and curriculum. The results of the final tests that were given to students after using the platform for five days showed improved knowledge of students in solving textual problems that were initially difficult for them.

Moreover, the environment in which the camps were held, the selection of students, and the activities that were held between the sessions showed an increased motivation and will for learning. The surveys that were given to students at the beginning of the project involved general questions related to studying mathematics in general, and students were asked about the types of tasks and exercises they find most difficult.

The results of the survey introduced the basis for creation of the interactive book. Namely, the word problems are most challenging to students because they need to “translate” them into mathematical symbols, in order to solve them. Moreover, when these problems have application in real situations, they make the students use their logics and critical thinking to find connections of mathematics with everyday life.

EVALUATION FORMS

In the following section there are the forms we used to assess and evaluate the students' knowledge, motivation and progress. The tests and the forms created can be used as sample tests and evaluation instruments by the teachers in mathematics in primary and secondary schools who would use the MATH-Labyrinth interactive book in the classroom or as extra-curricular activities.

They are given in order as they were administered during the mathematical camps:

- **Self-assessment**
- **Entry test**
- **Students' Progress**
- **Final test**
- **Evaluation of the interactive book**
- **Evaluation of the Mathematical camps**
- **Analysis of the results**

SELF-ASSESSMENT FORM

Name: ______________________________

1. State the reasons why you joined this summer mathematical camp:

____
____
____

2. Do you like doing math at school?

a) yes b) no c) other _________________________

3. What do you feel more motivated doing?

a) Mathematical symbolic problems b) word problems with real life application

4. On the scale from 1-5 how difficult do you think problem solving is? (1 is least difficult, 5 is the most difficult)

a) 1 b) 2 c) 3 d) 4 e) 5

5. What of the following you might use when solving problems? (you can choose more than one option)

a) internet
b) books
c) open educational resources (Geogebra)
d) peers (school mates)
e) the MATH-Labyrinth interactive book
f) the teacher
g) other ___________________________

6. If your teacher provides help for solving the problems, how many times you will ask for help?
a) I don't need any help
b) once or twice
c) three to six times
d) more than that

7. What are you expectations from this mathematical camp and the MATH-Labyrinth interactive book?
____
____
____

ENTRY TEST

Solve the problems:

1. The amount of 2200 euros is needed to be allocated to four people. The second person should get 500 euros more than the first, the third person should receive 300 euros less than the first one, and the fourth person should get twice more than the first person. How much money would each person get?

2. Four stores split the profit between themselves according to the following portions: the first store got $\frac{3}{8}$, the second one got $\frac{1}{4}$, the third one got $\frac{1}{5}$, and the fourth store got the rest of 140 000 euros. What is the total profit and how much did the first three stores get?

3. A pool has the form of a hexagonal prism with basic edge of 10 m and equal depth of 1,8 m. How many cubic meters of water are needed to fill in the pool?

4. How many grams of water should be poured into a solution of 260 grams with 75% sugar, to get a solution with 25% of sugar?

5. On a Monday, all prices in Isla's shop are 10% more than normal. On Friday all prices in Isla's shop are 10% less than normal. James bought a book on Monday for 5.50 euro. What would be the price of another copy of this book on Friday?

6. In class 3A, half of the students live in the same province, but outside the municipality, $\frac{1}{6}$ live outside the province and 8 live in the same municipality. How many students are in class?

EVALUATION OF THE STUDENTS' PROGRESS

CHECK LIST FOR EVALUATION OF THE STUDENTS' PROGRESS

Name of the student: ________________________________

	Is the student motivated?		What level is the student's knowledge			How many hints does he/she need?			Does the student use references while solving?		Does he use other OER to visualize?		How many times student requires help from the teacher?				Number of problems solved?	other
	yes	no	low	average	high	none	few	many	yes	no	rarely	often	0	1-2	3-4	more		
DAY 1																		
DAY 2																		
DAY 3																		
DAY 4																		
DAY 5																		

Overall comment by the teacher:

____

____

____

____

____

Signature: ____________________________

FINAL TEST

Solve the problems:

1. In class 3A, half of the students live in the same province, but outside the municipality, $\frac{1}{6}$ live outside the province and 8 live in the same municipality. How many students are in class?
2. On a Monday, all prices in Isla's shop are 15% more than normal. On Friday all prices in Isla's shop are 15% less than normal. James bought a book on Friday for 5.95 euro. What would be the price of another copy of this book on Monday?
3. How many grams of water should be poured into a solution of 260 grams with 75% sugar, to get a solution with 25% of sugar?
4. At the beginning of the physical education course, the teacher makes a survey among his students to decide what to do. He finds out that $\frac{1}{4}$ of the pupils would like to play volleyball, $\frac{1}{5}$ handball, half pupils want to practice running and only one pupil putting the shot. How many students want to play handball?
5. Four stores split the profit between themselves according to the following portions: the first store got 37,5%, the second one got 25%, the third one got 20%, and the fourth store got the rest of 140 000 euros. What is the total profit and how much did the first three stores get?
6. A flowerbed has the form of a pentagonal prism with basic edge of 6 m and equal depth of 1,2 m. How many cubic meters of water are needed to fill in the flowerbed?

EVALUATION OF THE INTERACTIVE BOOK

Please mark using 1 to 5 the following statements concerning the Interactive Book

(1 for least adequacy, 2 for little adequacy, 3 for medium adequacy, 4 for good adequacy and 5 for excellent adequacy in the implementation of the project)

The extent of adequacy of the relevant aspect for the Interactive Book	Mark
Useful	
Attractive and interesting	
Satisfies goals	
Accurate and scientifically correct	
Covers necessary material	
Contains illustrations, graphs, figures, photos etc	
Contains questions, exercises, support, instructions	

Please add any other comment concerning the Interactive Book:

……..……………………………………………………………………………………………
…………………………………………………………………………………………………..

…………………………………………………………………………………………………
…………………………………………………………………………………………………

EVALUATION OF THE MATHEMATICAL CAMPS

A. Previous Experience and Training of the teachers participating in the summer camps

	Previous Experience and Training for the participation	YES	NO	In case of YES, mark from 0 to 5 the extend of adequacy (0 indicating no adequacy, 5 indicating very satisfactory)	COMMENTS
1	Have you any previous experience in teaching problem solving?				
2	Have you any previous training on approaches and issues concerning teaching problem solving?				
3	Have you any previous experience in teaching mathematics at summer camps?				
4	Have you participated in a training activity for this summer camp?				
5	Did you have any involvement in the Math-Labyrinth project before the participation in the summer camp?				
6	Did you use the website of this project before the participation in the summer camp?				
7	Have you gone over the interactive book of this project before the participation in the camp?				
8	Have you gone over the Guidelines for training of this project before the participation in the camp?				
9	Do you find useful the Interactive book in teaching problem solving?				
10	Do you feel that the summer camp has been fruitful to the students in improving their skills in mathematics?				

B. Views of teachers on the outcomes of teaching problem solving skills during the summer camp?

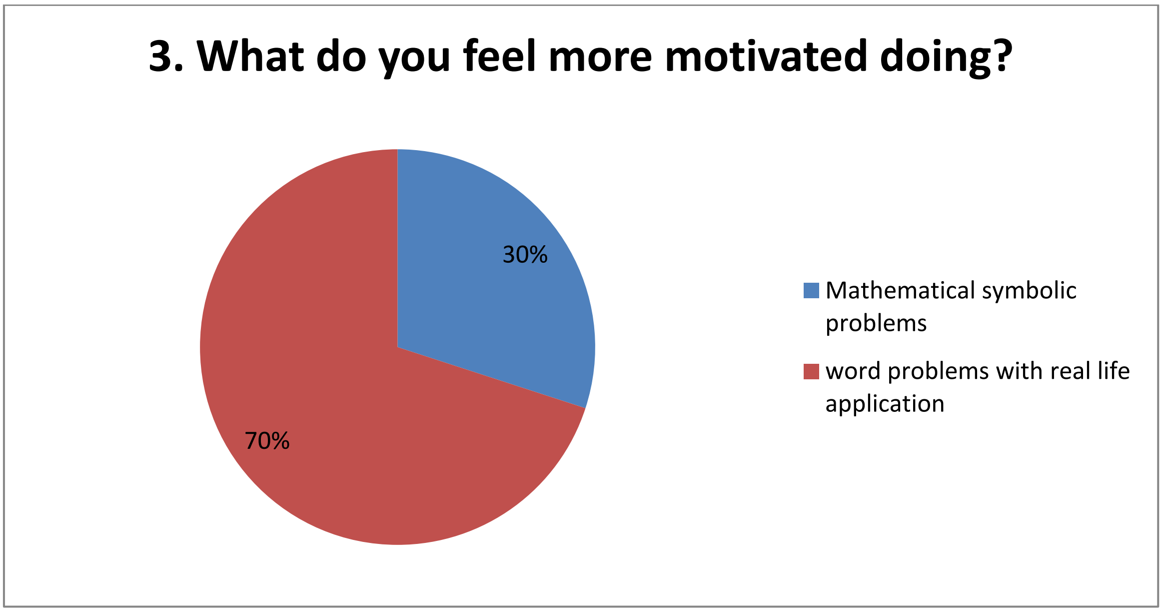
		YES	NO	In case of YES, mark from 0 to 5 the extend of adequacy (0 indicating no adequacy, 5 indicating very satisfactory)	COMMENTS
1	Was the course for the summer camps well designed?				
2	Was the course for the summer camps well accepted by the pupils?				
3	Did the course helped the pupils in acquiring skills for understanding the problem?				
4	Did the course helped the pupils in acquiring skills for developing a plan for solving problem?				
5	Did the course helped the pupils in acquiring skills for implementing a solution to the problem?				
6	Did the course helped the pupils in acquiring skills for assessing/ reviewing the process/ concepts/ outcomes of the solution to the problem?				
7	Did the course helped the pupils in acquiring skills for developing/ implementing various strategies for solving a problem?				

ANALYSIS OF THE RESULTS

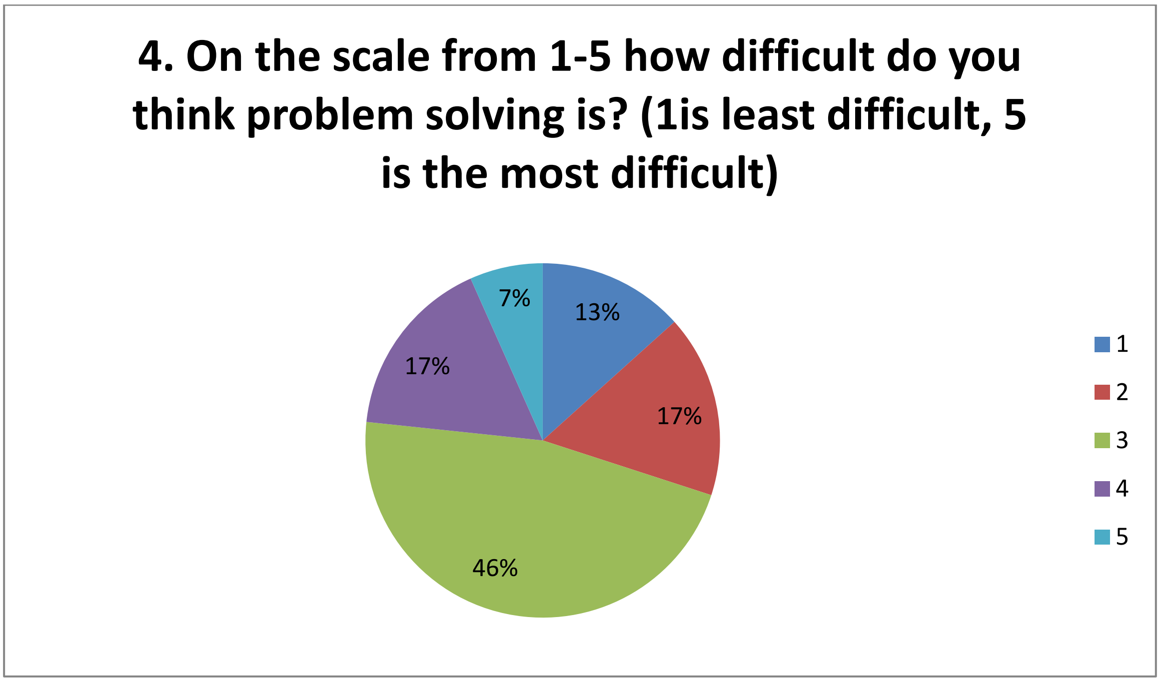
After the completion of the mathematical camps, the forms and instruments used during the 5 day-trainings have been analyzed and they showed the following results:

1. The students wanted to learn more about the Interactive book; learn new methods of problem solving, improve their knowledge in mathematics, etc.

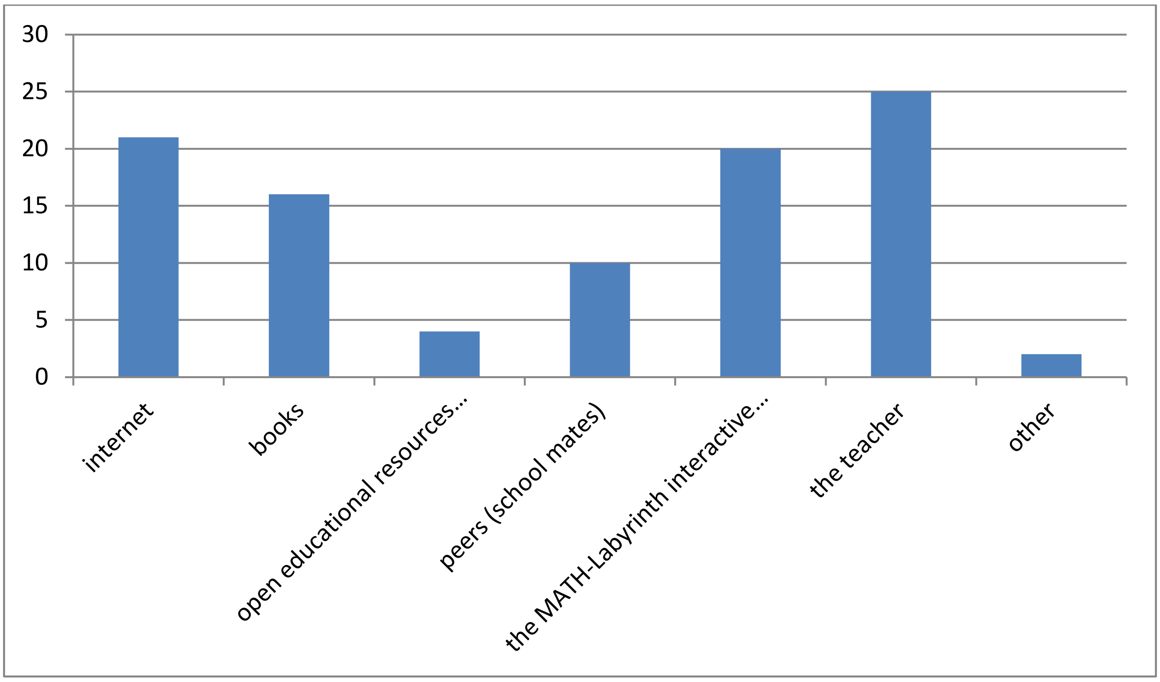
2. They feel motivated doing word problems more than doing problems and tasks with symbols.



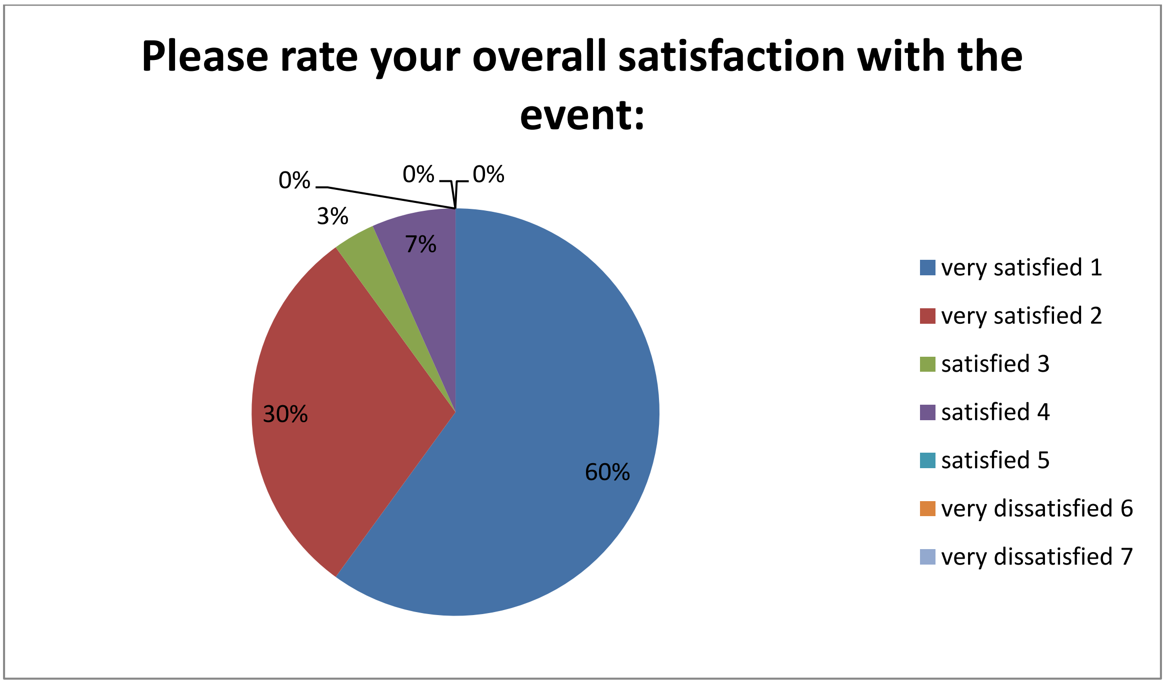
3. They perceive problem solving as most difficult tasks in doing math.

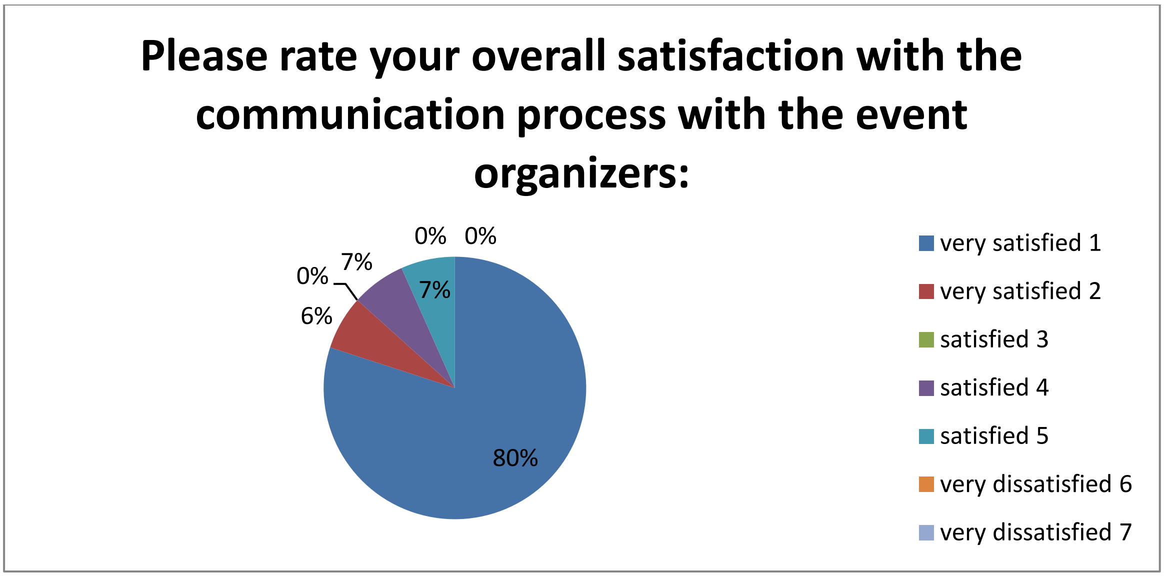


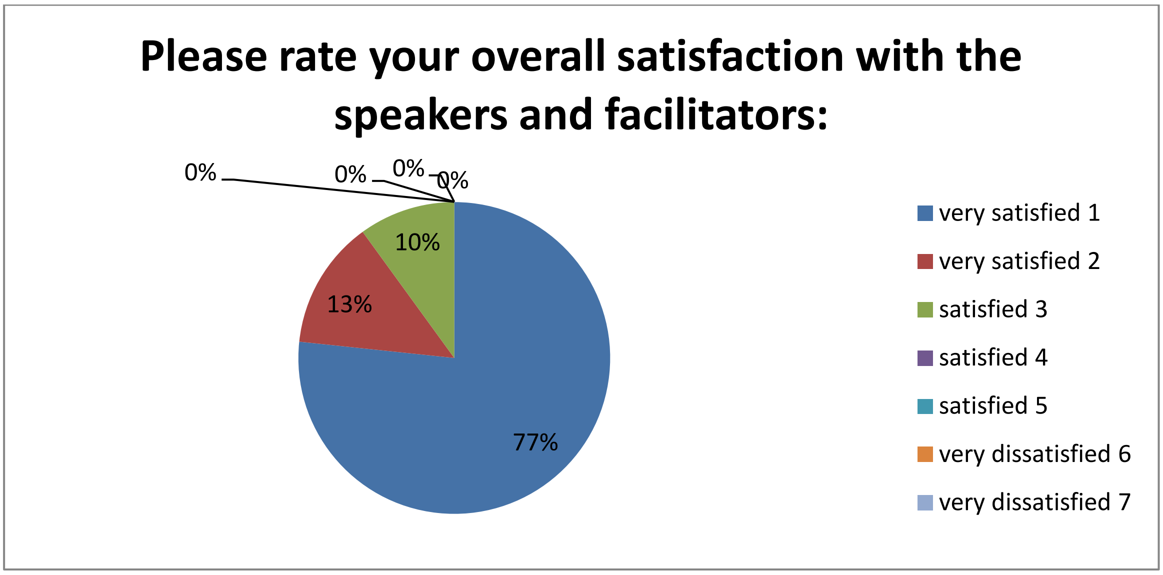
4. When students come across difficulties while doing real life word problems they ask for various help.

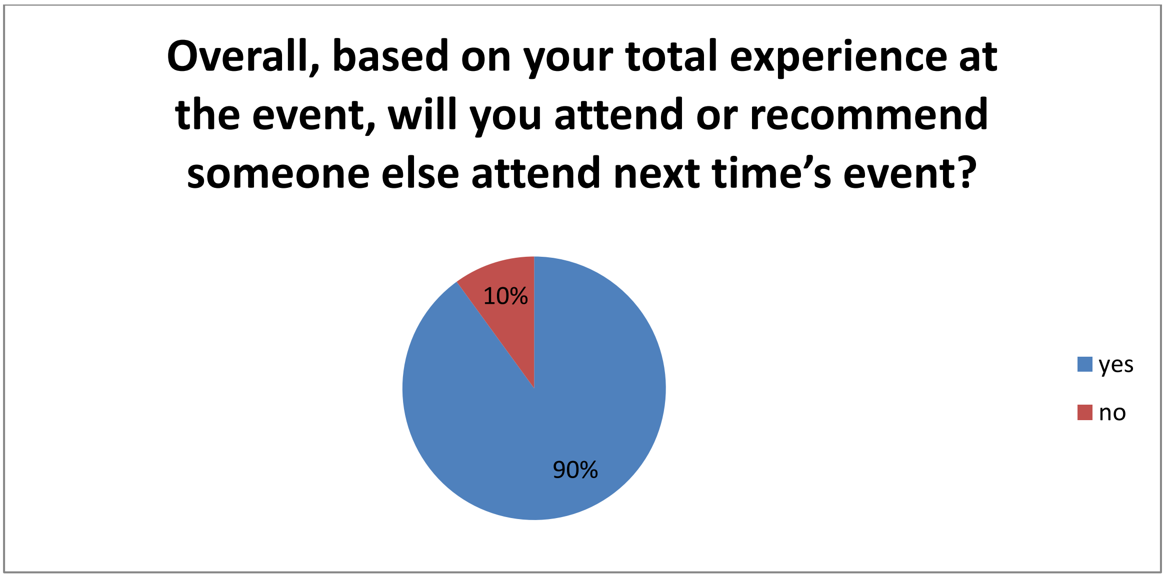


5. Students who attended the mathematical camps are satisfied with its contents, communication process, the location of the event, and they would recommend it to their peers and school mates.




Please rate your overall satisfaction with the communication process with the event organizers:
0%
0%
0%
7%
7%
6%
80%
very satisfied 1
very satisfied 2
satisfied 3
satisfied 4
satisfied 5
very dissatisfied 6
very dissatisfied 7


Please rate your overall satisfaction with the speakers and facilitators:
0%
0%
0%
0%
10%
13%
77%
very satisfied 1
very satisfied 2
satisfied 3
satisfied 4
satisfied 5
very dissatisfied 6
very dissatisfied 7


Overall, based on your total experience at the event, will you attend or recommend someone else attend next time's event?
10%
90%
yes
no

6. According to the analysis of the forms for evaluation of the students’ progress the following conclusions have been made by the teachers:

- **motivation of students**

Most of the students were motivated for solving real life problems from the interactive book every day during the camp. Some of them who have difficulties in mathematics were less motivated the first day, but their motivation increased from the second day, due to the interesting problems and the composition of the group of students.

- **number of hints/help used**

During the first two days of the camp students with less knowledge used a lot of hints, while those with better knowledge only a few. After the third day, all students significantly improved in terms of understanding the tasks and using a smaller number of hints. However, this is more notable among the students with average/high knowledge because they needed less time to understand and master the content. On the other hand, the students with lower knowledge would need a longer period of time to improve their mathematical skills and knowledge of problem solving.

- **use of open educational resources**

Initially, only those students who had previous experience with OER used them independently, and were able to deal with them. But, gradually the students understood how much OER, in particular GeoGebra (used in many examples in Math Labyrinth book), can help them for better understanding of the tasks and their better visualization. Students, regardless of success, have a desire to use OER.

- **number of problems solved**

The number of solved problems increased day by day and it was from 2 to 6 solved problems per day. After the second day, students were able to use the interactive book and without much help from the teachers. Students with medium or high knowledge were able to independently use the interactive book. It means that the interactive book can be a good support for independent student learning and developing skills for problem solving.

CONCLUSION

The collaborative work of the organizations involved in this project resulted in creation of a very innovative tool - an Interactive book with a step by step approach.

The design of the Interactive book is very appealing and easy to use. Its contents cover wide range of real-life problems relevant to the curricula, and age and interest of the students.

The approach of the method encourages low-achieving students to understand the matter of mathematics and increase their motivation and learning. By providing help at each stage of solving, it enables the students to use the book independently.

These types of problems have shown to be quite challenging for the students in many international skills tests. The MATH-Labyrinth method of teaching mathematics may give a solution to many PISA related negative results.

Mathematical problem solving can be difficult for teachers to teach as well. However, helping students to become good problem solvers is a very important goal, and challenging and exciting at the same time. Students are natural problem solvers. The teacher’s job is to develop the natural ability of students as problem solvers to its maximum extent, and to add to the already extensive repertoire of problem-solving techniques that students have at their disposal.

Useful Links

TEDEd Lessons Worth Sharing
https://ed.ted.com/lessons?category=mathematics

GEOGEBRA The graphing calculator for functions, geometry, algebra, calculus, statistics and 3D Math
https://www.geogebra.org/

Teaching Channel
https://www.teachingchannel.org/videos/real-world-math-examples

Real World Math is a collection of free math activities for Google Earth designed for students and educators.
http://www.realworldmath.org/

Math and Logic Problems to develop logical reasoning and problem solving skills
https://www.aplusclick.org/

The website is an extensive online resource providing tools and ideas for the professional development of mathematics educators across all phases, and is a dynamic means of communication between them.
https://www.ncetm.org.uk/ncetm/about-the-portal

Site with hundreds of free videos featuring Art of Problem Sovling
http://artofproblemsolving.com/videos

Project Learning mathematics through new communication factors
http://www.le-math.eu/index.php?id=14

The Virtual Math World
The virtual math lab is a project that allows students to visualize complex mathematical concepts through accessible and convenient software
http://cabinet.bg/index.php?status=applet&appletid=22&page=1

Top free resources for teaching and learning Mathematics
http://classroom-aid.com/educational-resources/mathematics/

The Math Open Reference Project
Mission: A free interactive math textbook on the web. Initially covering high-school geometry.
http://www.mathopenref.com/

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Polya's Problem Solving Techniques [Online] Available:http://scimath.unl.edu/conferences/documents/K-2ProblemSolvers.pdf [Accessed 14 April 2017]

Polya's Four Step Problem Solving Process [Online] Available:http://faculty.salisbury.edu/~dccathcart/mathreasoning/polya.html [Accessed 26 March 2017]

Meera (No date) Evaluation: What is it and why do it? [Online] Available: http://meera.snre.umich.edu/evaluation-what-it-and-why-do-it [Accessed 20 April 2017]

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ISBN 978-608-66136-0-0

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NATIONAL AGENCY
for European Educational
Programmes and Mobility

Funded by the
Erasmus+ Programme
of the European Union