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Systems of differential equations approximating the Lorenz system

Biljana Zlatanovska, Donco Dimovski

Abstract

By using modified Lorenz system from [1] as the system of differential equations of seventh order which approximated the Lorenz system, we obtained four new systems of differential equations of third, fourth, fifth and sixth order. Every new system of differential equations is obtained using the solutions of the third differential equation from the modified Lorenz system. The third differential equation of modified Lorenz system is homogeneous linear differential equation of fifth order with constant coefficients which can be solved. By computer simulations we compare the local behavior of modified systems of differential equations with the global behavior of the Lorenz system.

Keywords: Lorenz system, system of differential equations, modified Lorenz system, computer simulations.

1 Introduction

In [2] and [3] we have used power series combined with difference equations to find local approximations to the solutions of the Lorenz system of differential equations:

$$\begin{aligned} \dot{x} &= \sigma(y-x) \\ \dot{y} &= x(r-z)-y \\ \dot{z} &= xy-bz \end{aligned} \qquad (1)$$

with parameters σ, r and b. For initial values $a_0=x(0)$, $b_0=y(0)$, $c_0=z(0)$. We assume that the solutions x(t), y(t), z(t) of the system (1) are expanded as Maclaurin series with the coefficients a_n, b_n, c_n.

By [2], [3] and [1] after mathematical transformations with the initial values $a_0=x(0)$, $b_0=y(0)$, $c_0=z(0)$, $c_p = \overset{(p)}{z}(0)$, $p \in \{1,2,3,4\}$ and for $A=1+\sigma+b$, $B=\sigma(r-c_0)-a_0^2$, $C=\sigma a_0 b_0$, $D=-\sigma^2 b_0^2$, it was obtained modified

Lorenz system ([1]),

$$\begin{aligned} \dot{x} &= \sigma(y-x) \\ \dot{y} &= x(r-z)-y \\ \overset{(5)}{z} &= -(A+b)\overset{(4)}{z} + (B-Ab)\overset{(3)}{z} - (C-Bb)\overset{(2)}{z} + (D-Cb)\dot{z} - Dbz \end{aligned} \qquad (2)$$

2 Systems of differential equations

The third equation of the system (2) is homogenous linear differential equation of fifth order with constant coefficients and its characteristic equation has solutions $\lambda_1 = -b$, $\lambda_{2/3/4/5} = \lambda(A,B,C,D,b)$. Let, we suppose that all solutions of characteristic equation λ_i, i=1,2,3,4,5 are real solutions.

For the solutions $\lambda_{1/2/3/4/5}$ the system (2) of seventh order can be transformed in the following systems of differential equations (SDE):

$$\begin{aligned} \dot{x} &= \sigma(y-x) \\ \dot{y} &= x(r-z)-y \\ \dot{z} - \lambda_1 z &= 0 \end{aligned} \qquad (3)$$

$$\begin{aligned} \dot{x} &= \sigma(y-x) \\ \dot{y} &= x(r-z)-y \\ \overset{(2)}{z} - (\lambda_1+\lambda_2)\dot{z} + \lambda_1\lambda_2 z &= 0 \end{aligned} \qquad (4)$$

$$\begin{aligned} \dot{x} &= \sigma(y-x) \\ \dot{y} &= x(r-z)-y \\ \overset{(3)}{z} - (\lambda_1+\lambda_2+\lambda_3)\overset{(2)}{z} + (\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3)\dot{z} - \lambda_1\lambda_2\lambda_3 z &= 0 \end{aligned} \qquad (5)$$

$$\begin{aligned} &\dot{x} = \sigma(y-x) \\ &\dot{y} = x(r-z)-y \\ &\overset{(4)}{z} - (\lambda_1+\lambda_2+\lambda_3+\lambda_4)\overset{(3)}{z} + \\ &(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_2\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4)\overset{(2)}{z} - \\ &(\lambda_1\lambda_2\lambda_3+\lambda_1\lambda_2\lambda_4+\lambda_1\lambda_3\lambda_4+\lambda_2\lambda_3\lambda_4)\dot{z} + \lambda_1\lambda_2\lambda_3\lambda_4 z = 0 \end{aligned} \qquad (6)$$

with the initial values a_0=x(0),b_0=y(0),c_0=z(0), $\dot{z}(0) = c_1$, $\overset{(2)}{z}(0) = c_2$, $\overset{(3)}{z}(0) = c_3$.

3 Computer simulations for the SDE

In this section, we will look via computer simulations the local behavior for the SDE (2), (3), (4), (5), (6) and we will compare with global behavior of the Lorenz system,(fig.1). For given parameters σ, r, b, the procedure for looking at the local behavior of the SDE is the same as in [4], fig.2.

Example: For the parameters σ=2, r=31, b=1 and the initial values a_0=-3, b_0=1, c_0=-5 and λ_1=-1, λ_2≈-0.301, λ_3≈-10.147, λ_4≈0.210.

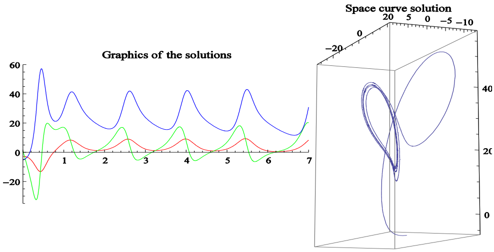
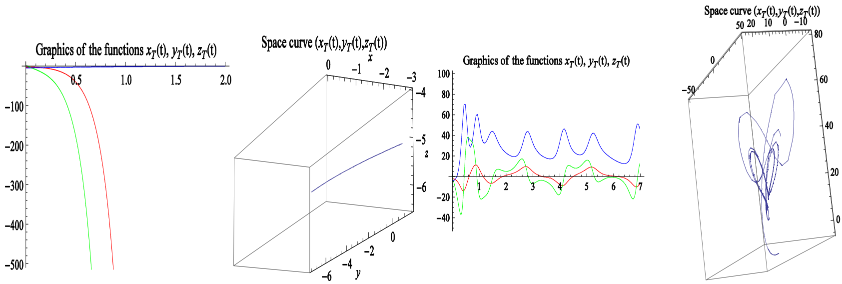
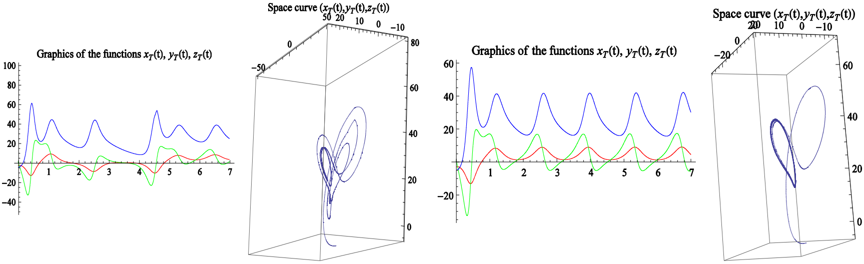


Figure 1: Results obtained by Mathematica for the Lorenz system (1) of time interval [0,7]

a) the systems (3) and (4) of time intervals [0,2] and [0,7] respectively



b) the systems (5) and (6) of time interval [0,7]



c) the system (2) of time interval [0,7]

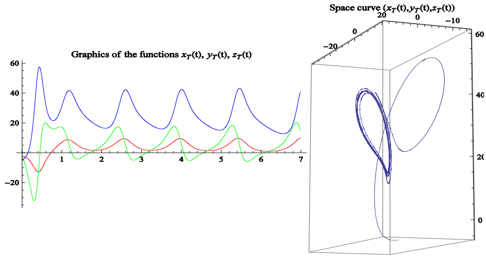


Figure 2: The solutions $x_T(t)$, $y_T(t)$, $z_T(t)$ for the systems (2), (3), (4), (5) and (6) with time step T=0.05

4 Conclusion

The local behavior of the system (2) is closest to the behavior of the Lorenz system for a small time step.

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