UNIVERSITY OF NOVI SAD
TECHNICAL FACULTY ”MIHAJLO PUPIN”
ZRENJANIN
REPUBLIC OF SERBIA

VIII INTERNATIONAL CONFERENCE OF

INFORMATION TECHNOLOGY AND DEVELOPMENT OF EDUCATION

ITRO 2017

PROCEEDINGS OF PAPERS

VIII MEĐUNARODNA KONFERENCIJA

INFORMACIONE TEHNOLOGIJE I RAZVOJ OBRAZOVANJA

ITRO 2017

ZBORNIK RADOVA

ZRENJANIN, JUNE 2017

Publisher and Organiser of the Conference:
University of Novi Sad, Technical faculty „Mihajlo Pupin”, Zrenjanin, Republic of Serbia

For publisher:
Dragica Radosav, Ph. D, Professor, Dean of the Technical faculty „Mihajlo Pupin”, Zrenjanin, Republic of Serbia

Editor in Cheaf - President of OC ITRO 2017:
Dragana Glušac, Ph. D, Professor,

Proceedings editors:
Marjana Pardanjac, Ph. D, Professor, Ivan Tasic, Ph. D, Professor; Dijana Karuovic, Ph. D, Professor; Erika Eleven, M.Sc, Assistant; Dusanka Milanov MSc, Assistant

Lecturer:
Erika Tobolka, Ph. D, Professor

Printed by:
Printing office Donat Graf d.o.o, Mike Alasa 52, Beograd

Circulation: **50**
ISBN: 978-86-7672-302-7

By the resolution no. 142-451-2481/2017-01/02, Autonomous Province of Vojvodina, Provincial Secretariat For Science and Technological Development donated financial means for printing this Conference Proceedings.

The Conference is supported by the Autonomous Province of Vojvodina

CIP - Каталогизација у публикацији
Библиотека Матице српске, Нови Сад

37.01:004(082)
37.02(082)

INTERNATIONAL Conference on Information Technology and Development of Education ITRO (8 ; 2017 ; Zrenjanin)
Proceedings of papers / VIII International Conference on Information Technology and Development of Education ITRO 2017 = Zbornik radova = VIII Međunarodna konferencija Informacione tehnologije i razvoj obrazovanja ITRO 2017, June 2017, Zrenjanin. - Zrenjanin : Technical Faculty "Mihajlo Pupin", 2017 (Beograd : Donat graf). - XIII, 290 str. : ilustr. ; 30 cm

Tekst štampan dvostubačno. - Tiraž 50. - Str. VI: Introduction / Dragana Glusac. - Bibliografija uz svaki rad.

ISBN 978-86-7672-302-7

a) Информациона технологија - Образовање - Зборници b) Образовна технологија - Зборници
COBISS.SR-ID *315769095*

PARTNERS INTERNATIONAL CONFERENCE

South-West University „Neofit Rilski”
Faculty of Education, Blagoevgrad,
Republic of Bulgaria

SOUTH WEST UNIVERSITY
“NEOFIT RILSKI”

Faculty of Electrical Engineering and Informatics
Department of Computers and Informatics of Kosice
Slovak Republic

University Goce Delcev Stip
Republic of Macedonia

УНИВЕРЗИТЕТ
„ГОЦЕ ДЕЛЧЕВ“
ШТИП

THE SCIENCE COMMITTEE:

Dragoslav Herceg, Ph.D, Professor, Faculty of Science, Novi Sad, Serbia – Chairman
Marina Čičin Šain, Ph.D, Professor, University of Rijeka, Croatia
Anton Vukelić, Ph.D, Professor, Faculty of Philosophy, Croatia
Ion Dzitac, Ph.D, Professor, Department of Mathematics-Informatics, Aurel Vlaicu University of Arad, Romania
Sashko Plachkov, Ph.D, Professor, South-West University "Neofit Rilski" /Department of Education, Blagoevgrad, Republic of Bulgaria
Sulejman Meta, Ph.D, Professor, Faculty of Applied Sciences, Tetovo, Macedonia
Márta Takács, Ph.D, Professor, Óbuda University, John von Neumann Faculty of Informatics, Budapest, Hungary
Nina Bijedić, Ph.D, Professor, Applied mathematics, Bosnia and Herzegovina
Viorel Negru, Ph.D, Professor, Department of Computer Science, West University, Timisoara, Romania
Mirjana Segedinac, Ph.D, Professor, Faculty of Science, Novi Sad, Serbia
Milka Oljača, Ph.D, Professor, Faculty of Philosophy, Novi Sad, Serbia
Dušan Starčević, Ph.D, Professor, Faculty of Organizational Sciences, Belgrade, Serbia
Dobrivoje Mihajlović, Ph.D, Professor, Faculty of Organizational Sciences, Belgrade, Serbia
Kosta Voskresenski, Ph.D, Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Josip Ivanović, PhD, Professor, Hungarian Language Teacher Training Faculty, Subotica, Serbia
Ivanka Georgieva, Ph.D, South-West University "Neofit Rilski", Faculty of Engineering, Blagoevgrad, Republic of Bulgaria
Miodrag Ivković, Ph.D, Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Momčilo Bjelica, Ph.D, Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Dragica Radosav, Ph.D, Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Dragana Glušac, Ph.D, Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Dijana Karuović, Ph.D, Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Ivan Tasić, Ph.D, Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Vesna Makitan, Ph.D, Assistant Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Marjana Pardanjac, Ph.D, Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Snežana Babić Kekez, Ph.D, Professor,Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Erika Tobolka, Ph.D, Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Stojanov Željko, Ph.D, Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Brtka Vladimir, Ph.D, Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Kazi Ljubica, Ph.D, Assistant Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Berković Ivana, Ph.D, Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Nikolić Milan, Ph.D, Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia

THE ORGANIZING COMMITTEE:

Dragana Glusac, Ph.D, Professor, Technical Faculty “Mihajlo Pupin” Zrenjanin, R. of Serbia - Chairman of the Conference ITRO 2017
Dragica Radosav, Ph.D, Professor, Technical Faculty “Mihajlo Pupin” Zrenjanin, R. of Serbia
Dijana Karuovic, Ph.D, Professor, Technical Faculty “Mihajlo Pupin” Zrenjanin, R. of Serbia
Marjana Pardanjac, Ph.D, Professor, Technical Faculty “Mihajlo Pupin” Zrenjanin, R. of Serbia
Ivan Tasic, Ph.D, Professor, Technical Faculty “Mihajlo Pupin” Zrenjanin, R. of Serbia
Vesna Makitan, Ph.D, Ass. Professor, Technical Faculty “Mihajlo Pupin” Zrenjanin, R. of Serbia
Erika Tobolka, Ph.D, Professor, Technical Faculty “Mihajlo Pupin” Zrenjanin, R. of Serbia
Erika Eleven, MSc, Assistant, Technical Faculty “Mihajlo Pupin” Zrenjanin, R. of Serbia
Dusanka Milanov, MSc, Assistant, Technical Faculty “Mihajlo Pupin” Zrenjanin, R. of Serbia

All rights reserved. No part of this Proceeding may be reproduced in any form without written permission from the publisher.

The editor and the publisher are not responsible either for the statements made or for the opinion expressed in this publication.

The authors are solely responsible for the content of the papers and any copyrights, which are related to the content of the papers.

With this publication, the CD with all papers from the International Conference on Information Technology and Development of Education, ITRO 2017 is also published.

INTRODUCTION

The Technical Faculty "Mihajlo Pupin", Zrenjanin, of the University of Novi Sad, the Republic of Serbia organizes VIIIth International Scientific Professional Conference "Information Technologies and Development of Education 2017" (ITRO 2017). The Conference will be held on 22nd June 2017 at the Technical Faculty "Mihajlo Pupin" in Zrenjanin, Serbia.

The Conference "Information Technologies and Development of Education 2017" (ITRO 2017) is organized due to the needs to connect science, profession and education through topics and content concept, first of all concerning the teaching process as base of information society. The tendencies of developed countries are in accordance with the efforts of UNESCO to improve this area related to the needs of life and work in the XXIst century. It is necessary to assess the state, detect the problems and perspectives of the development of education by competent professionals and teachers as well as the influence of the development of education on the development of the society as a whole.

The central topic of the meeting is the model of dual education as base for creating good base for the development of industry. Thus, our aim is to gather the representative entities who are able constructively contribute to establishing link between the educational system and industry as follows: Chamber of Commerce of Serbia – Centre for Dual Education, Ministry of Education, Science and Technological Development, Union of Employers of Serbia, ZREPOK – Business Organization of Zrenjanin and Companies that run their business in the region, directors of grammar schools and secondary vocational school, members of the academic communities and other participants who are interested in the topics.

The main topics of the scientific professional conference are:

- Model of dual education
- Teaching based on the concept of entrepreneurship

Other thematic areas of the Conference:

- Theoretical and methodological questions of contemporary Pedagogy
- Digital didactics media
- Contemporary communication in teaching
- Curriculum of contemporary teaching
- Developing teaching
- E-learning
- Management in Education
- Teaching methods of natural and technical subjects
- Information-communication technologies

The Chairman of the Organizing Committee of the ITRO 2017 Prof. Dragana Glušac opened the Conference. The participants were addressed by the vice dean of the Technical Faculty »Mihajlo Pupin«, Prof. Dijana Karuović; provincial secretary for science, higher education and scientific Research prof. Zoran Milošević, and the vice-major of Zrenjanin Mr. Dusko Radisic.

There were total of 143 authors that took part at the Conference from 12 countries, 2 continents: 82 from the Republic of Serbia and 61 from foreign countries such as: Macedonia, Bulgaria, Slovakia, Austria, Cyprus, Albania, Hungary, Spain, Bosnia and Herzegovina, USA, Portugal.

The Proceedings of papers contains 60 papers and it has been published in the English language.

President of the Organizing Committee
Prof. dr Dragana Glusac

We are very grateful to:

Autonomous Province of Vojvodina

for donated financial means which supported printing of the Conference Proceedings and organizing of the Conference.

CONTENTS

M. Mabić , F. Dedić , N. Bijedić, D. Gašpar
DATA MINING AND CURRICULUM DEVELOPMENT IN HIGHER EDUCATION 1

S. Brkanlić, M. Ćirić, E. Breso
THE IMPORTANCE OF COMMUNICATION SKILLS OF EMPLOYEESIN HIGHER EDUCATION INSTITUTION FOR ACHIEVING STUDENTS SATISFACTION 7

B. Sobota, L. Jacho, Š. Korečko, P. Pastornický
EXPERIMENTAL VIRTUAL REALITY SPACE FOR SMART ENVIRONMENT DEVELOPMENT 11

B. Delipetrev,E. Deleva
HYBRID MOBILE AND CLOUD APPLICATION FOR MACHINE MAINTENANCE 15

D. Mitova, Ly. Zoneva
INTERACTIVE ENVIRONMENT FOR TECHNOLOGY AND ENTREPRENEURSHIP LEARNING THROUGH THE MEANS OFINFORMATION EDUCATIONAL RESOURCES IN THE SECONDARY EDUCATION 20

S. Babić-Kekez, M. Josić, E. Eleven
PUPILS' WORKLOAD BY EDUCATIONAL CONTENT OF TECHNICAL AND IT EDUCATION 25

B. Sobota, D. Petríková, Š. Korečko, L. Jacho, P. Pastornický
AUGMENTED REALITY AND SYMBOLIC-TEXT METHOD IN AN EDUCATION OF DISABLED CHILDREN 29

M. Kocaleva, B. Zlatanovska, A. Stojanova, N. Stojkovikj, V. Gicev
WAVE EQUATION WITH DIRICHLET BOUNDARY CONDITIONS 34

I.Georgieva, F.Tsvetanov
PREREQUISITES FOR DUAL EDUCATION IN ENGINEERING 38

Lj. Popović, S. Popov, Đ. Ćosić
A GIS BASED APPROACH FOR HYDROLOGICAL CONFLICTS ESTIMATION 43

B. Kiss, D. Radosav, E. Tobolka, E. Eleven, M. Kavalić
MODERNIZATION OF ELEMENTARY SCHOOL EMPLOYEES ADVANCED TRAINING 49

D. Krstev, A. Krstev, R. Minovski, B. Krstev
EXPLORING EDUCATIONAL DILEMMAS USING THE SYSTEM DYNAMICS AND ARCHETYPES 57

I. Tasić, D. Glušac, D. Karuović, N. Tasić
PEDAGOGICAL ASPECTS OF E – LEARNING 61

Lj. Stanojević, A. Veljović, M. Ranđelović, M. Papić
THE EFFECT OF WEB-BASED CLASSROOM RESPONSE SYSTEM ON STUDENTS LEARNING OUTCOMES: RESULTS FROM PROGRAMMING COURSE 67

K. Runcev, N. Koceska, T. A. Pacemska
TANGIBLE USER INTERFACE FOR PRE-SCHOOL CHILDREN LEARNING ... 73

M. D. Blagojević, B. D. Kuzmanović, M. Ž. Papić
PREFERABLE E-LEARNING TOOLS – AN ACTION RESEARCH 77

A. Felbab, D. Radosav, Ž. Eremić, T. Sekulić, E. Tobolka
THE IMPORTANCE OF DUAL EDUCATION AND ITS REPRESENTATION ON THE TERRITORY OF MIDDLE BANAT 80

Basri Saliu
INTERACTION ON GOOGLE CLASSROOM AND GROUP COMMUNICATION: A CASE STUDY OF AN ESP GROUP IN CST AT SOUTH EAST EUROPEAN UNIVERSITY 86

V. Brtka, V. Ognjenović, I. Berković, E. Brtka, G. Jotanović, V. Makitan
THE METHOD FOR THE FORMATION OF GROUPS OF STUDENTS IN A COLLABORATIVE LEARNING 91

N. Aleksić, A. Mišković, A. Antonijević
THE IMPACT ON VERBAL AND NONVERBAL COMMUNICATION, AS ONE OF THE MOST IMPORTANT FACTORS OF QUALITY TEACHING STUDENT – TEACHER........ 95

D. Cvetković, B. Medić, M. Mijatović, M. Mijatović
SELF-STUDY WITH LANGUGAGE LEARNIG SOFTWARE IN THE WORKPLACE: WHAT HAPPENS?........ 101

D. Glušac, N. Ljubojev, D. Radosav
PARENTS' PERCEPTION OF THE NEEDS FOR IMPLEMENTING MEASURES FOR CHILD PROTECTION ON THE INTERNET 105

G. Jauševac, Ž. Stojanov, G. Jotanović
RANKING EDUCATIONAL WEB SITES FOR WEB DESIGN BY USING FUZZY SCREENING METHOD 113

Ž. Jokšić, D. Tadić, E. Brtka, M. Pardanjac, S. Vranješ
TRAINING OF MEDICAL STAFF USING THE INTERNET SERVICE 117

J. Kovacević, I. Marković, M. Tot
MIND MAP AS A TEACHING TOOL 120

A. Krstev, B. Krstev
APPLICATION OF MATLAB REDEFINED AND MODIFIED APPLICATIONS TO SOLVE PROBLEMS OF INTERACTION OF CHEMICAL ELEMENTS AND THEIR IMPACT ON THE ENVIRONMENT 124

M. Lutovac, Dž. Veljagić, Z. Jovanović
INFLUENCE OF INTERNET ON TOURISM INDUSTRY 128

S. Morača, V. Milanko, M. Pardanjac
VALUE NETWORKS AND DUAL EDUCATION PROCESS 135

J. Ollé, Ž. Namestovski
INSTRUCTIONAL DESIGN IN AN ONLINE ENVIRONMENT 141

T.A. Pacemska, B. J. Tuneska, G. Makrides, L.K. Lazarova, M. Miteva, S. Pachemska, Z. Trifunov
INCREASING MOTIVATION FOR LEARNING MATHEMATICS THROUGH DEBATE 143

S. Plachkov, E. Tosheva
APPLICATION CLOUD-BASED SERVICES IN TECHNICAL EDUCATION 148

V. Premčevski
NEAR FIELD COMMUNICATION, QR CODE AND BARCODE SCANNER COMMUNICATION WITH WINDOWS FORM APLICATION 151

D. Serafimovski, A. Krstev, Z. Zdravev
CUSTOMIZABLE MOBILE COMPONENTS AS SUPPORTING TOOLS FOR BLENDED LEARNING 155

B. Sobota, Š. Korečko, P. Pastornický, L. Jacho
A VIEW TO THE IMPACT OF SOME VR-TECHNOLOGIES IN AN EDUCATION OF DISABLED PEOPLE 159

Cs. Szabó, J. Saraiva
FOCUSING SOFTWARE ENGINEERING EDUCATION ON GREEN APPLICATION DEVELOPMENT 165

D. Tadić, Ž. Jokšić, M. Pardanjac, D. Milanov, M. Ćoćkalo-Hronjec
USE OF MODERN COMMUNICATION IN SCHOOLS IN SERBIA 170

M. Votipka, I. Sekulić, B. Vukmanović, B. Santo, D. Karuović
ADVANCED ADS WEBSITE 174

Z. Zeljković, J. Stojanov
AN APPROACH FOR INCREASING PUPIL'S MOTIVATION - MATHEMATICAL TOURNAMENT 178

A. Felbab, M. Pardanjac, E. Tobolka, E. Eleven
MOTIVATION AS AN INCENTIVE FOR STUDENTS TO ACHIEVE BETTER RESULTS WITH THE USE OF MODERN TECHNOLOGY 182

Z. Trifunov, T.A.Pacemska, L. K. Lazarova, M. Miteva, G. Makrides, P. Trifunov, H. Leova, M. Lazarevska
A METHOD FOR INCREASING THE LEVEL OF KNOWLEDGE IN MATHEMATICS 186

A. Krstev, D. Serafimovski, B. Krstev, A. Donev
BIG DATA ANALYSIS AND APPLICATION OF EXPERIMENTAL RESEARCH IN THE HIGHER EDUCATION PROCESS 191

D. Cvetković, B. Medić, M. Mijatović, M. Mijatović
BLOGS IN AUTONOMOUNS LEARNING ENCOURAGING REFLECTIVE PRACTICES 196

M. Bursać, R. Vulović, M. Milosavljević
COMPARATIVE ANALYSIS OF THE OPEN SOURCE TOOLS INTENDED FOR DATA ENCRYPTION 202

S. Čolić, M. Kavalić
NEGATIVE ASPECTS OF USING IT TECHNOLOGIES IN SERBIAN EDUCATION SYSTEM 207

B. Zlatanovska, M. Kocaleva, A. Stojanova, N. Stojkovikj, E. Deleva
EXAMPLES OF FOLD BIFURCATION IN A ONE-DIMENSIONAL SYSTEMS .. 211

T. Novaković, J. Bondžić , S. Popov, Đ. Ćosić
FLOOD SIMULATION FOR EXPECTED DAMAGE CALCULATION................215

V. Tešin, J. Bushati
ACTOR SYSTEMS IN ONLINE GAMING..220

I. Tasić, D. Glušac, M. Ćoćkalo-Hronjec, J. Jankov
THE IMPACT OF PROFESSIONAL DEVELOPMENT OF TEACHERS OF TECHNICAL AND INFORMATION EDUCATION ON THE QUALITY OF TEACHING..223

M. Kocaleva, A. Stojanova, N. Koceska
REVIEW: USING PHYSIOLOGICAL PARAMETERS FOR EVALUATING USER EXPERIENCE..228

G. Jotanovic, V. Brtka, D. Mandic, G. Jausevac, V. Ognjenovic
IMPROVEMENT OF THE ASSESSMENT LEVEL OF STUDENT'S ICT COMPETENCIES USING TRIANGULAR MEMBERSHIP FUNCTION................233

Lj. Kazi, Z. Kazi, D. Karuović, T. Lojović, O. Latinović Stevanov, D. Vidović
PRESCHOOL CHILDRENS' EXPERIENCES WITH COMPUTERS: A CASE STUDY...237

M. Knezevic, N. Bobinac, D. Vukic, E. Tobolka
INFORMATION TECHNOLOGY IN THE EDUCATION SYSTEM.......................242

M. Bakator, S. Borić, D. Radosav, G. Radić
CLOUD COMPUTING MODELS FOR E-LEARNING 248

M. Bakator, S. Borić, D. Radosav, G. Radić
INNOVATIVE MODEL OF DUAL EDUCATION THROUGH STUDENT-ORGANIZATION NETWORKS...253

F. Tsvetanov, S. Sokolov, G. Hristov, I. Georgieva
INFORMATION EXCHANGE MODELS FOR INTERNET OF THINGS TECHNOLOGY 258

M. Sladić, S. Bajić
USING PHOTOGRAMMETRY AND LASER SCANNING FOR MONITORING OF ARCHITECTURAL HERITAGE..264

N. Stojkovikj, A. Stojanova, M. Kocaleva, B. Zlatanovska, E. Karamazova
SIMULATION OF QUEUING SYSTEM BASED ON ANYLOGIC 268

S. Stankovic
IMPORTANCE OF CRITICAL THINKING IN THE AGE OF INTERNET 274

T. A. Pachemska, R. Timovski, M. Lapevski, A. Rushiti
TWO-LAYER QUALITY ANALYSIS OF UNIVERSITY STUDY PROGRAM SUBJECTS USING DEA AND AHP 277

D. Nikić, D. Radosav, D. Karuović, D. Glušac
C# ENCIRCLEMENT DEVELOPMENT IN SECONDARY SCHOOL LEVEL EDUCATION 284

M. Kovač, N. Siker, D. Karuović
CREATING GAMES USING JAVA AND PYTHON 287

Examples of fold bifurcation in a one-dimensional systems

B. Zlatanovska*, M. Kocaleva*, A. Stojanova*, N. Stojkovikj* and E. Deleva*
* Faculty of computer science, Goce Delcev University, Stip, Macedonia
{biljana.zlatanovska, mirjana.kocaleva, aleksandra.stojanova, natasa.maksimova, eleonora.210117}@ugd.edu.mk

Abstract – In this paper will be analyzed a fold bifurcation (or saddle-node bifurcation) in one-dimensional systems, which depend of one parameter r. The analysis will be made via two examples. The examples will analyze the graphical presentations and bifurcation diagram. Finally, we will show that the models of the examples can be represented in one of the normal forms for a fold bifurcation.

I. INTRODUCTION

The fold bifurcations (or saddle-node bifurcation) in a one-dimensional systems $\dot{x} = f(x,r), x \in R,$ which depend of a parameter $r \in R$ is discussed in mathematical literature as [1], [2], [3], [4], [5], [6], [7], [8].

In the dynamics of the one-dimensional systems $\dot{x} = f(x,r), x \in R, r \in R$ interesting is the behavior on the system in dependence of the parameter $r \in R$. The fixed points in these systems can exist or can disappear and their stability can change. These qualitative changes on the dynamics of the system are called bifurcations and the values for the parameter $r=r_C$ for which comes to these changes are called a bifurcation point. In other words, for $r=r_C$ occurs a bifurcation, if the qualitative changes of the system are different for $r>r_C$ and $r<r_C$. Then the parameter r is called a bifurcation (control) parameter. Usually for presenting a bifurcation, we use a bifurcation diagram, which is the graphical presentation on the dependence of x from r. In other side, for marking the vector field on the line for the differential equations $\dot{x} = f(x,r)$ is the graphical presentation on the dependence of $\dot{x}$ from x where for $\dot{x} > 0$ the arrows of vector field are targeted to the right and for $\dot{x} < 0$ the arrows of vector field are targeted to the left.

In this paper, we will consider two examples of a fold bifurcation that belong to group on the local bifurcations where the change of stability on fixed point is limited to small regions around her in the phase space. Because, a fold bifurcation is defined as a local bifurcation where the two fixed points in continuous dynamical system collide and are destroyed (disappear).

The first example is similar with an example in paper [1] on page 35. The second example is unsolved exercise with the title "Fold bifurcation in ecology" in paper [2] on page 104.

The normal forms of the fold bifurcation for a differential equation that depend on one parameter are $\dot{x}=r+x^2$ (*i*) and $\dot{x}=r-x^2$ (*ii*) where r is control parameter, [1], [2], [3], [7]. In (i) for $r < 0$ and in (ii) for $r > 0$ there are two fixed points where one is stable, but another is unstable. In (i) and (ii) for $r = r_c = 0$ the two fixed points collide and appears a fixed point that is semi-stable fixed point. In (i) for $r > 0$ and in (ii) for $r < 0$, there are not fixed points (they disappeared) and the system is unstable. A bifurcation point appears for $r_c = 0$ because in her appears change in behavior of the system where the vector fields for $r < 0$ and $r > 0$ are different.

Close to the fold bifurcation a differential equation which describes the dynamics for one-dimensional system can be reduced to a normal forms (i) or (ii) by [1],[2].

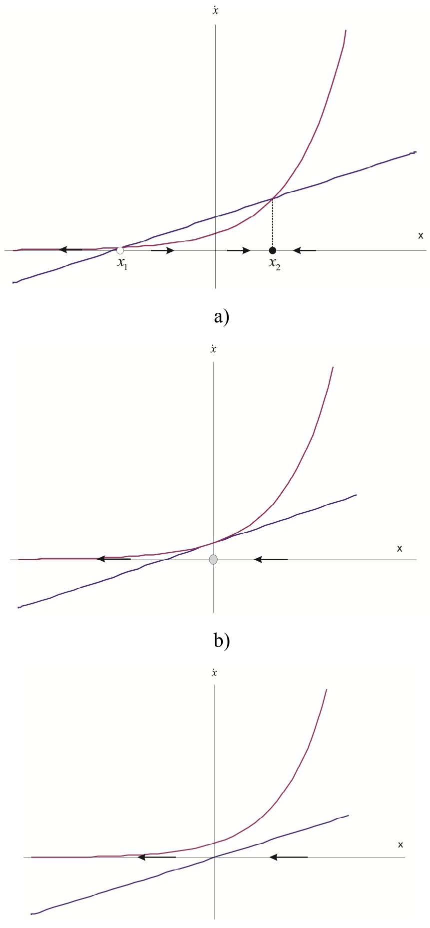
In graphical presentation on the paper, the stable fixed point will be marked with a black point, the unstable fixed point with white point and semi-stable fixed point with the grey point.

II. SIMPLE EXAMPLE FOR A FOLD BIFURCATION

One simple example for a fold bifurcation on a differential equation depending on one control parameter $r \in R$ is

$$\dot{x} = r + x - e^x, x \in R \qquad (1)$$

In this example, the fixed points of the system will be found with the geometric approach, because the differential equation $\dot{x} = 0$ does not have the explicit solution for $r \in R$. An intersection of the functions $r+x$ and e^x are the fixed points for the system (1), where exist a three cases: intersection in two points, intersection in one point and the functions do not have an intersection. An intersection of the functions is presented in Figure 1 with marked vector field. The arrows of vector field are targeted to the right for $r+x>e^x (\dot{x}>0)$ and the arrows of vector field are targeted to the left for $r+x<e^x (\dot{x}<0)$. The two fixed points of the system (1) where x_1 is unstable fixed point and x_2 is stable fixed point are shown also in Figure 1, a). The two fixed points of the system (1) collide and appears one fixed point (semi-stable) $x=0$ which is shown in Figure 1, b). The system does not have a fixed points and it is unstable, Figure 1, c).



a)

b)

c)

Figure 1. Fold bifurcation for (1)

The fixed point $x=0$ on the Figure 1, b) is a bifurcation point. This bifurcation point is found analytical with an equation

$$r+x=e^x \Leftrightarrow \frac{d(r+x)}{dx}=\frac{d(e^x)}{dx}$$

$$\Leftrightarrow 1=e^x \Leftrightarrow x=0$$

For $x=x^*=0$ and $\dot{x}=0$ in the equation (1), we obtained the value for the control parameter $r=r_c=1$. This value for the control parameter $r=r_c$ is a fold bifurcation for the system (1), because the vectors fields for $r<1$ and $r>1$ are different. This can be seen in Figure 1.

To show that the system (1) can be represented in one of the normal forms ((i) or (ii)) given in the introduction we used the development of the function $e^x = 1+x+\frac{x^2}{2!}+o(x^3)$ in Taylor series (to the third order) in neighborhood of the bifurcation point $x^* = 0$. With using (1), we obtain

$$\dot{x}=r+x-(1+x+\frac{x^2}{2!}+o(x^3))$$

$$=(r-1)-\frac{x^2}{2}+o(x^3)$$

Really, the differential equation $\dot{x}=(r-1)-\frac{x^2}{2}+o(x^3)$ corresponds with the normal forms mentioned in the introduction.

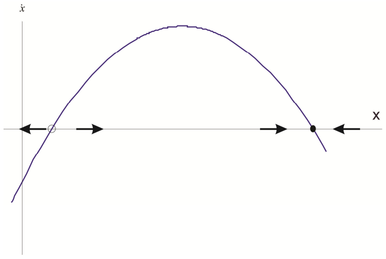
III. A FOLD BIFURCATION IN ECOLOGY

A simple ecological model (as a model for single population) is given with following differential equation

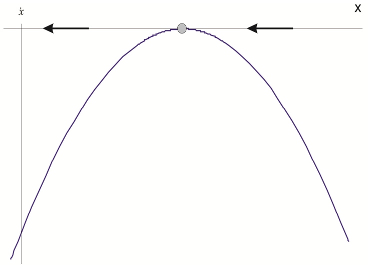
$$\dot{x} = a\,x\,(1-\frac{x}{K})-r \qquad (2)$$

where x(t)>0 is the population number in the time *t*. a is the intrinsic growth rate of the population, *K* is the carrying capacity of the population and *r* is the harvest rate as control parameter. We will focus only on the mathematical aspect.

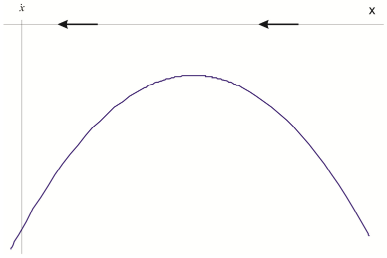
The fixed points are obtained of the equation $\dot{x} = a\,x\,(1-\frac{x}{K})-r=0$. There are three cases: two fixed points, one fixed point and no fixed points. In Figure 2 is marked the fixed point (if there is some) and vector field. The two fixed points of the system (2) where $x_1=\frac{aK-\sqrt{a^2K^2-4aKr}}{2a}$ is unstable fixed point and $x_2=\frac{aK+\sqrt{a^2K^2-4aKr}}{2a}$ is stable fixed point are shown in Figure 2, a). The two fixed points of the system (2) collide and appears one fixed point (semi - stable) $x = \frac{K}{2}$ is shown in Figure 2, b). The system does not have a fixed points and it is unstable, Figure 2, c).



a) for $r < r_c$



b) for $r = r_c$

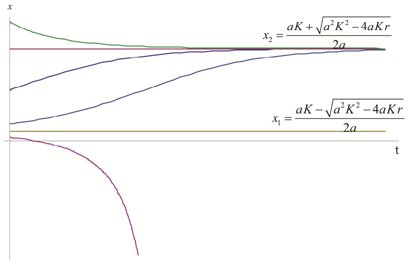


c) for $r > r_c$

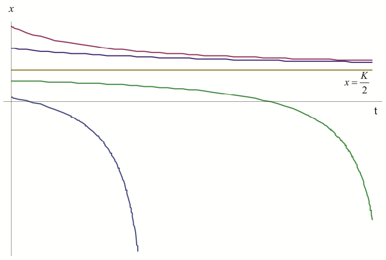
Figure 2. Fold bifurcation for the model (2)

The fixed point $x = \frac{K}{2}$ in Figure 2, b) is a bifurcation point, which will be marked with $x = x^* = \frac{K}{2}$. For $x^* = \frac{K}{2}$ and $\dot{x} = 0$ in the model (2), we obtain the value for the control parameter $r = r_c = \frac{aK}{4}$. This value is a fold bifurcation for the system (2), because the vector field for $r < \frac{aK}{4}$ and $r > \frac{aK}{4}$ are different. This can be seen in Figure 2.

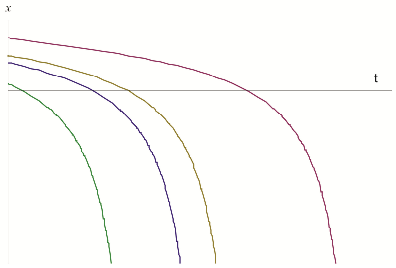
The dependence of x (t) for different initial values for the differential equation (2) is shown in Figure 3. In Figure 3, a) an unstable fixed point x_1 and a stable fixed point x_2 are shown. In Figure 3, b) a semi-stable fixed point (the bifurcation point) $x^* = \frac{K}{2}$ for the values of the control parameter $r_c = \frac{aK}{4}$ with the stable points $x > \frac{K}{2}$ and unstable points $x < \frac{K}{2}$ are shown. Moreover, in Figure 3, c) instability on (2) is shown.



a) for $r < r_c$



b) for $r = r_c$



c) for $r > r_c$

Figure 3. The integral curves ***for*** the model (2)

In Figure 4 are shows a bifurcation diagram for the system (2), which presents depends of x from the control parameter *r*, *x(r)*. For $x > \frac{K}{2}$, the system (2) is stable, for $x < \frac{K}{2}$, the system (2) is unstable and for $x^* = \frac{K}{2}$, x* is a bifurcation point for the values of the control parameter $r_c = \frac{aK}{4}$.

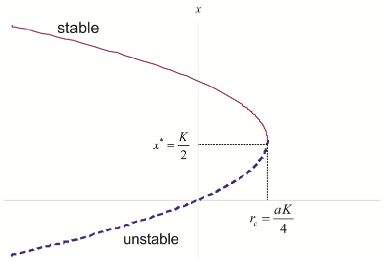


Figure 4. Bifurcation diagram of fold bifurcation for the model (2)

Finally, we come to the following conclusion.

1. When the function has two fixed points then the value of the control parameter is $r < r_c$: If the population $x(t)$ is close to the unstable fixed point x_1 of left then the population $x(t)$ is dying out. If $x(t)$ is close to the unstable fixed point x_1 of right then the population $x(t)$ grows and it is approaching to the stable fixed point x_2. If $x(t)$ is close to the stable fixed point x_2 of right then the population $x(t)$ is reducing and it is approaching to the stable fixed point x_2.
2. When the function has one fixed point then the value of the control parameter is $r = r_c$: If $x(t)$ is close to the semi-stable fixed point $x^* = \frac{K}{2}$ of left then the population $x(t)$ is dying out. If $x(t)$ is close to the semi-stable fixed point $x^* = \frac{K}{2}$ of right then the population $x(t)$ is reducing and it is approaching to the stable fixed point $x^* = \frac{K}{2}$. The point $x^* = \frac{K}{2}$ has maximum value for the population, which depends of the K - the carrying capacity.
3. When the function does not have fixed point then the value of the control parameter is $r > r_c$: The system (2) is instable and the population is dying out.

To show that the system (2) can be represented in one of the normal forms ((i) or (ii)) given in the introduction we used the function $f(x,r) = a\,x(1-\frac{x}{K}) - r$ as a function of two variables x and r. In order to analyze the dynamics of the system (2) in neighborhood of the bifurcation point $x^* = \frac{K}{2}$ and when $r_c = \frac{aK}{4}$, the function $f(x,r)$ is developed in Taylor series (to the third order)

$$\dot{x} = f(x,r) = f(x^*,r_c) + (x-x^*)\frac{\partial f}{\partial x}|_{(x^*,r_c)}$$

$$+(r-r_c)\frac{\partial f}{\partial r}|_{(x^*,r_c)} + \frac{1}{2}(x-x^*)^2\frac{\partial^2 f}{\partial x^2}|_{(x^*,r_c)} + ...$$

where the square member $(r-r_c)$ is discarded. The members $f(x^*,r_c)$ and $\frac{\partial f}{\partial x}|_{(x^*,r_c)}$ are zeros, because the point $x^* = \frac{K}{2}$ is e fixed point and because tangential condition of fold bifurcation, respectively. Really,

$$f(x^*,r_c) = a x^*(1-\frac{x^*}{K}) - r_c = 0$$

and $\frac{\partial f}{\partial x}|_{(x^*,r_c)} = a - \frac{2ax^*}{K} = 0$. The other expressions are $\frac{\partial^2 f}{\partial x^2}|_{(x^*,r_c)} = -\frac{2a}{K}$ and $\frac{\partial f}{\partial r}|_{(x^*,r_c)} = -1$ and we obtain

$$\dot{x} = (r-r_c)\frac{\partial f}{\partial r}|_{(x^*,r_c)} + \frac{1}{2}(x-x^*)^2\frac{\partial^2 f}{\partial x^2}|_{(x^*,r_c)} + ...$$

$$= -(r-\frac{aK}{4}) - \frac{a}{K}(x-\frac{K}{2})^2 + ...$$

The differential equation

$$\dot{x} = -(r-\frac{aK}{4}) - \frac{a}{K}(x-\frac{K}{2})^2 + ...$$

correspond with the normal forms mentioned in the introduction.

REFERENCES

[1] D. Kuzmanovic, N.Vasovic, S. Kostic, S. Simic, I. Franovic, I. Grozdanovic, K. Todorovic-Vasovic, B. Rankovic-Plazinovic (2013) Uvod u teoriju haosa, pp.21-54, 485-486, ISBN 978-86-7395-317-5, Univerzitet u Beogradu

[2] Y. A. Kuznetsov (1998) Elements of Applied Bifurcation Theory, Second Edition, Springer, USA, ISBN 0-387-98382-1, pp.79-111

[3] K.T.Alligood, T.D.Sauer, J.A.Yorke (2000) Chaos: An Introduction to Dynamical Systems, pp.448-455, Springer, USA

[4] C.Chicone (1999) Ordinary Differential Equations with Applications, pp.483-487, Springer, USA

[5] M.W.Hirsch, S.Smale, R.L.Devaney (2004) Differential Equations, Dynamical Systems and An Introduction to Chaos, Second Edition, pp.176-182, Elsevier, USA

[6] Shui-Nee Chow, C.Li, D.Wang (2007) Normal Forms and Bifurcation of Planer Vector Fields, pp.177-213, Cambridge University (2007)

[7] S.Wiggins (2003) Inroduction to Applied Nonlinear Dynamical Systems and Chaos, Second Edition,pp.363-366, Springer

[8] V. I. Arnold (1992) Ordinary Differential Equations, Translated from the Russian by Roger Cooke Springer, USA, pp. 13-35