V Congress
of Mathematicians of Macedonia

September 24-27, 2014, Ohrid, R. Macedonia

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UNION OF MATHEMATICIANS
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Edited by:
Aleksa Malčeski
Slagjana Brsakoska
Risto Malčeski
Vesna Celakoska-Jordanova

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VISUALIZATION OF DISCRETE RANDOM VARIABLES

Zoran Trifunov[1], Elena Karamazova[2]

Abstract. In this paper, by use of information technology and existing definitions, a discrete random variable will be introduced, with emphasis on variables modeling probability situations with only two outcomes. The event can be repeated finite or infinite number of times. By analyses of examples, the Bernoulli, binomial and geometric random variables will be applied. Examples of discrete random variable with a geometric distribution will be given, which can be represented visually by using GeoGebra. The given problem will be visually presented by an applet developed in GeoGebra that will make the visual representation of the problem, i.e. the areas of definition and the favorable events easier. Then it will be solved mathematically.

1. INTRODUCTION

There are two types of random variables: random variables whose set of values is discrete (discontinuous or infinite) which for any $x_i \in R_X$, $P\{X = x_i\} \neq 0$, and random variables whose values set is an interval of real numbers, where $P\{X = x\} = 0, \forall x \in \mathbb{R}$.

The random variable X is of discrete type, if there is a discrete set $R_X \subseteq \mathbb{R}$ such that $P\{X \in R_X\} = 1$.

Example 1. Number of defective products under control of production, number of traffic accidents at a certain time, etc.

Random variables of discrete type usually are given with set of possible values $R_X = \{x_1, x_2, \ldots\}$ and sequence of real numbers $p_1, p_2, \ldots,\ 0 < p_i < 1, \sum_i p_i = 1,$ such that $P\{X = x_i\} = p_i$. Sets R_X and sequence numbers $p_i, i = 1, 2, \ldots,$ determine the law of probability distribution of the random variable X of the form $X : (\begin{smallmatrix} x_1 & x_2 & \cdots & x_n \\ p_1 & p_3 & \cdots & p_n \end{smallmatrix}),$ $\sum_{i=1}^{n} p_i = 1$ where $x_1, x_2, \ldots, x_n$ might be values of the random variable X and $p_1, p_2, \ldots, p_n$ probability that random variable X has for $x_1, x_2, \ldots, x_n$ accordingly.

Generally, the distribution function of a random variable X with the set values $R_X = \{x_1, x_2, \ldots\}$ is determined in the following manner

$$F_X(x) = \sum_{i: x_i < x} P\{X = x_i\},\ x \in R_X,$$

Key words and phrases. discrete random variable, the law of distribution, the cumulative distribution function

where counting is done by all i whose values x_i are less than the fixed x. If X receives n different values arranged by size $x_1, x_2, \ldots, x_n$ then

$$F_X(x) = P\{X < x\} = \sum_{x_i < x} p_i ,$$

i.e.

$$F_X(x) = \begin{cases} 0, & x < x_1, \\ p_1, & x_1 \le x < x_2, \\ p_1 + p_2, & x_2 \le x < x_3, \\ p_1 + \ldots + p_{n-1}, & x_{n-1} \le x < x_n, \\ 1, & x \ge x_n. \end{cases}$$

2. VISUAL REPRESENTATION OF BINOMIAL, BERNOULLI AND GEOMETRIC DISTRIBUTION IN GEOGEBRA

Random variable X has a binomial distribution with parameters p and n, if the set of values X, $R_X = \{0, 1, 2, \ldots, n\}$ and

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad k \in R_X. \tag{1}$$

Write $X \sim B(n, p)$ where n is the number of independent experiments, and probability p to a realization of an event. Example of visually presented random variable with binomial distribution for $p = 0,47, n = 90,$ using the software GeoGebra have in figure1.

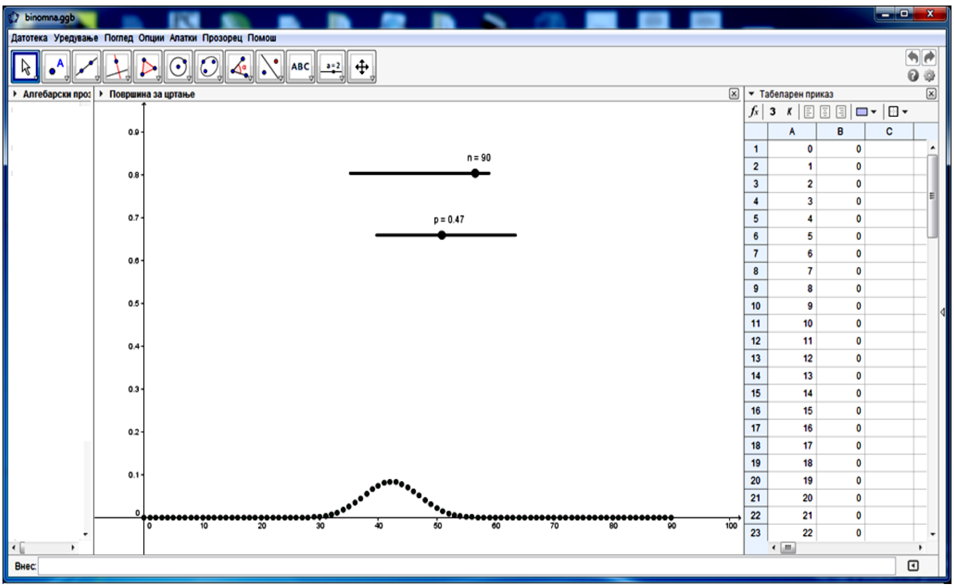

Figure 1

For $n=1$ the distribution $B(1, p)$ is called Bernoulli distribution with a probability of realization of an event p and $1-p$ if the event is not realized. Example of visually presented Bernoulli random variable for $p=0,4$ in software GeoGebra in figure 2:

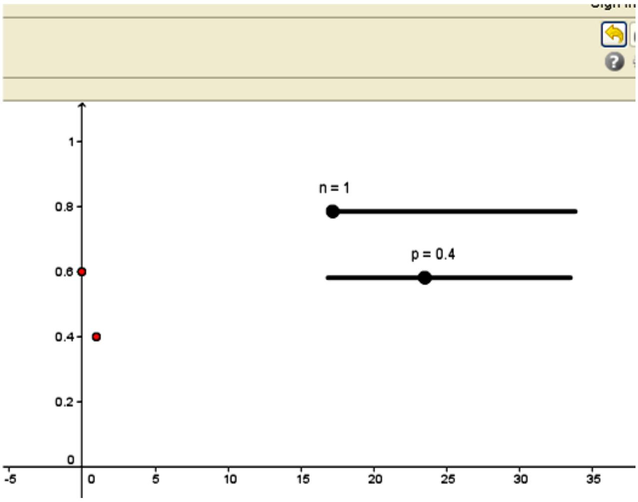

Figure 2

Random variable X has a geometric distribution with parameter p if

$$P\{X=k\}=(1-p)^{k-1}p, \quad k=1,2... \quad . \qquad (2)$$

An example of a random variable with geometric distribution for $p=0,3,$ in software GeoGebra presented in figure 3:

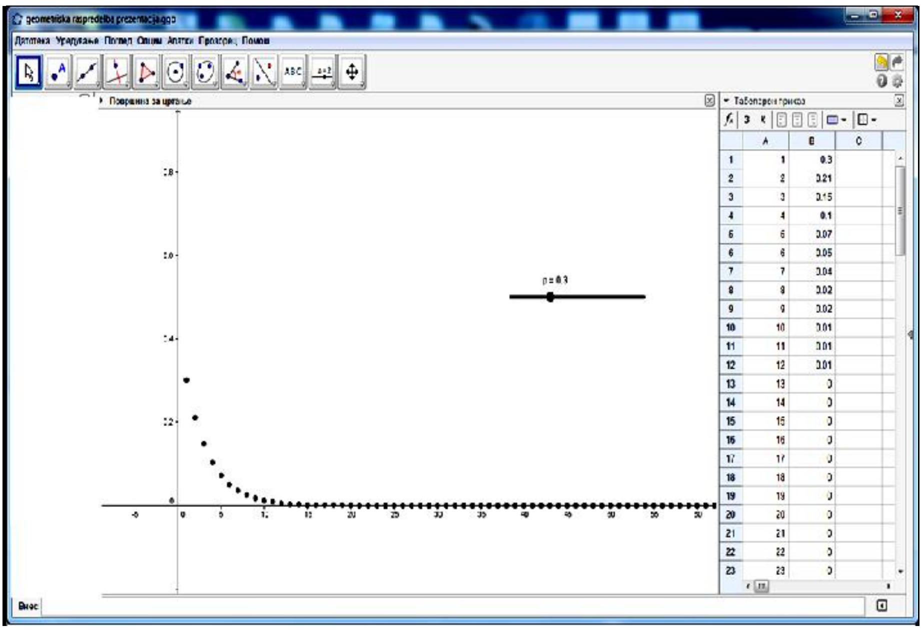

Figure 3

Figure 4 presents an applet for visualizing geometric and binomial distribution made in GeoGebra by which we can see how the change of n and p affect distributions. We use two spaces for drawing and two sliders. The first window represents a random variable with binomial distribution, and the second window a random variable with geometric distribution. One of the slides denoted by p, has a value of 0 to 1, step 0.01 which is the probability that the event A will occur. The second slider denoted by n has a value of 0 to 100, step 1 which is a number of repetitions of favorable events in 100 repetitions of the experiment. The slides are connected to the two windows and two graphs of distributions. By changing the value of p we visually represent the change of distributions and their graphical representation.

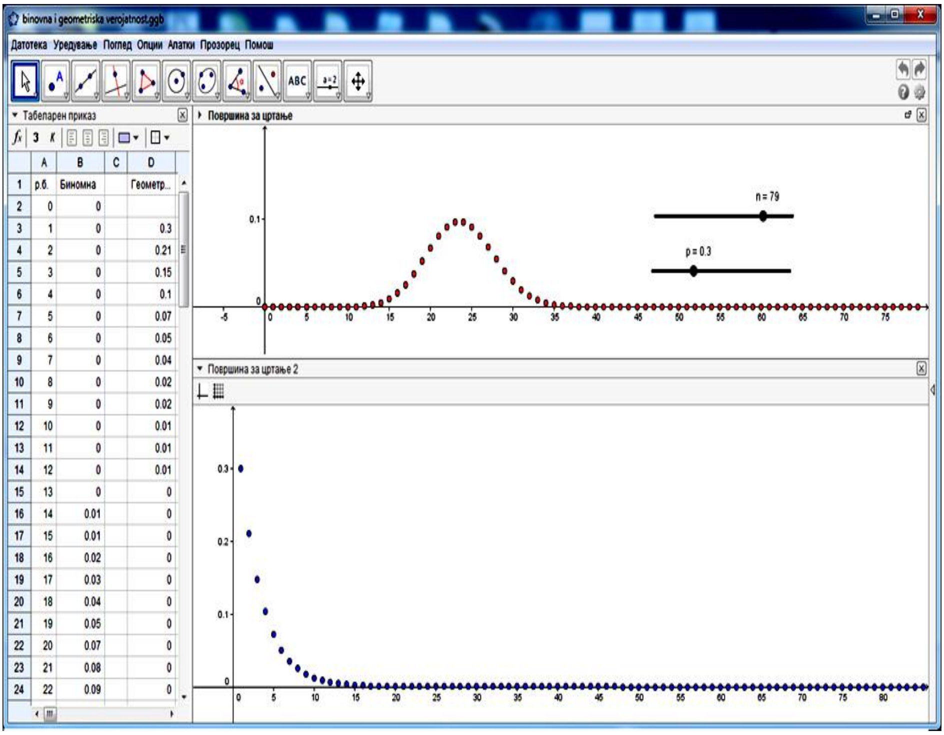

Figure 4

The following will give examples of discrete random variables of the same experiment that vary according to the number of repetitions of the experiment. Let the probability to hit the target in the shooting of a shooter be 0.7. Let

a) Y : be the number of target hits in one shooting

b) X : be the number of target hits, in 5 shootings,

c) Z : be the number of the shooting when the target is hit, if the shootings continue until the target is hit.

We will determine the type of the random variables, their laws of probability distributions and the cumulative distribution function. Using the previous applet we could visually present the law of distribution for the random variable. Let A be an event target hit.

$$p = P(A) = 0,7,\ P(\overline{A}) = 1 - P(A) = 1 - 0,7 = 0,3.$$

Y : be the number of target hits in one shooting. The random variable Y has a Bernoulli distribution.

$$R_Y = \{0,1\},\ P\{Y = 0\} = P(\overline{A}) = 0,3,\ P\{Y = 1\} = \mathrm{P}(A) = 0,7\,.$$

So the law of probability distributions PDF of the random variable Y is given in figure 5.

Y	P(Y)
0	0,3
1	0,7

Figure 5

The law of probability distributions PDF of the random variable Y presented in GeoGebra is given in figure 6.

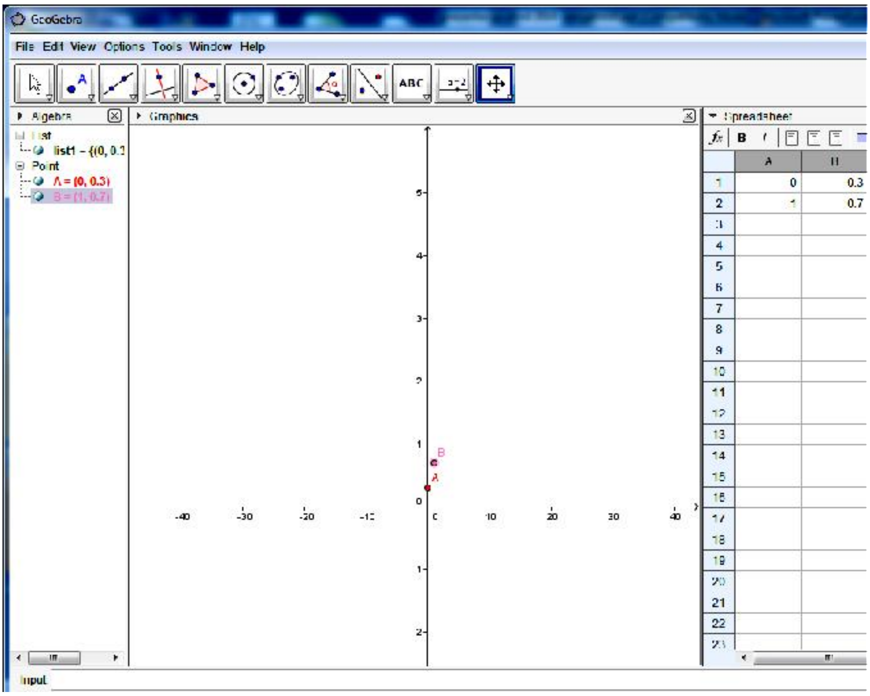

Figure 6

For the cumulative distribution function CDF of the random variable Y we have:

$$F(y)=\begin{cases}0, & y<0,\\ 0,3, & 0,3\le y<1,\\ 1, & y\ge 1.\end{cases}$$

The cumulative distribution function CDF of the random variable Y presented in GeoGebra is shown in figure 7:

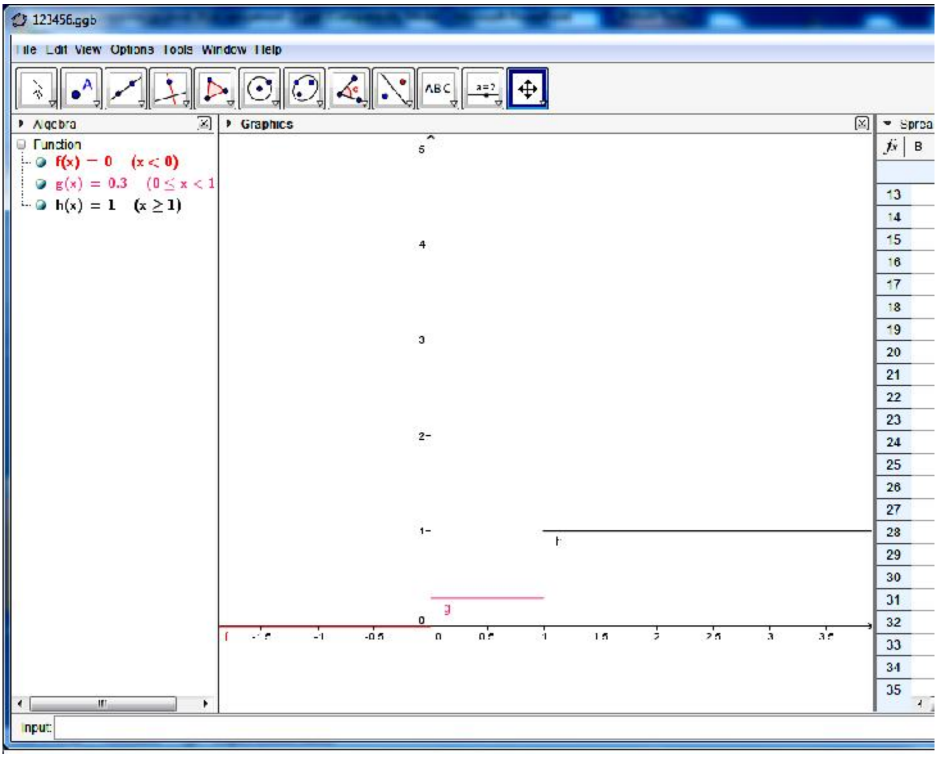

Figure 7

b) X is the random varible the number of target hits, in 5 shootings. The random variable X has a binomial distribution.

$$R_X=\{0,1,2,3,4,5\},$$

$$P\{X=0\}=\binom{5}{0}0,7^0\cdot 0,3^5=1\cdot 0,3^5=0,3^5=0,00243$$

$$P\{X=1\}=\binom{5}{1}0,7^1\cdot 0,3^4=5\cdot 0,7\cdot 0,3^4=0,02835$$

$$P\{X=2\}=\binom{5}{2}0,7^2\cdot 0,3^3=10\cdot 0,49\cdot 0,027=0,1323$$

$$P\{X=3\}=\binom{5}{3}0,7^3\cdot 0,3^2=10\cdot 0,343\cdot 0,09=0,3087$$

$$P\{X=4\}=\binom{5}{4}0,7^4\cdot 0,3^1=5\cdot 0,2401\cdot 0,3=0,36015$$

$$P\{X=5\}=\binom{5}{5}0,7^5\cdot 0,3^0=1\cdot 0,16807\cdot 1=0,16807$$

Therefore, the law of probability distributions PDF of the random variable X is given in figure 8.

X	$P(X)$
0	0,00243
1	0,02835
2	0,1323
3	0,3087
4	0,36015
5	0,16807

Figure 8

The law of probability distributions PDF of the random variable X presented in GeoGebra is given in figure 9.

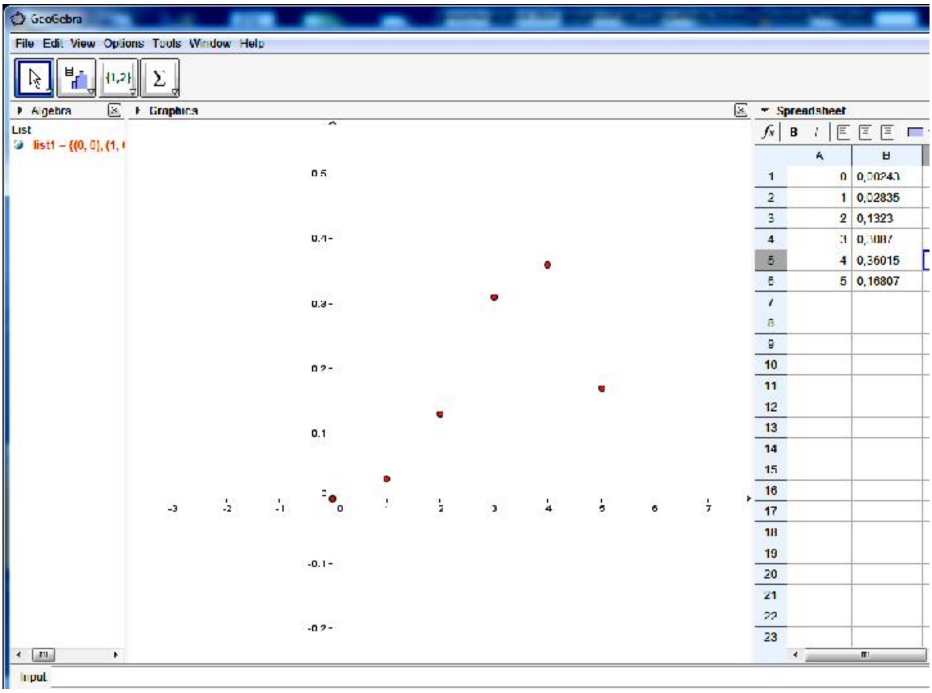

Figure 9

The cumulative distribution function CDF of the random variable X is:

$$F(x)=\begin{cases}0 & x<0\\ 0,00243 & 0\le x<1\\ 0,03078 & 1\le x<2\\ 0,16308 & 2\le x<3.\\ 0,47178 & 3\le x<4\\ 0,83193 & 4\le x<5\\ 1 & 5\le x\end{cases}$$

The cumulative distribution function CDF of the X presented in GeoGebra is given in figure 10.

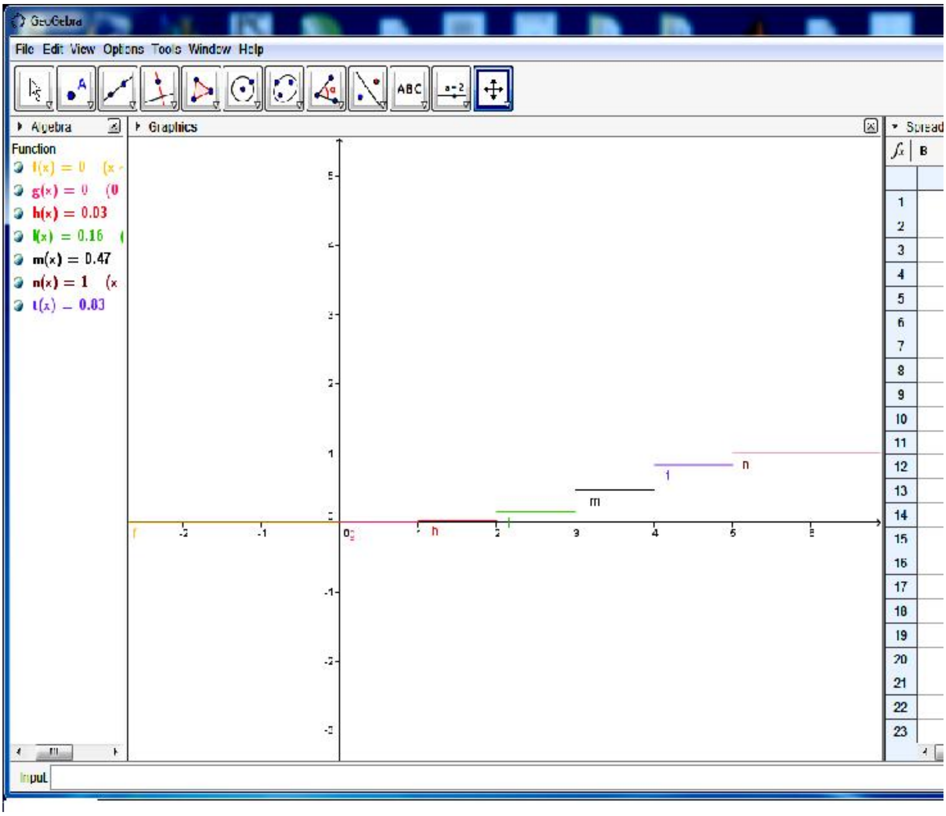

Figure 10

c) Z is the number of the shooting when the target is hit, if the shootings continue until the target is hit. The random variable Z has a geometric distribution. The set of values is

$1-$ target hit at the first attempt

$2-$ target hit at the second attempt

……………………………………

$n-$ target hit at the n th attempt

……………………………………

$P\{Z=1\}=P(A)=0,7$,

$P\{Z=2\}=P(\overline{A}A)=0,3\cdot 0,7=0,21$,

$P\{Z=3\}=P(\overline{A}\overline{A}A)=0,3^2\cdot 0,7=0,063$,

………………………………………….

$P\{Z=n\}=P(\underbrace{\overline{A}\overline{A}...\overline{A}}_{n-1}A)=0,3^{n-1}\cdot 0,7$,

………………………………………..

$$P\{Z=1\}+P\{Z=2\}+...+P\{Z=n\}+...=0,7+0,3\cdot 0,7+0,3^2\cdot 0,7+...+0,3^{n-1}\cdot 0,7+...$$
$$=0,7(1+0,3+0,3^2+...+0,3^{n-1}+...)$$
$$=0,7\cdot\frac{1}{1-0,3}=1.$$

3. CONCLUSION

This paper visually represents discrete random variables with binomial, Bernoulli and geometric distribution through the software GeoGebra. With the visual representation of random variables we want to achieve our goal of students easily learning and recognizing random variables in solving tasks.

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[1] *Goce Delcev University,*
email: zoran.trifunov@ugd.edu.mk
[2] *Goce Delcev University,*
email: elena.gelova@ugd.edu.mk