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OBTAINING FUNCTIONS FROM FOURIER SERIES WITH MATLAB

UDK: 517.5:004.434MATLAB
Original Research

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Abstract - Fourier series represent a very important tool for solving problems in any field of science that uses sinusoidal signals, such as engineering, physics, applied mathematics, and chemistry. So, good understanding of Fourier series is crucial for the process of learning the basics of these scientific fields. The theory of these series is complicated but their application is simple. Matlab as program package is suitable for easily plotting trigonometric series and the most convenient way for understanding their characteristics. In this paper we present a program written in Matlab that plot partial sums of three trigonometric series, as a way of finding periodic functions that series represent. We also give a mathematical proof for obtaining one of periodic functions that corresponds with our graphical representation.

Keywords: Fourier series, Mathlab, periodic functions

I. INTRODUCTION

Mathematics is important to the extent to which it support and contributes to the purposes of general education [1]. Mathematical skills are crucial for a wide array of analytical, technological, scientific, security and economic applications. Mathematics forms the basis of most scientific and industrial research and development. Increasingly, many complex systems and structures in the modern world can only be understood using mathematics and much of the design and control of high-technology systems depends on mathematical inputs and outputs [2].

Mathematics is of central importance to modern society. All its need to be done is to understand the logic hidden behind. Mathematical calculations give way to analyze results of every experiment, and can help to make precise conclusions [3].

Fourier analysis is a way of mathematics analysis that has many applications in different scientific fields like physics, engineering, microwave circuit analysis, control theory, optical measurements and also in chemistry [4] [10].

The study of Fourier series is a branch of Fourier analysis. Fourier series were introduced by Joseph Fourier (1768–1830) for the purpose of solving the heat equation in a metal plate. The Fourier series and methods of numerically approximating it have been active areas of research since then [4] [5] [6].

Fourier series are used to approximate complex functions in many different parts of science and math. They are helpful in their ability to imitate many different types of waves: x-ray, heat, light, and sound. Fourier series are used in many cases to analyze and interpret a function which would otherwise be hard to decode and understand. These series has many applications in electrical engineering, vibration analysis, acoustics, optics, signal processing, image processing, quantum mechanics, and econometrics [5] [10].

In this paper we give a brief introduction to the Fourier series as a base of different scientific fields, we present a way of plotting the series in Matlab which can be helpful for their better understanding and studying. From mathematical point of view this presentation of trigonometric series with Matlab can be helpful for easily obtaining functions from given trigonometric series, which can be more difficult problem than the reversed one i.e. obtaining Fourier series from a given function.

II. FUNDAMENTALS OF FOURIER SERIES

In mathematics, a Fourier series decomposes any periodic function or periodic signal into the sum of a set of simple oscillating functions, namely sines and cosines. In other words, Fourier series can be used to express a function in terms of the frequencies (harmonics) it is composed of [5].

In engineering and technical sciences many practical problems gave periodic functions which are usually complex. We know that the function is periodic if it is defined for each real number x, and if there is a positive number T, called a period of f (x) i.e. $f(x+T) = f(x), \forall x$ and the smallest period T>0 named fundamental period of the function, according to [6] [7].

For simplicity and easier operation by periodic functions, we presented them with elementary periodic functions such as sine and cosine function who have a fundamental period 2π. This presentation of periodic functions like trigonometric series leads to Fourier series. Fourier series are very important practical tool because they allow the solution of ordinary (ODEs) and partial differential equations and they are also more practical than development of functions in Taylor (Maclaurin) series.

The particular conditions that a function *f* (x) must fulfill in order that it may be expanded as a Fourier series are known as the Dirichlet conditions, and can be summarized by the following points:

1. the function must be periodic;
2. it must be single-valued and continuous, except possibly at a finite number of finite discontinuities;
3. it must have only a finite number of maxima and minima within one periodic;
4. the integral over one period of $|f(x)|$ must converge.

Therefore, Fourier series of function $y = f(x)$ is defined as follows: If $y = f(x)$ is periodic and integrable function of the interval $[-\pi, \pi]$, could be expressed as the sum of a trigonometrical series of the above form

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx \qquad (0),$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx \qquad (1),$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx \qquad (2)$$

for each positive integer n.

A trigonometric series of the form

$$a_0 + \sum_{n=1}^{+\infty}(a_n \cos nx + b_n \sin nx) \qquad (3)$$

with coefficients a_0, a_n and b_n given by the integrals (0), (1) and (2) is referred to the Fourier series of the function *f* (x) [6] [7].

That is the way for obtaining Fourier's series from periodic functions *f* (x) of period 2π. The reverse way is more complicated and that is the reason way we used Matlab for plotting trigonometric series of n terms to obtain a periodic functions they derived from. For good understanding of Fourier series is essential to know both ways.

III. Graphical Representation of Fourier Series With Matlab

We are using three partial sums as example to show how a periodic function can be obtained from partial sum when n is big enough to recognize a function. To demonstrate plotting of Fourier series with Matlab we will use these three partial sums:

$$\frac{1}{3}\pi^2 - 4(\cos x - \frac{1}{4}\cos 2x + \frac{1}{9}\cos 3x - \frac{1}{16}\cos 4x + -...)\_(4)$$

$$\frac{4}{\pi}(\sin x + \frac{1}{3}\sin 2x + \frac{1}{5}\sin 5x + \frac{1}{7}\sin 7x + ...)\_\_\_\_\_\_(5)$$

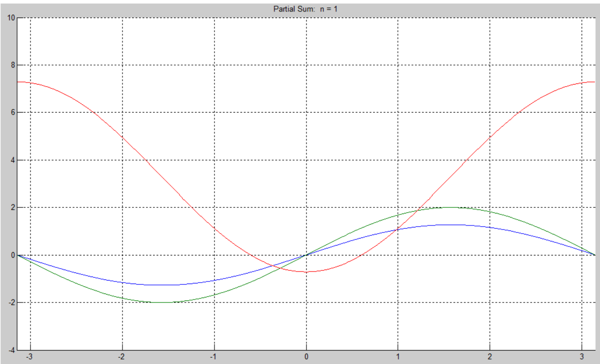
$$2(\sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x - \frac{1}{4}\sin 4x + -.........)\_\_\_\_\_(6)$$

By their graphical presentation we will try to find periodic function that these series represent. First, we will present all three sums together in the same graphic for different n, and then we will plot separately in different graphs in order to obtain a clear view.

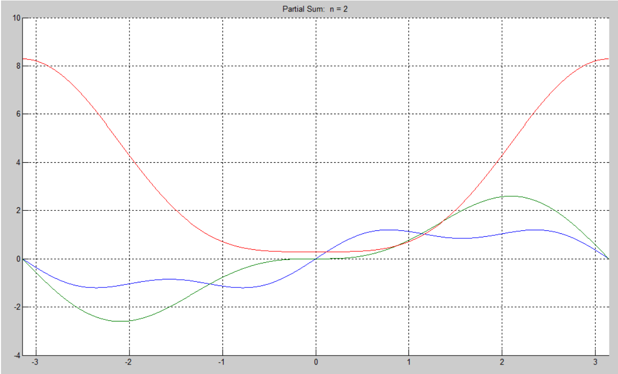
For plotting all three partial sums given in (4), (5) and (6) we are using this simple code written in Matlab.

```
x=linspace(-pi,pi,1000);
partial_sum1=0;
partial_sum2=0;
partial_sum3=1/3*pi.^2;

grid on;hold on;
axis([-pi pi -4 10])

for n=1:1:100

partial_sum1=partial_sum1+(4/((2*n-1)*pi))*sin((2*n-1)*x);
partial_sum2=partial_sum2-2/n*(-1).^n*sin(n*x);     partial_sum3=partial_sum3+(4/n.^2*cos(n*x)*(-1).^n);
    handle1=plot(x,partial_sum1,x,partial_sum2,x,partial_sum3);

   title(['Partial Sum:  n = ',num2str(n)])
   pause
   set(handle1,'Visible','off');

end
```

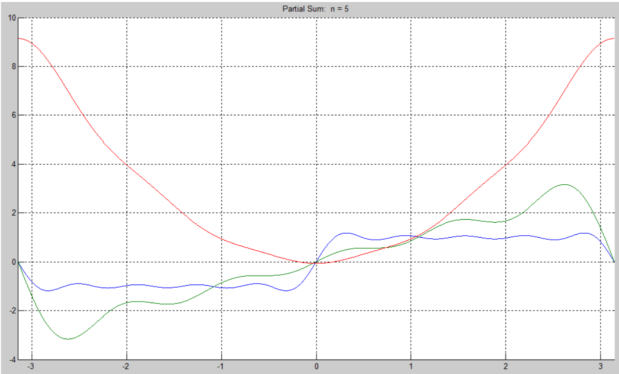
According to given code we first plot partial sums in period from –π to π, and terms n are from 1 to 100. We present partial sums for different terms. Partial sums for terms of 1, 2, 5, 21 and 100 are given in Figure 1, a), b), c), d), e).



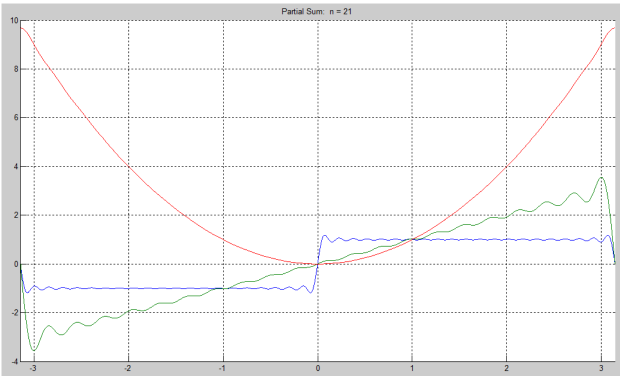
a) n=1



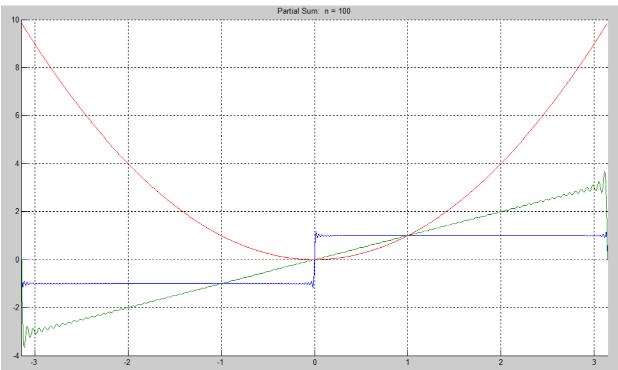
b) n=2



c) n=5



d) n=21



e) n=100

Figure 1. Graphical presentation of trigonometric series given in (4) (5) (6) ((4) red, (5) blue and (6) green curve) for n = 1, 2, 5, 21 and 100, and period from $-\pi$ to π.

From the Figure 1 is clearly seen that the larger the number of terms that are included in this series, the better the approximation to the periodic function is.

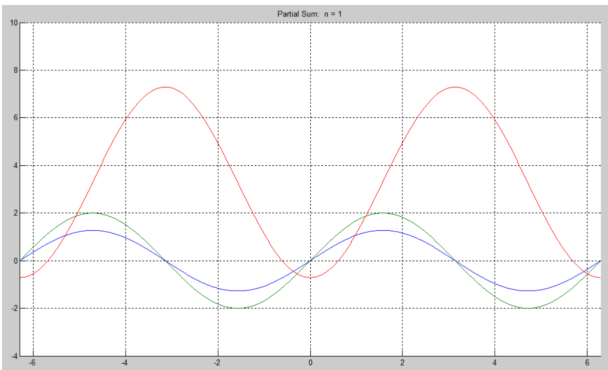
Therefore, when n is large enough we can guess the periodic functions that these trigonometric series present. According to Figure 1 partial sum (4) corresponds to periodic function $f(x) = x^2, x \in [-\pi, \pi]$, partial sum (5) corresponds to periodic function $f(x) = \begin{cases} 1 & x \in [0, \pi] \\ -1 & x \in [-\pi, 0] \end{cases}$, and partial sum (6) corresponds to function $f(x) = x$, $x \in [-\pi, \pi]$.

The 100-term plot gives a very good approximation to the functions, although it appears to overshoot the value at the discontinuity (and it also has some wiggles in it). This overshoot is an inevitable effect at a discontinuity, known as the Gibbs phenomenon [6]. Gibbs phenomenon can be seen in graphical presentation of partial sums (4) and (5) for large terms (in our case, n=21 and n=100).

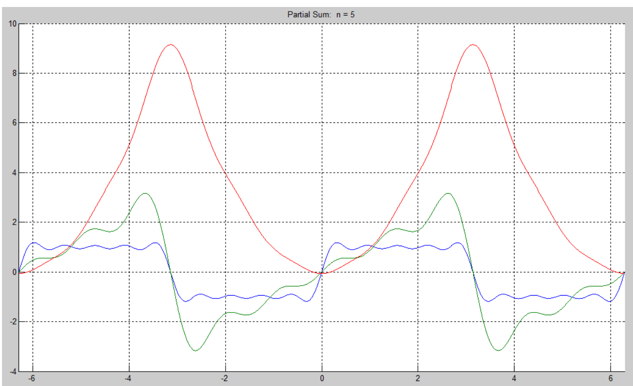
In order to obtain more clear view of trigonometric nature of partial sums and periodic nature of functions, we are presenting trigonometric series in 2T period, or x axis are from -2π to 2π. For this purpose we change only a small piece from the code given above.

```
x=linspace(-pi,pi,1000);
axis([-pi pi -4 10])
is changed to:
x=linspace(-2*pi,2*pi,1000);
axis([-2*pi 2*pi -4 10])
```

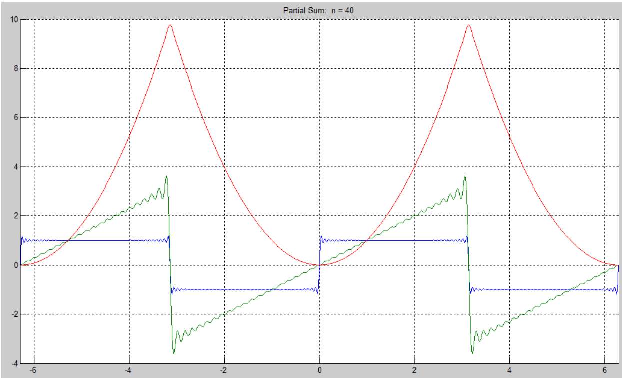
Partial sums according to this change are shown in Figure 2.



a) n=1



b) n=5



c) n=40

Figure 2. Graphical presentation of the partial sums (4) (5) (6) ((4)-red, (5)-blue and (6)-green curve) for 1,2 and 40 terms and x-axes from -2π to 2π.

In Figure 1 and Figure 2 all previous conditions of curves are deleted.

In order to obtain a precise view of partial sum and the way of curve's changing and at the same time keep previous conditions of partial sum we plot each of three partial sums in separate graphic. All previous conditions of curve are shown in different color.

Following each curve represented by different color can be observed the appearance of partial trigonometric sum for each n.

This can be done with a minor change in code by adding different color in each term.

```
colors = 'rgbmy';
```

and instead of

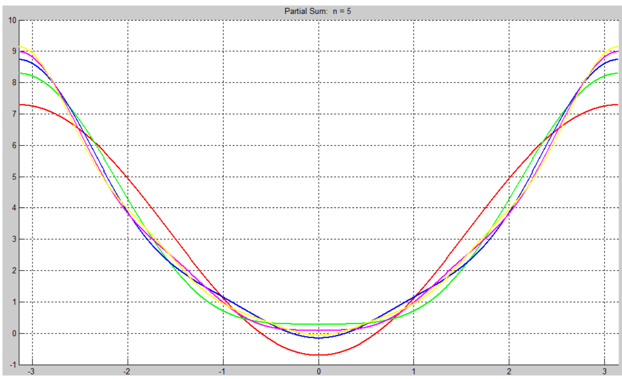
```
pause set(handle1,'Visible','off');,
```

is added:

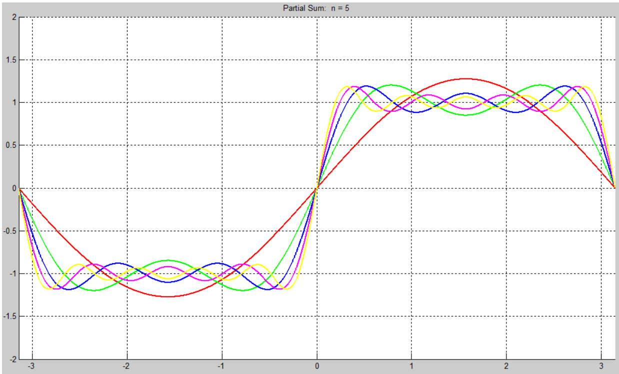
```
pause set (handle1,'Color',colors(n));,
```

where *handle1* is used for plotting partial sum and is different for different trigonometric series (4), (5) and (6) .

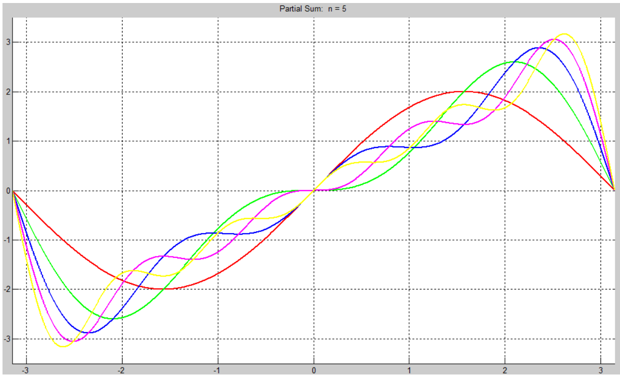
All first five partial sums of trigonometric series (4) (5) (6) are printed in Figure 3 a), b), c) accordingly.



a)



b)



c)

Figure 3. Graphical presentation of first 5 term partial sum a) of (4) b) of (5) c) of (6)
red color for n=1, green for n=2, blue for n=3, violet for n=4, and yellow for n=5.

Figure 3 is useful in process of understanding of trigonometric partial sum and Fourier series.

IV. Mathematical Proof

Using graphical representations for the given Fourier series we managed to obtain periodic functions, but their correctness will be confirmed with mathematical proof. For (6) we are going to obtain periodic function starting from given trigonometric series.

Partial sum (6) can be shown as:

$$f(x) = 2\sum_{n=1}^{+\infty}(-1)^{n-1}\frac{\sin nx}{n} = -2\sum_{n=1}^{+\infty}(-1)^{n}\frac{\sin nx}{n}.$$

Our aim is to show that $f(x) = x$ on the interval $[-\pi, \pi]$ is the function that is obtained by the given Fourier series.

According to De Moivre's formula, formula that is important because it connects complex numbers and trigonometry $e^{inz} = \cos nz + i \sin nz$ and according to equation

$$\log(1+z) = -\sum_{n=1}^{+\infty}\frac{(-1)^n}{n} z^n \quad \text{for } z \in C$$

we obtain:

$$-\log(1+e^{ix}) = \sum_{n=1}^{+\infty}(-1)^n\frac{e^{inx}}{n} =$$

$$\sum_{n=1}^{+\infty}(-1)^n\frac{\cos nx}{n} + \sum_{n=1}^{+\infty}(-1)^n\frac{\sin nx}{n} i =$$

$$= \mathrm{Re}(-\log(1+e^{ix})) + i\,\mathrm{Im}(-\log(1+e^{ix}))$$

We start with the Fourier series to obtain the function

$$f(x) = -2\sum_{n=1}^{+\infty}(-1)^n \frac{\sin nx}{n} =$$

$$2Arg(1+e^{ix}) = 2\,arctg\,\frac{\sin x}{1+\cos x} =$$

$$2\,arctg\,\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}} = 2arctg\,\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} =$$

$$2\cdot\frac{1}{2}x = x$$

Obtaining functions from series (4) and (5) is more complex, therefore we will use a reverse process as a proof. We will start from periodic functions we obtained to find Fourier series they represent.

First example is to find the Fourier series of the function $f(x) = x^2, x\in[-\pi,\pi]$. Because this function is even function, the coefficient b_n has zero value. Thus the coefficients of the Fourier series are

$$b_n = 0,$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{2}{2\pi}\int_0^{\pi} x^2\,dx = \frac{1}{\pi}\cdot\frac{x^3}{3}\Bigg|_0^{\pi} = \frac{\pi^2}{3},$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx = \frac{2}{\pi}\int_0^{\pi} x^2\cos nx\,dx =$$

$$= \frac{4}{n^2}\cos n\pi$$

For $n = 2k$ is obtained $a_{2k} = \frac{4}{(2k)^2}\cos 2k\pi = \frac{4}{4k^2}$ and for $n = 2k-1$ is obtained

$$a_{2k-1} = \frac{4}{(2k-1)^2}\cos(2k-1)\pi = -\frac{4}{(2k-1)^2}.$$

Hence the Fourier series for the function $f(x) = x^2, x\in[-\pi,\pi]$ is

$$f(x) = \frac{\pi^2}{3} + 4\sum_{n=1}^{+\infty}\frac{(-1)^n\cos nx}{n^2}.$$

That is equivalent to partial sum (4).

The other example is to find the Fourier series of the function $f(x) = \begin{cases} -1, -\pi \le x < 0 \\ 1, \quad 0 \le x < \pi \end{cases}$. In this example we have to find all the coefficients i.e. a_0, a_n and b_n where n=1,2,3…

The coefficients of the Fourier series are

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx = 0,$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{1}{\pi}[\int_{-\pi}^{0} -\cos nx\,dx +$$

$$+\int_{0}^{\pi} \cos nx\,dx] = 0,$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}[\int_{-\pi}^{\pi} -\sin nx\,dx +$$

$$+\int_{-\pi}^{\pi} \sin nx\,dx] = \frac{2}{n\pi}(1-\cos n\pi).$$

For $n = 2k$ is obtained $b_{2k} = \frac{2}{(2k)\pi}(1-\cos 2k\pi) = 0$.

and for $n = 2k-1$ is obtained $b_{2k-1} = \frac{2}{(2k-1)\pi}[1-\cos(2k-1)\pi] = \frac{4}{(2k-1)\pi}$.

Hence the Fourier series for the function $f(x) = \begin{cases} -1, -\pi \le x < 0 \\ 1, \quad 0 \le x < \pi \end{cases}$ is

$$f(x) = \frac{4}{\pi}\sum_{n=1}^{+\infty}\frac{\sin(2n-1)x}{2n-1}.$$

That is equivalent to partial sum (5).

And the last and already proven example with reverse process, is to find the Fourier series of the function $f(x) = x, x \in [-\pi, \pi]$. Because this function is odd function, the zero coefficients in this case are: $a_0 = 0$ and $a_n = 0, \forall n$. We only need to find a b_n coefficient

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} x\,\sin nx\,dx =$$

$$= -\frac{2}{\pi}\cos n\pi.$$

For $n = 2k$ is obtained $b_{2k} = -\frac{2}{(2k)}\cos 2k\pi = -\frac{2}{2k}$

and for $n = 2k-1$ is obtained $b_{2k-1} = -\frac{2}{(2k-1)}\cos(2k-1)\pi = \frac{2}{(2k-1)}$.

Hence the Fourier series for the function $f(x) = x, x \in [-\pi, \pi]$ is

$$f(x) = 2\sum_{n=1}^{+\infty}(-1)^{n-1}\frac{\sin nx}{n}.$$

This partial sum is equivalent to partial sum (6).

These three proves confirms graphically obtained functions.

V. CONCLUSION

Fourier series are infinite series that represent periodic functions in terms of cosines and sines. Because these series can be used in solving different problem in many scientific fields, very important is process of their understanding. We gave a brief introduction to Fourier series and tried to facilitate their understanding with graphical presentation using Matlab. Also, with these graphical representations we tried to facilitate the way of obtaining periodic functions knowing the Fourier series. This process, using only mathematical equations can be complex and difficult to accomplish, especially to students. Therefore, we tried to easier this process by obtaining functions using Matlab and plotting partial sums. From graphical representation of trigonometric series we noticed that with the large number of terms included in this series, approximation to the periodic function can be very close.

We also gave and mathematical proof that obtained functions from our graphical representation of Fourier series are correct periodic functions.

By using Matlab, students' process of learning and understanding Fourier series is significantly improved. Also this kind of presenting Fourier series is more interesting, easy to use and learn and therefore is easy acceptable by students.

Matlab can be used as tool for different more or less complex mathematical computations with little effort, but in this paper we focused only on obtaining periodic functions knowing the Fourier series. By using Matlab students can benefit in more subjects.

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