# MULTICRITERIA ANALYSIS AND OPTIMIZATION

# APPLICATION OF THE AHP IN THE PROCESS OF SELECTION OF THE BEST MATHEMATICAL SOFTWARE SOLUTIONS

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**Abstract**

Modern lifestyle and technology development naturally comes with a greater use of software packages and solutions in everyday life, as well as in education. This often results with numerous new software solutions on the market, usually similar between each other. These similarities lead to dilemma what is the software solution that can be recognized as most suitable for use. This paper presents the way to select the most suitable mathematical software package that could be used in teaching, with respect of technical and mathematical study courses in higher education. The selection is made by applying the mathematical method analytical hierarchical process (AHP). AHP is multi – criteria decision making method which is widely used in resolving the conflict situations for decision making processes. In order to achieve more realistic and relevant assessment, the criteria used for the decision making and selection performance are given by the international standard for software product quality ISO 9126. This combination of AHP method and ISO 9126 standard provides a good basis for optimization problem in a specific selection issue - selection of mathematical educational software.

**Keywords:** multi – criteria decision making, Analytic hierarchy process (AHP), mathematical software

## 1. Introduction

The IT development becomes increasingly important link in everyday activities. It also can be found in simplest daily activities, as well as in highly developed scientific problems. This rapid development contributes to appearance of various kinds of new software with increasing number. This leads to difficulties in selecting the most appropriate software. One way we suggest, in order of simplification of the making decision procedure is with appliance of mathematical method for multi criteria decision. One of these methods is the method of analytic hierarchy process (AHP).

The selection of basis criteria that will lead to the final decision will be crucial in the selection process. The determination of the most appropriate criteria can usually be a problem. In terms of software, the standard ISO 9126 provides a good framework for evaluation and selection of the best software solution. The presence of such a number of criteria that often leads to conflict situations in decision process imposes the application of mathematical methods for decision. AHP method as multi – criteria technique provides a solid basis for the resolution of such complex situations. AHP structures the problem in a hierarchical structure which allows the decision maker to recognize the mutual dependence of the criteria and thus to determine the dependencies of one over another [6,7, 8].

The aim of this paper is to select the most adequate mathematical software that would be used for teaching purposes in higher education institutions, more precisely in mathematical and technical study courses / subjects. For this purpose, five mathematical software packages are selected as starting propose: Matlab, Maple, SPSS, Mathematica and Statistica. The features of the standard ISO 9126 as will be used as evaluation factors in AHP method.

## 2. Methodological basis

### 2.1. Defining the criteria for choosing the most appropriate mathematical software product

The criteria that will be defined below are used as factors in the selection of the best and most appropriate mathematical software solution. This criteria create the hierarchical structure of the AHP and based on them will be a decision. The criteria are following: **Functionality, Reliability, Usability, Efficiency, Maintainability, Portability.**

### 2.2. Mathematical model

First step in the process proposed for selecting the optimal mathematical software is definition of the evaluate criteria used to select the optimal mathematical software. Figure 1 schematically illustrates the developed AHP model for particular problem in this paper.

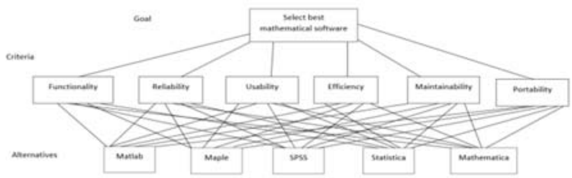


**Figure1.** AHP hierarchy model.

In the step, the elements of a particular level are compared pair – wise, with respect to a specific element in the immediate upper level. A judgment matrix is formed and used for computing the priorities of the corresponding elements. First, a criterion is compared pair – wise with respect to the goal. The judgment matrix, denoted as $A$ will be formed using the comparison. Let $A_1, A_2, \dots, A_n$ be the set of stimuli. The quantified judgments on pairs of stimuli $A_i, A_j$ are represented by

$$\boldsymbol{A = [a_{ij}], \quad i, j = 1, 2 \dots, n} \qquad (1)$$

The comparison of any two criteria $C_i$ and $C_j$ with respect to the goal is made using the questions of the type: which one of the two criteria $C_i$ and $C_j$ is more important and what is the quantity of it. Saaty suggests the use of a nine – point scale to transform the verbal judgments into numerical quantities representing the values of $a_{ij}$. Larger number assigned to the pair – wise comparison means larger differences between criteria levels. The entries $a_{ij}$ are governed by the following rules:

$$\boldsymbol{a_{ij} > 0, \quad a_{ji} = \frac{1}{a_{ij}}, \quad a_{ii} = 1 \ for\ all\ i} \qquad (2)$$

This scale can be applied with ease to criteria that can be defined numerically as well as to those that cannot be defined numerically. Thus, relative importance scale is presented. The decision maker is supposed to specify judgments of the relative importance of each contribution of criteria towards achieving the overall goal.

With the numerical judgments $a_{ij}$ recorded in the matrix $A$ , the problem now is to recover numerical weights $(W_1, W_2, \dots, W_n)$ of the alternatives from the matrix. For doing this, the following equation is considered:

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \dots & \vdots \\ \vdots & \vdots & \dots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \cong \begin{bmatrix} W_1/W_1 & W_1/W_2 & \dots & W_1/W_n \\ W_1/W_1 & W_1/W_2 & \dots & W_1/W_n \\ \vdots & \vdots & \dots & \vdots \\ \vdots & \vdots & \dots & \vdots \\ W_n/W_1 & W_n/W_2 & \dots & W_n/W_n \end{bmatrix} \qquad (3)$$

Moreover, lets multiply both matrices in equation (3) on the right with the weights vector $W = (W_1, W_2, \dots, W_n)$, where $W$ is a column vector. The result of this multiplication of the matrix of pair – wise ratios with $W$ is $nW$, hence it follows:

$$\boldsymbol{AW = nW} \qquad (4)$$

This is a system of homogenous linear equations. It has a non – trivial solution if and only if the determinant of $A - nI$ vanishes, that is, n is eigenvalue of $A$ . $I$ is a $n \times n$ identity matrix. Saaty's method computes $W$ as the principal right eigenvector of the matrix $A$ - that is

$$\boldsymbol{AW = \lambda_{max} W} \qquad (5)$$

where $\lambda_{max}$ is the principal eigenvalue of the matrix$A$. If matrix $A$ is a positive reciprocal one then $\lambda_{max} \geq n$. If $A$ is consistency matrix, eigenvector $X$ can be calculated by

$$\boldsymbol{A - (\lambda_{max} I)X = 0} \qquad (6)$$

Here, using the comparison matrix, the eigenvectors were calculated by equations (5) and (6).

#### Consistency of the Comparison Matrix

The eigenvector method yields a natural measure of consistency. Saaty [6] defines the consistency index (CI) as

$$CI = \lambda_{max} - n/(n-1) \quad (7)$$

where $\lambda_{max}$ the maximum eigenvalue and n is the number of factor in the judgment matrix. Accordingly, Saaty defined the consistency ratio (CR) as

$$CR = CI / RI \quad (8)$$

For each size of matrix *n*, random matrices were generated and their mean CI value, called the random index (RI). RI represents the average consistency index over numerous random entries of same reciprocal matrices and its value is taken from table 2 where the first row (n) indicates the number of rows i.e. matrix size, whereas second row is Random consistency index. The consistency ratio CR is a measure of how a given matrix compares to a purely random matrix in terms of their consistency indices. A value of the consistency ratio CR ≤ 0.1 is considered acceptable. Larger values of CR require the decision-maker to revise his judgments. Results of the consistency test and the CR of the comparison matrix form the available interview and previous data are all ≤ 0.1 indicating 'consistency'.

| n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| RI | 0 | 0 | 0,52 | 0,89 | 1,11 | 1,25 | 1,35 | 1,40 | 1,45 | 1,49 |

Last step in AHP method is calculation of the overall level hierarchy weight, in order to select the best mathematical software. The composite priorities of the alternatives are then determined by aggregating the weights throughout the hierarchy.

## 3. Results

In addition, the results from the conducted research are shown and explained. As previously stated, the purpose of this research was to obtain the most appropriate mathematical software which could be used for teaching the study subjects in higher education with a mathematical or technical basis. Due to space limitations and the calculations volume, only the most important results are shown. Also, the criteria and alternatives are shown with short marks with legend: K1 – functionality, K2 – reliability, K3 – efficiency, K4 – usability, K5- maintainability and K6 – portability. For the alternatives, short marks are used with legend: A1 – Matlab, A2 - Maple, A3 – SPSS, A4 –Statistica and A5 – Mathematica.

| | K1 | K2 | K3 | K4 | K5 | K6 | Weights |
|---|---|---|---|---|---|---|---|
| K1 | **1** | 2 | 4 | 3 | 1/3 | 1 | 0,199776 |
| K2 | 1/2 | **1** | 1/2 | 1 | 1/2 | 1/3 | 0,08542 |
| K3 | 1/4 | 2 | **1** | 3 | 1/5 | 1/3 | 0,108705 |
| K4 | 1/3 | 1 | 1/3 | **1** | 1/3 | 1/3 | 0,070595 |
| K5 | 3 | 2 | 5 | 3 | **1** | 1 | 0,299622 |
| K6 | 1 | 3 | 3 | 3 | 1 | **1** | 0,235882 |
| Sum | 6,083333 | 11 | 13,83333 | 14 | 3,366667 | 4 | |

**Table 3.** Pairwise comparison matrix of the main criteria with respect to the Goal

From Table 3 can be concluded that largest i.e. most important value with respect to the goal belongs to the criteria K5 (maintainability); $\lambda_{max}$ = 6,481, consistency index CI is 0,096; consistency ratio CR is 0,077. It is lower than 0,10, so the conclusion is that the level of inconsistency is accepted.

| | A1 | A2 | A3 | A4 | A5 | Weights |
|---|---|---|---|---|---|---|
| A1 | **1** | 3 | 2 | 3 | 4 | 0,397057 |
| A2 | 1/3 | **1** | 2 | 2 | 3 | 0,222928 |
| A3 | 1/2 | 1/2 | **1** | 1/2 | 3 | 0,147842 |
| A4 | 1/3 | 1/2 | 2 | **1** | 2 | 0,160222 |
| A5 | 1/4 | 1/3 | 1/3 | 1/2 | **1** | 0,071951 |
| Sum | 2,416667 | 5,333333 | 7,333333 | 7 | 13 | |

**Table 4.** Pairwise comparison matrix for the alternatives with respect to the functionality

From Table 4 can be concluded that largest i.e. most important value with respect to functionality have the alternative A1 (Matlab); $\lambda_{max}$ = 5,252; consistency index CI is 0,063; consistency ratio CR is 0,056. It is lower than 0,10, so the conclusion is that the level of inconsistency is accepted.

| | A1 | A2 | A3 | A4 | A5 | Weights |
|---|---|---|---|---|---|---|
| A1 | **1** | 4 | 3 | 2 | 3 | 0,389862 |
| A2 | 1/4 | **1** | 1/2 | 1/5 | 1/3 | 0,066364 |
| A3 | 1/3 | 2 | **1** | 2 | 1/2 | 0,15713 |
| A4 | 1/3 | 5 | 1/2 | **1** | 1/2 | 0,168859 |
| A5 | 1/3 | 3 | 2 | 2 | **1** | 0,217785 |
| Sum | 2,25 | 15 | 7 | 7,2 | 5,333333 | |

**Table 5.** Pairwise comparison matrix for the alternatives with respect to the reliability

From Table 5 can be concluded that largest i.e. most important value with respect to reliability have the alternative A1 (Matlab); $\lambda_{max}$ = 5,387; consistency index CI is 0,096; consistency ratio CR is 0,087. It is lower than 0,10, so the conclusion is that the level of inconsistency is accepted.

| | A1 | A2 | A3 | A4 | A5 | Weights |
|---|---|---|---|---|---|---|
| A1 | **1** | 3 | 2 | 4 | 2 | 0,364212 |
| A2 | 1/3 | **1** | 1/3 | 1/2 | 1/5 | 0,070962 |
| A3 | ½ | 3 | **1** | 2 | ½ | 0,178741 |
| A4 | ¼ | 2 | 1/2 | **1** | 1/3 | 0,100646 |
| A5 | ½ | 5 | 2 | 3 | **1** | 0,285439 |
| Sum | 2,583333 | 14 | 5,833333 | 10,5 | 4,033333 | |

**Table 6.** Pairwise comparison matrix for the alternatives with respect to the efficiency

From Table 6 can be concluded that largest i.e. most important value with respect to efficiency have the alternative A1 (Matlab); $\lambda_{max}$ = 5,155; consistency index CI is 0,038; consistency ratio CR is 0,034. It is lower than 0,10, so the conclusion is that the level of inconsistency is accepted.

| | A1 | A2 | A3 | A4 | A5 | Weights |
|---|---|---|---|---|---|---|
| A1 | **1** | 2 | 2 | 3 | 1 | 0,278881 |
| A2 | ½ | **1** | 1/2 | 5 | 1/3 | 0,155535 |
| A3 | ½ | 2 | **1** | 2 | ½ | 0,170081 |
| A4 | 1/3 | 1/5 | 1/2 | **1** | 1/5 | 0,067232 |
| A5 | 1 | 3 | 2 | 5 | **1** | 0,328271 |
| Sum | 3,333333 | 8,2 | 6 | 16 | 3,033333 | |

**Table 7.** Pairwise comparison matrix for the alternatives with respect to the usability

From Table 7 can be concluded that largest i.e. most important value with respect to usability have the alternative A5 (Mathematica); $\lambda_{max} = 5{,}273$; consistency index CI is 0,068; consistency ratio CR is 0,061. It is lower than 0,10, so the conclusion is that the level of inconsistency is accepted.

| | A1 | A2 | A3 | A4 | A5 | Weights |
|---|---|---|---|---|---|---|
| A1 | **1** | 5 | 4 | 4 | 3 | 0,450982 |
| A2 | 1/5 | **1** | 1/5 | 1/3 | 1/3 | 0,056081 |
| A3 | ¼ | 5 | **1** | 2 | 1/3 | 0,162163 |
| A4 | ¼ | 3 | 1/2 | **1** | 1/2 | 0,112162 |
| A5 | 1/3 | 3 | 3 | 2 | **1** | 0,218613 |
| Sum | 2,033333 | 17 | 8,7 | 9,333333 | 5,166667 | |

**Table 8.** Pairwise comparison matrix for the alternatives with respect to the maintainability

From Table 8 can be conclude that largest i.e. most important value with respect to maintainability have the alternative A1 (Matlab); $\lambda_{max}$ = 5,366; consistency index CI is 0,091; consistency ratio CR is 0,082. It is lower than 0,10, so the conclusion is that the level of inconsistency is accepted.

| | A1 | A2 | A3 | A4 | A5 | Weights |
|---|---|---|---|---|---|---|
| A1 | **1** | 3 | 3 | 4 | 2 | 0,388907 |
| A2 | 1/3 | **1** | 5 | 3 | 2 | 0,263481 |
| A3 | 1/3 | 1/5 | **1** | 1/3 | 1/2 | 0,072937 |
| A4 | 1/4 | 1/3 | 3 | **1** | 1/2 | 0,112813 |
| A5 | 1/2 | 1/2 | 2 | 2 | **1** | 0,161861 |
| Sum | 2,416667 | 5,033333 | 14 | 10,33333 | 6 | |

**Table 9.** Pairwise comparison matrix for the alternatives with respect to the portability

From Table 9 can be concluded that largest i.e. most important value with respect to portability have the alternative A1 (Matlab); $\lambda_{max} = 5{,}341$; consistency index CI is 0,085; consistency ratio CR is 0,077. It is lower than 0,10, so the conclusion is that the level of inconsistency is accepted.

| | | K x A1 | | | K X A2 | | | K x A3 | | | K x A4 | | | K x A5 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| K1 | 0,1998 | A1 | 0,3971 | 0,0793 | A2 | 0,2229 | 0,0445 | A3 | 0,1478 | 0,0295 | A4 | 0,1602 | 0,032 | A5 | 0,072 | 0,014 |
| K2 | 0,0854 | A1 | 0,3899 | 0,0333 | A2 | 0,0664 | 0,0057 | A3 | 0,1571 | 0,0134 | A4 | 0,1689 | 0,0144 | A5 | 0,2178 | 0,019 |
| K3 | 0,1087 | A1 | 0,3642 | 0,0396 | A2 | 0,071 | 0,0077 | A3 | 0,1787 | 0,0194 | A4 | 0,1006 | 0,0109 | A5 | 0,2854 | 0,031 |
| K4 | 0,0706 | A1 | 0,2789 | 0,0197 | A2 | 0,1555 | 0,011 | A3 | 0,1701 | 0,012 | A4 | 0,0672 | 0,0047 | A5 | 0,3283 | 0,023 |
| K5 | 0,2996 | A1 | 0,451 | 0,1351 | A2 | 0,0561 | 0,0168 | A3 | 0,1622 | 0,0486 | A4 | 0,1122 | 0,0336 | A5 | 0,2186 | 0,066 |
| K6 | 0,2359 | A1 | 0,3889 | 0,0917 | A2 | 0,2635 | 0,0622 | A3 | 0,0729 | 0,0172 | A4 | 0,1128 | 0,0266 | A5 | 0,1619 | 0,038 |
| SUM | | | | 0,3988 | | | 0,1479 | | | 0,1402 | | | 0,1223 | | | 0,191 |

**Table 10.** Synthesising to obtain the final result

| | |
|---|---|
| A1 | 0,398763818 |
| A5 | 0,190861807 |
| A2 | 0,147851934 |
| A3 | 0,140186177 |
| A4 | 0,122336264 |

**Table 11.** Final result

After the final summary of the results and performing all the calculations from the final table 11, we can see that the best software solution that should be used is the software package Matlab.

## 4. Conclusion

This research shows practical way of mathematical way of decision making procedure. For that purpose, scientific and mathematical method AHP was used. In terms of deciding factors and making the right decisions, characteristics of International quality software standard ISO 9126 were applied. The decision was conducted in way of selection of the most appropriate and best software solution that would apply in teaching on technical and mathematical subjects in higher education institutions. After the analysis and calculations, it can be concluded that in competition of five software packages, **Matlab** is detected as most appropriate.

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