

FEM AIDED DESIGN OF PERFORMANCE CHARACTERISTICS OF SINGLE PHASE MOTORS

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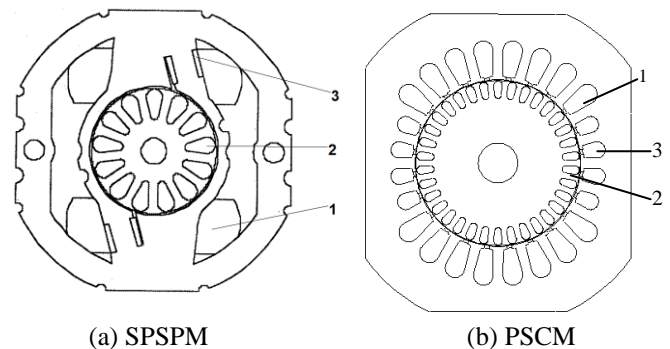
Abstract: Paper proposes methodology for calculation of parameters and performance characteristics of single-phase motors based on double-field revolving theory and method of symmetrical components. Permanently split capacitor motor and single-phase shaded pole motor are investigated and their parameters and performance characteristics are calculated and verified by experiment under different operating regimes. Once, the parameters are calculated they are used as input data in numerical models of the motors based on Finite Element Method (FEM). FEM models are used for calculation of magnetic flux density and its distribution in motors' cross-section as well as in the air gap, since these parameters are difficult to be calculated or exactly predicted by analytical methods. Proposed models enable motor performance under different operating regimes to be analyzed and accurately predicted.

Keywords: Finite element method, method of symmetrical components, performance characteristics, single-phase motors.

INTRODUCTION

Single phase motors have wide application in many household devices since most small power (generally below 2 kW) induction machines have to operate with single phase a.c. power supplies that are readily available at homes and remote rural areas. As the name suggests, this type of motor has only one stator winding (main winding) while the rotor is a squirrel cage type. The single-phase induction motor is not a self-starting. When the motor is connected to single-phase supply the main winding carries, the altering current, which produces a pulsating magnetic field. As the main magnetic field is pulsating, torque necessary for the motor rotation is not generated. Consequently, it is necessary to have start/auxiliary winding. At single-phase shaded-pole motor (SPSPM) it is a short-circuit coil placed in stator poles while in permanently split capacitor motor (PSCM) it is a permanently connected capacitor in motor auxiliary winding. In this paper, two single-phase motors are investigated: permanently split capacitor motor (PSCM), type FMR 35/6 and single-phase shaded pole motor (SPSPM) type AKO-16, both product of company MikronTech (Fig.1). Based on motor exact geometry, obtained from the producer, parameters of the motors are calculated. Method of symmetrical components based on double-field revolving theory is implemented as mathematical tool for calculation of motor performance characteristics. Obtained values of characteristics from mathematical models are compared with data from experiment performed at producer premises. FEM models of motors are constructed, based on motor geometry, calculated parameters and characteristics of implemented materials. Magnetic flux density distribution in

cross-sections of the motors is obtained using time-harmonic approach, meaning all quantities are oscillating at frequency $f=50$ Hz and currents in rotor windings are freely induced.



1–main stator winding, 2–squirrel cage winding, 3–auxiliary winding

Fig.1 – Cross section of single phase motors.

PERFORMANCE CHARACTERISTICS

During recent years, an extensive number of models of single-phase machines are constructed in order steady-state and transient performance characteristics to be determined. Some authors propose motor model based on d-q transformation mainly for calculation of single-phase motors' transient characteristics [1-3]. Others propose method of revolving field polygon technique mainly applied for calculation of performance characteristics of single phase split-phase motors where split phase (auxiliary) winding is replaced by three phase winding enabling motor to be analyzed as symmetrical three phase motor under different operating regimes [4]. In this paper, calculation of performance characteristics of single-phase motors is based on double-field revolving theory and methods of symmetrical components as a result of the existence of two stator windings mutually coupled one to another which produces elliptical electromagnetic field in machine air-gap. The unsymmetrical magnetomotive forces (mmf), currents and voltages corresponding to the two windings of single phase induction motor denoted as general vector A may be decomposed into two symmetrical systems (Fig. 2) consisted of forward and backward components (f&b) of the symmetrical systems [5]. On that way, all the electromagnetic processes inside the machine are analyzed as in the symmetrical three phase induction machine and all motor characteristics are obtained as sum of forward (direct "+") and backward (inverse "-") components.

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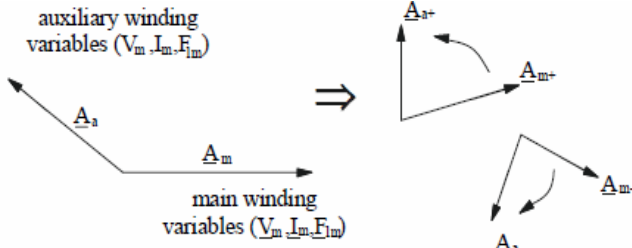


Fig.2 – Symmetrical components of a two-phase system.

From Figure 2:

$$\underline{A}_{a+} = j \underline{A}_{m+} ; \underline{A}_{a-} = j \underline{A}_{m-} . \quad (1)$$

The superposition principle yields:

$$\underline{A}_m = \underline{A}_{m+} + \underline{A}_{m-} ; \underline{A}_a = \underline{A}_{a+} + \underline{A}_{a-} . \quad (2)$$

From (1) through (2):

$$\underline{A}_{m+} = \frac{1}{2} (\underline{A}_m - j \underline{A}_a) , \quad (3)$$

$$\underline{A}_{m-} = \frac{1}{2} (\underline{A}_m + j \underline{A}_a) = \underline{A}_{m+}^* . \quad (4)$$

The first step in performance characteristics calculation is to determine all motor parameters: R_{sm} -main stator winding resistance, X_{sm} -main winding leakage reactance, R_{sa} -auxiliary stator winding resistance, X_{sa} -auxiliary stator winding leakage reactance, X_{mm} -the magnetizing reactance, R_{rm} -rotor winding resistance, X_{rm} -rotor winding leakage reactance, based on the motor dimensions and cross-section obtained from the producer (Fig.1). Afterwards direct and inverse impedances of main stator winding \underline{Z}_{m+} and \underline{Z}_{m-} are determined as well as mutual impedance between main and auxiliary winding \underline{Z}_a^m and consequently they are used for calculation of direct and inverse components of current in main stator winding \underline{I}_{m+} and \underline{I}_{m-} . Direct and inverse components of rotor winding impedance \underline{Z}_{r+} and \underline{Z}_{r-} are calculated as well [6]. They are used for calculation of direct and inverse components of rotor currents \underline{I}_{r+} and \underline{I}_{r-} .

Supply current is calculated from:

$$\underline{I}_s = \left| \underline{I}_{m+} + \underline{I}_{m-} + j \frac{\underline{I}_{m+} - \underline{I}_{m-}}{a} \right| . \quad (5)$$

Currents in main stator winding- \underline{I}_m , auxiliary winding- \underline{I}_a and rotor winding- \underline{I}_r are calculated respectively:

$$\underline{I}_m = \left| \underline{I}_{m+} + \underline{I}_{m-} \right| , \quad \underline{I}_r = \left| \underline{I}_{r+} + \underline{I}_{r-} \right| , \quad (6)$$

$$\underline{I}_a = \left| j \frac{\underline{I}_{m+} - \underline{I}_{m-}}{a} \right| , \quad (7)$$

where a is the reduction ration of auxiliary winding to main winding i.e. ratio between the number of turns of auxiliary stator winding- N_a and main winding- N_m .

Developed electromagnetic power is calculated from parameters of rotor winding and motor slip- s :

$$P_{em} = \frac{2R_{rm} |I_{r+}|^2}{s} - \frac{2R_{rm} |I_{r-}|^2}{2-s} . \quad (8)$$

Mechanical power is calculated from:

$$P_{mech} = P_{em} (1-s) . \quad (9)$$

The motor output power is expressed as:

$$P_2 = \frac{P_{mech}}{1.015} . \quad (10)$$

The motor output power is obtained from the mechanical power decreased by the value of stray losses, often difficult for measurement and IEEE-125 specifies them as 0.9-1.8% of the motor output power.

Direct and inverse components of motor torque are found from:

$$M_+ = \frac{2p}{\omega_1} I_{m+}^2 [\text{Re}(\underline{Z}_+) - R_{sm}] , \quad (11)$$

$$M_- = -\frac{2p}{\omega_1} I_{m-}^2 [\text{Re}(\underline{Z}_-) - R_{sm}] , \quad (12)$$

where p is the number of pair of poles and ω_1 is the angular frequency [rad/s]. Z_+ and Z_- are direct and inverse impedances obtained from main and rotor winding parameters as well as from magnetizing reactance [5].

Torque is obtained form:

$$M = M_+ + M_- \quad (13)$$

Input power is calculated from:

$$P_1 = V_s |I_s| \cos \varphi . \quad (14)$$

V_s is the motor supply voltage, I_s is the supply current and $\cos \varphi$ is the power factor.

Power factor is calculated from:

$$\cos \varphi = \frac{\text{real}(I_s)}{I_s} . \quad (15)$$

Efficiency factor is calculated from:

$$\eta = \frac{P_2}{P_1} . \quad (16)$$

Obtained values of motor characteristics at rated load operating regime are compared with data from experiment for the purpose of verification of proposed methodology and they are presented in Table I for PSCM at rated load operating point or motor slip- s of 0.04, and in Table II for SPSPM.

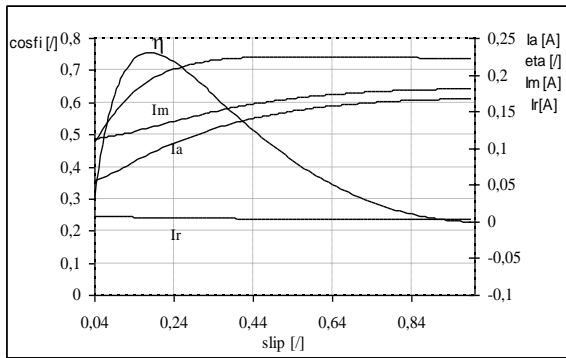
Table I
Comparison - calculated and measured data of PSCM.

Parameter	Analytical approach	Measur.
Rated torque M_n [Nm]	0.412	0.402
Rated supply current I_s [A]	1.6	1.32
Maximum output power P_2 [W]	215	210
Maximal torque M_{max} [Nm]	0.766	0.80

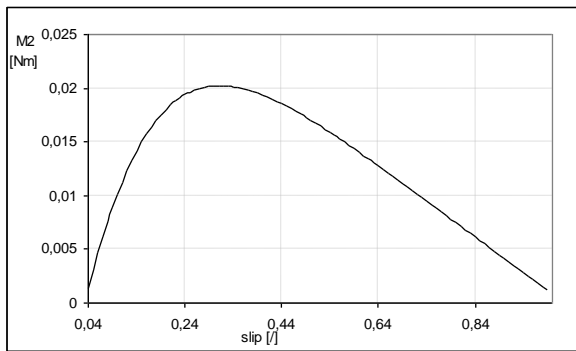
In Fig.3 are presented performance characteristics of SPSPM for currents in all windings, efficiency factor η and power factor- $\cos \varphi$ for different motor slips.

Table II*Comparison-calculated and measured data of SPSPM*

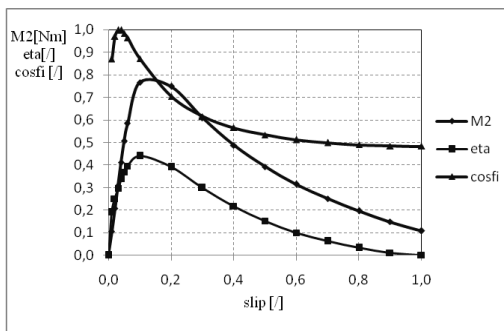
Parameter	Analyt.	Measur.
Rated current I_1 [A]	0.125	0.1259
Input power P_1 [W]	18	18.114
Power factor $\cos\phi$ [/]	0.6545	0.6538
Main st. wind. resistance R_{sm} [Ω]	498	493 Ω
No-load current I_0 [A]	0.111	0.1134
No load input power P_0 [W]	13.8	10.21
Short-circuit current I_k [A]	0.14	0.181
Short circuit power P_k [W]	19.87	29
Short-circuit p. f. $\cos\phi_k$ [/]	0.6683	0.7389

**Fig.3** – Performance characteristics of currents, power factor and efficiency of SPSPM.

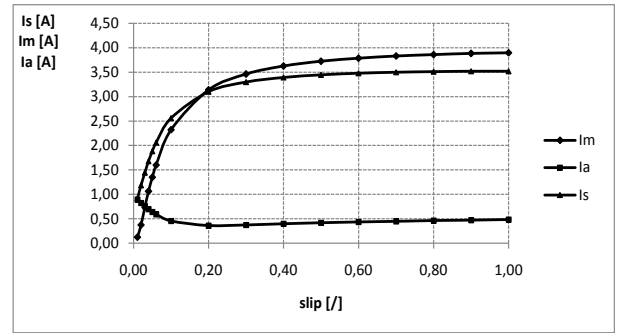
Characteristic of output torque of SPSPM is presented in Fig.4.

**Fig.4** – Performance characteristics of torque of SPSPM.

In Fig.5 are presented characteristics of output torque- M_2 , efficiency factor- η and power factor- $\cos\phi$ for PSCM.

**Fig.5** – Performance characteristics of output torque, power factor and efficiency of PSCM.

Currents in all motor windings of PSCM: main, auxiliary and rotor winding are presented in Fig.6.

**Fig.6** – Performance characteristics of currents of PSCM.

FEM MODELS

Throughout the recent years, FEM has proved itself as valuable tool in electrical machine analysis when calculating parameters and characteristics of the variety of electromagnetic devices [7-8]. The analysis of the electromagnetic phenomena inside single-phase machines is always a challenging task due to the existence of the two stator windings mutually electromagnetically coupled, which together with the rotor winding produce an elliptic electromagnetic field in the motor air gap. Therefore, a special attention is paid on the proper motor modeling regarding current distribution in motor FEM model, by taking into account the currents in the main and the auxiliary stator winding and their phase displacement. Another important issue is the proper modeling of different operating regimes such us: no-load, rated load or locked rotor. Therefore, the rotor bars conductivity is adjusted to the motor slips correspondingly, i.e. the motor operating regimes. Motors are analyzed for time-harmonic case, i.e. the currents are input only in the stator windings while in the rotor winding, the current is freely induced at frequency $f=50$ Hz due to the specific motor modeling. On that way, the analysis of electromagnetic phenomena inside the motor is closer to the real electromagnetic processes inside the machine when it is supplied with voltage 220 V, 50 Hz. In order to determine the magnetic vector potential \mathbf{A} , it is necessary for the whole domain i.e. motor's cross-section to be divided into numerous elements. When analyzing induction machines, considering their AC excitation, the air gap magnetic field is always a time-varying quantity. In the materials with non-zero conductivity, the eddy currents are induced, consequently the field problem turns into magneto-dynamic i.e. non-linear time-harmonic problem. When the rotor is moving, the rotor quantities oscillate at slip frequency. In this case, the rotor bars conductivity σ is adjusted correspondingly to the slip. Consequently, the following partial equation is going to be solved numerically:

$$\nabla_x \left(\frac{1}{\mu(\mathbf{B})} \nabla_x \mathbf{A} \right) = -\sigma \mathbf{A} + \mathbf{J}_{src} - \sigma \nabla V, \quad (17)$$

where \mathbf{J}_{src} represents the applied current sources. The additional voltage gradient ΔV in 2-D field problems is constant over the conduction body.

In Fig. 7 and 8 is presented magnetic flux density distribution at cross-section of both motors for no-load and rated operating regime, respectively.

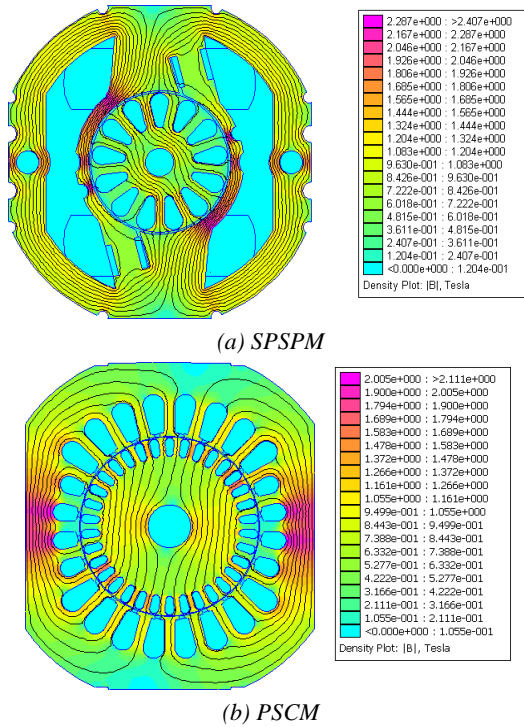


Fig.7 – Magnetic flux density distribution in motor cross-section--no load.

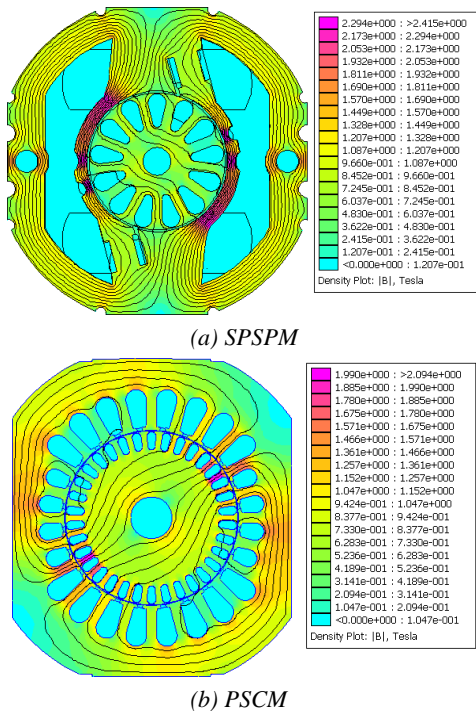


Fig.8 – Magnetic flux density distribution in motor cross-section--rated load.

In Fig. 9 is presented magnetic flux density distribution in air gap for both motor models at rated load. From presented results in Figs. 7 and 8 it is evident the presence of high values of magnetic flux density especially for SPSPM in the area of stators' bridge which can be lowered by implementing high quality magnetic materials or soft magnetic powders at critical points of motor construction.

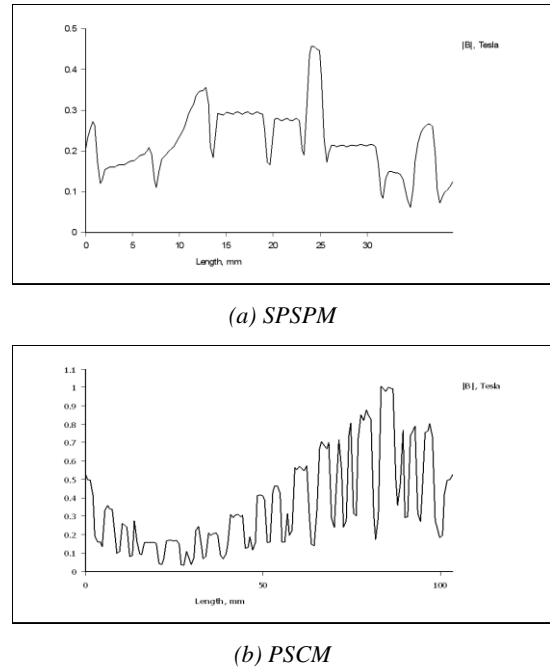


Fig.9 – Flux density distribution in air gap--rated load.

CONCLUSION

Two types of single-phase motors (shaded pole and permanently split capacitor) are analyzed under different operating regimes. Method of symmetrical components is applied as mathematical tool for obtaining parameters and characteristic of the motors. Proposed mathematical modes of the motors and obtained results from them are verified by experiments. Complete set of performance characteristics for different motor operating regimes i.e. different motor slips are plotted enabling motors operating regimes to be easily predicted and analyzed. Obtained parameters and characteristics are used as input data in FEM based motor models. FEM models are analyzed for time-harmonic case, when 220 V, 50 Hz is applied at motor power supply. As a result, of FEM models magnetic flux density in motor cross-section and air gap is obtained. Usually, values of magnetic flux density in the air gap during motor design are predicted, based on designer experience and reference values. Obtained models enable their accurate calculation, which improves overall motor design.

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