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ON THE USE OF MATHEMATICA IN ENGINEERING EDUCATION

B. Zlatanovska, L. Lazarova, A.Stojanova
Goce Delcev University/Faculty of computer science, Stip, Republic of Macedonia
biljana.zlatanovska@ugd.edu.mk, limonka.lazarova@ugd.edu.mk, aleksandra.stojanova@ugd.edu.mk

Abstract - So far, Mathematica has been used mainly either in teaching as a support for classical courses, or in research as a computing environment. However, it could be assigned a wider and more profound role in mathematics and mathematical education. Many problems that include functions of two variables are too difficult for the students because they cannot visualize these surfaces or give them geometric interpretation. The aim of this paper is to demonstrate how students and educators to visualize the functions of two variables and to help to the students in solving some problems with these functions can use Mathematica.

I. INTRODUCTION

It is not possible to achieve the objectives and skills of a modern mathematics course at the secondary or university level without resorting to graphic concepts. These concepts can be more easily apprehended when the students work with a large number and variety of graphics, in an interactive way, with the support of the appropriate technology. Obviously, calculations with the support of technology are not a replacement for paper and pencil calculations, and they should be properly combined with other methods of calculation, including mental calculation. Students should be prepared for an intelligent dialogue with the tools they have available. There are a great variety of tools and teaching methods available to lecturers who are providing instruction to engineering students in today's colleges and universities. The choices made among these many options are often due to the particular backgrounds and interests of an instructor. The students in the classroom also bring a diversity of experiences and learning styles to the student-teacher relationship. Keeping in mind the responsibility of both instructors and students to communicate effectively with one another as well as to prepare adequately for learning outside of the classroom, engineering instructors should be interested in considering different ways of presenting course material. [8]

So far, Mathematica has been used mainly either in teaching as a support for classical courses, or in research as a computing environment. However, it could be assigned a wider and more profound role in mathematics and mathematical education, [4]. The aim of this paper is to demonstrate how students and educators to visualize the functions of two variables and to help to the students in solving some problems with these functions can use Mathematica. This will improve the teaching-and-learning process precisely by providing teachers and students alike with new ways to explore some of the main mathematical subjects, at the secondary and university levels, specifically in the areas of precalculus and differential calculus.

Wolfram's Mathematica is a powerful (Computer Algebra Systems) CAS used in scientific, engineering, and mathematical fields and in other areas of technical computing. In what concerns the work presented in this paper, Mathematica graphics are completely integrated into its dynamic interactive language. Any visualization can immediately be animated or made interactive using a single command and developed into sophisticated, dynamic visual applications. Creating interactive visual models with Mathematica allows students to explore hard to-understand concepts, test theories, and quickly gain a deeper understanding of the materials being taught firsthand. The students can explore changes to text, functions, formulas, matrices, graphics, tables, or even data, [3].

Mathematica can deliver integrated and affordable high-end dynamic visualization and simulation capabilities. With nVizx™ [5], it enables users to take large volumes of natural and synthetic data, add complex calculations and generate high-resolution, dynamic images for analysis and presentation. Its capabilities in visualization are [6]: transparency, texturing, interactive 3D models, data driven real-time 3D animations, true parametric curves and surfaces plots, fast visualization of large files and etc. Very similar to Mathematica is the Programming packet

Matlab that has the same capabilities in visualization [7].

II. VISUALIZATION OF THE TWO VARIABLE FUNCTION

Students do not have difficulties for solving problems, which include function with one variable, because this function is easier for graphic illustration. They can do this with pencil on paper. However, when the number of independent variables is larger, then they cannot give geometric interpretation. Because of that, they have problems in some mathematical calculations. They cannot visualize and imagine these surfaces, and because of that, they cannot solve problems with these functions. In mathematics, difficulties often appear because of the lack of geometric visualization.

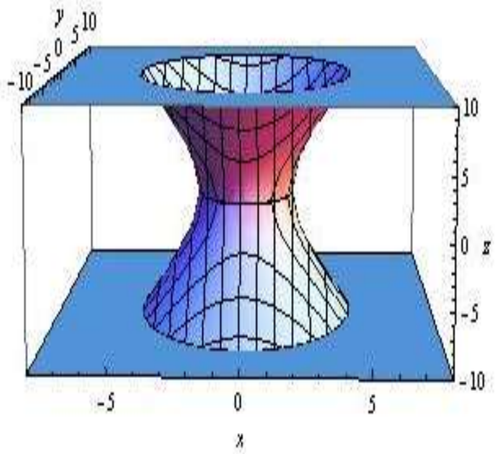
In Multivariable Calculus in engineering studies, students should learn and understand basic mathematical concepts, which have been already studied in one variable Calculus. The Multivariable Calculus is so important for engineering students because they applied these concepts in other engineering areas.

Students can draw the function with one variable with pencil and understand all the important facts about these real functions because their graphics are lines or curves, which are subset to the real plane.

They can distinguish the equation for circular or ellipse, but they cannot distinguish 3D figures because they cannot imagine these surfaces. Because of that, the use of some computational programs for visualization is necessary in the classroom. Mathematica Wolfram is very adequate programming package for using in the classrooms. It is easy for using, because it has simple commands for solving and plotting.

For visualization of function with two variables in Mathematica, the user should use the command Plot3D and to accent the PlotRange for three axes x, y and z .

On the figure 1 is represented a single hyperboloid $\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{25}=1$.



Mathematica has possibility for rotating of the graphics, so this single hyperboloid can be represented from different point of view:

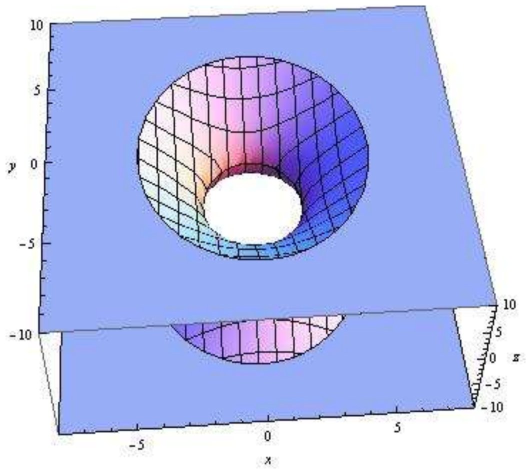


Figure 1. The single hyperboloid $\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{25}=1$.

We should notice that for plotting of two variables function in Mathematica, the functions should be t given in explicit form.

On the figure 2 is given double hyperboloid $\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{25}=-1$, from different point of view.

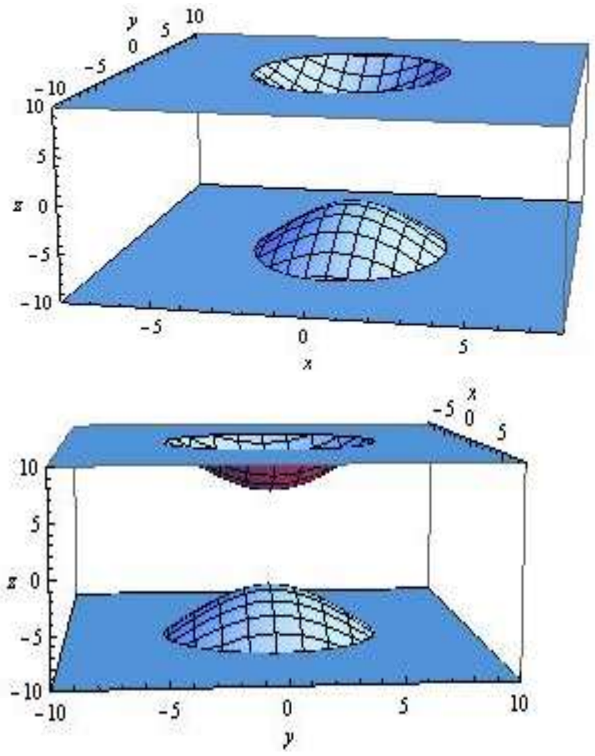


Figure 2. Double hyperboloid $\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{25}=-1$.

On the figure 3 is given hyperbolic paraboloid $z=\frac{x^2}{12}-\frac{y^2}{16}$.

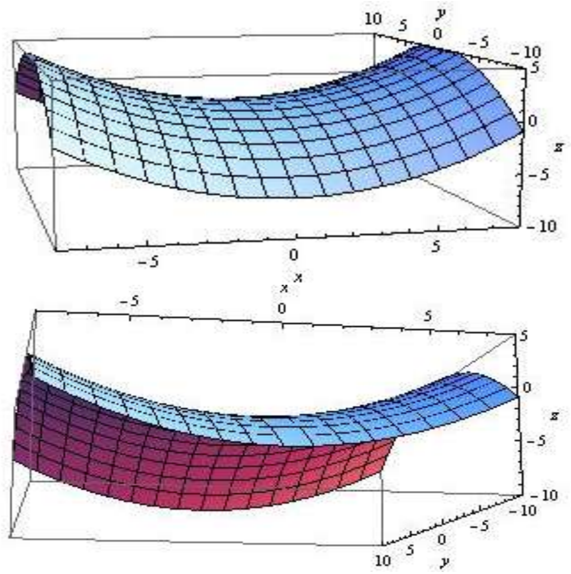


Figure 3. Hyperbolic paraboloid $z=\frac{x^2}{12}-\frac{y^2}{16}$.

These graphical representations could be very helpful for the students when they must calculate the volume of the surfaces area and 3D shapes. From the graphical illustration, they can determine the limits of the surfaces and then can make the necessary calculations in the double integrals. When the surfaces are shown graphically the students can calculate easily some line and surface integrals. That is not as much as easy when they should solve some problems, which are given analytically.

The plane tangent to a surface at a point and the normal line to a surface at a point are difficult to understand without a graphical visualization. Mathematical formulas for the plane tangent to a surface at a point $M_0(x_0,y_0,z_0)$ and for the normal line to a surface at a point $M_0(x_0,y_0,z_0)$ are

$$\left(\frac{\partial z}{\partial x}\right)_{M_0}(x-x_0)+\left(\frac{\partial z}{\partial y}\right)_{M_0}(y-y_0)=z-z_0 \qquad (1)$$

and

$$\frac{x-x_0}{\left(\frac{\partial z}{\partial x}\right)_{M_0}}=\frac{y-y_0}{\left(\frac{\partial z}{\partial y}\right)_{M_0}}=\frac{z-z_0}{-1}$$ respectively.

In these formulas for tangent plane and normal line of some surface the first partial derivatives appear.

On the figure 4 by a code

```
f1 = Plot3D[{x^2/2 − y^2, 2 * x + 2 * y − 1}, {x, −8,8}, {y, −10,10}, PlotRange
     → {{−8,8}, {−10,10}, {−50,50}}, AxesLabel → {x, y, z}]; f2
     = ParametricPlot3D[{2t + 2, 2t − 1, −t + 1}, {t, −50,50}]; Show[{f1, f2}]
```

is represented a hyperbolic paraboloid $S:z=\frac{x^2}{2}-y^2$. The tangent plane of the surface is given with the equation $\Sigma: z=2x+2y-1$ and the normal line of the surface is the line $n:\frac{x-2}{2}=\frac{y+1}{2}=\frac{z-1}{-1}$ at a point $M_0(2,-1,1)$.

The code for the first partial derivatives $z'_x=\frac{\partial z}{\partial x}, z'_y=\frac{\partial z}{\partial y}$ of the function $z=f(x,y)$ in terms of the variables x and y is $\partial_x(\frac{x^2}{2}-y^2)$ and $\partial_y(\frac{x^2}{2}-y^2)$, respectively. The code for calculation of the value of the first partial derivatives $z'_x=\frac{\partial z}{\partial x}, z'_y=\frac{\partial z}{\partial y}$ at the point M(2,-1,1) is $\partial_x(\frac{x^2}{2}-y^2)/.x\to 2$ $\partial_y(\frac{x^2}{2}-y^2)/.y\to -1$.

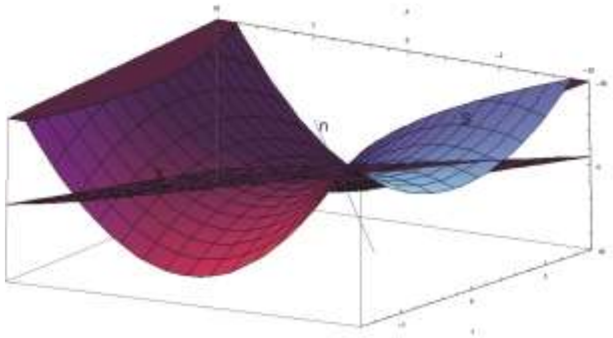


Figure 4. The hyperbolic paraboloid

$S: z = \dfrac{x^2}{2} - y^2$, the plane tangent to the surface $\Sigma: z = 2x + 2y - 1$, the normal line to the surface $n: \dfrac{x-2}{2} = \dfrac{y+1}{2} = \dfrac{z-1}{-1}$ at a point $M_0(2,-1,1)$.

The intersection of two surfaces or a surface with a plane is very complex for explanation by the teacher as well as for a perception by the students. The graphical visualization with Mathematica gives a simpler and a great way for an explanation by the teachers an as well as easier understanding by the students. Therefore, on the figure 5 is represented an intersection of a sphere $(x-5)^2 + (y+3)^2 + z^2 = 25$ with the plane $z = x - 2y - 5$ by the code

```
Plot3D[{√25 − (x − 5)² − (y + 3)², −√25 − (x − 5)² − (y + 3)², x − 2y
        − 5}, {x, −10,10}, {y, −10,10}, AxesLabel → {x, y, z}, PlotRange
        → {{−10,10}, {−10,10}, {−10,20}}]
```

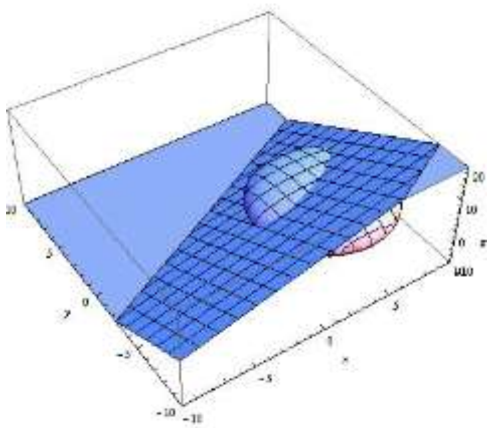


Figure 5. The intersection of a sphere $(x-5)^2 + (y+3)^2 + z^2 = 25$ with the plane $z = x - 2y - 5$.

On the figure 6 is represented an intersection of a sphere $(x-5)^2 + (y+3)^2 + z^2 = 25$ the hyperbolic paraboloid $z = x^2 - y^2$ by the code

```
Plot3D[{√25 − (x − 5)² − (y + 3)², −√25 − (x − 5)² − (y + 3)², x² −
y²}, {x, −10,10}, {y, −10,10}, AxesLabel → {x, y, z}, PlotRange →
{{−10,10}, {−10,10}, {−10,10}}]
```

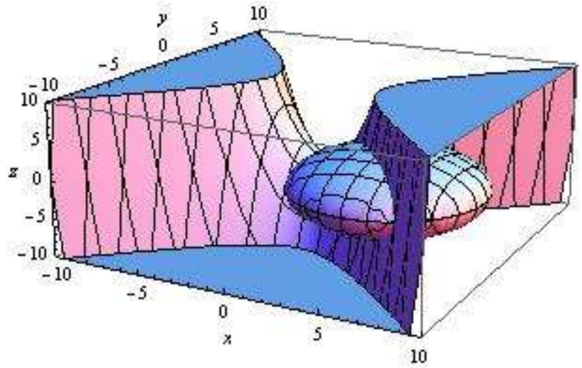


Figure 6. The intersection of a sphere $(x-5)^2 + (y+3)^2 + z^2 = 25$ with the hyperbolic paraboloid $z = x^2 - y^2$.

The intersection of two surfaces or of a surface with a plane is connected with the conditional extremes of the function with two variables. Because, the conditional extreme – maximum or minimum is smallest or biggest point respectively of the surface $z = f(x,y)$ with the plane or the surface $\varphi(x,y) = 0$, which is condition [10].

The point M(1,1,2) is an intersection of the eliptic paraboloid $z = x^2 + y^2$ with the plane $x + y - 2 = 0$. This point is a conditional extreme – minimum for the elliptic paraboloid $z = x^2 + y^2$ with condition $x + y - 2 = 0$, although the point M(1,1,2) is not an extreme for the elliptic paraboloid $z = x^2 + y^2$.

The code for this case is

```
b2 = Plot3D[{√z − x², −√z − x², −x − 2}, {x, −10,10}, {z, −20,30}, PlotRange
        → {{−10,10}, {−30,30}, {−10,10}}, AxesLabel → {x, z, y}, ViewPoint → {0,2,5}]
```

and the representation is given on the figure 7.

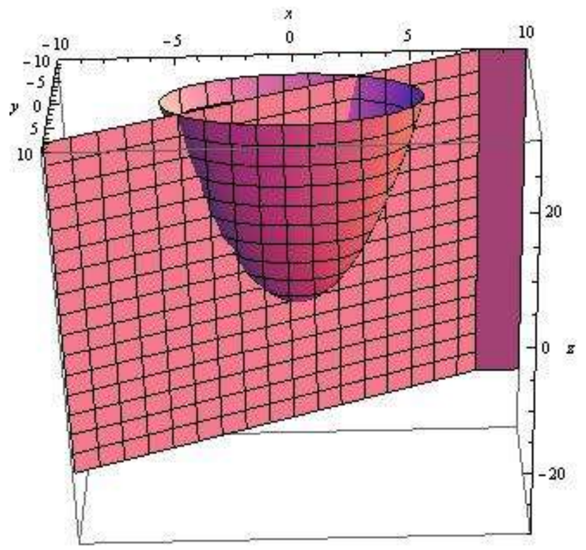


Figure 7. The conditional extreme – minimum M(1,1,2) for the elliptic paraboloid $z = x^2 + y^2$ on condition $x + y - 2 = 0$.

Determining of the first partial derivatives $z'_x = \frac{\partial z}{\partial x}, z'_y = \frac{\partial z}{\partial y}$ for the function $z = f(x, y)$ in terms of the variables x and y at a point is simple. However, a geometric representation is difficult for understanding.

By [9], [10], the geometric representation for a partial derivatives z'_x at the point (x_0, y_0): The surface which geometrically represent the function with two variables $z = f(x, y)$ cut with the plane $y = y_0$ (because when we determine the first partial derivative of the function with two variables in terms of the variable x, the variable y we treat as a constant). If in the plane $y = y_0$, we choose coordinate axes parallel to Ox and Oz axis then we obtain planar curve $z = f(x, y_0) = \varphi(x), y = y_0$. The first derivative $\varphi'(x_0)$ is equal on tangent of an angle between the tangent line $z - z_0 = z'_x(x_0, y_0)(x - x_0), y = y_0$ on the curve at the point $(x_0, \varphi(x_0))$ and positive part of an abscissa axis Ox. Because, the first partial derivative z'_x at the point (x_0, y_0) to the function with two variables $z = f(x, y)$ is equal on a tangent of an angle between positive part of an abscissa axis Ox and the tangent of the curve, which is an intersection on a surface $z = f(x, y)$ and the plane $y = y_0$.

For example, the geometric representation for a partial derivatives z'_x at the point (x_0, y_0) of the single hyperboloid $S: \frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{25} = 1$ at the point (2,3):

The single hyperboloid $S: \frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{25} = 1$ has the first partial derivative $z'_x = \frac{5x}{4\sqrt{\frac{x^2}{4} + \frac{y^2}{9} - 1}}, z'_x(2,3) = \frac{5}{2}$ with a code

$$\partial_x\left(\pm 5 * \sqrt{\frac{x^2}{4} + \frac{y^2}{9} - 1}\right)$$

The intersection of the surface $S: \frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{25} = 1$ with the plane y=3 is given with the curve $\Gamma: \frac{x^2}{4} - \frac{z^2}{25} = 0, y = 3$ in the plane y=3 for the surface S. The point M(2,3,5) is a point of the surface S and of the curve Γ. The tangent line t in real space Oxyz at the point M has the equation $t: z - 5 = \frac{5}{2}(x - 2), y = 3$,where $z'_x(2,3) = \frac{5}{2}$. The tangent of an angle between the tangent t on the curve Γ and the positive part of axis Ox is equal to the first partial derivative $z'_x(2,3) = \frac{5}{2}$ of the S at the point M. In the figure 8 with a code

$$a1 = \text{Plot3D}[\{3 * \sqrt{-\frac{x^2}{4} + \frac{z^2}{25} + 1}, -3 * \sqrt{-\frac{x^2}{4} + \frac{z^2}{25} + 1}, 3\}, \{x, -8, 8\}, \{z, -20, 20\}, \text{AxesLabel} \to \{x, z, y\}, \text{PlotRange} \to \{\{-8, 8\}, \{-20, 20\}, \{-10, 10\}\}]; a2 = \text{ParametricPlot3D}[\{\frac{2}{5} * t + 2, 3, 5 + t\}, \{t, -100, 100\}]; \text{Show}[\{a1, a2\}]$$

is represented the single hyperboloid $S: \frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{25} = 1$, the plane y=3, the intersection $\Gamma: \frac{x^2}{4} - \frac{z^2}{25} = 0, y = 3$ and the tangent line $t: z - 5 = \frac{5}{2}(x - 2), y = 3$ at the point M(2,3,5).

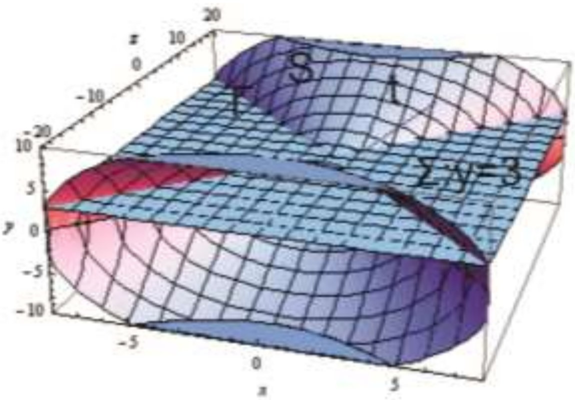


Figure 8. The single hyperboloid

$S:\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{25}=1$, the plane y=3, the intersection Γ: $\frac{x^2}{4}-\frac{z^2}{25}=0, y=3$ and the tangent $t: z-5=\frac{5}{2}(x-2), y=3$ at the point M(2,3,5).

In the figure 9 with a code

$$\mathrm{a1} = \mathrm{Plot3D}[\{3*\sqrt{-\frac{x^2}{4}+\frac{z^2}{25}+1}, -3*\sqrt{-\frac{x^2}{4}+\frac{z^2}{25}+1}, 3\}, \{x,-8,8\}, \{z,-20,20\}, \mathrm{AxesLabel} \to \{x,z,y\}, \mathrm{PlotRange} \to \{\{-8,8\},\{-20,20\},\{-10,10\}\}]; \mathrm{a2} = \mathrm{ParametricPlot3D}[\{\frac{2}{5}*t+2, 3, 5+t\}, \{t,-100,100\}]; \mathrm{a3} = \mathrm{Plot3D}[0, \{x,-8,8\}, \{y,-10,10\}, \mathrm{AxesLabel} \to \{x,y,z\}]; \mathrm{Show}[\{\mathrm{a1},\mathrm{a2},\mathrm{a3}\}]$$

is represented the angle α between the tangent t on the curve Γ: $\frac{x^2}{4}-\frac{z^2}{25}=0, y=3$ in the plane y=3 with the positive part of Ox axis, which is the first partial derivative $z'_x(2,3)=\frac{5}{2}$ of S at the point M(2,3,5).

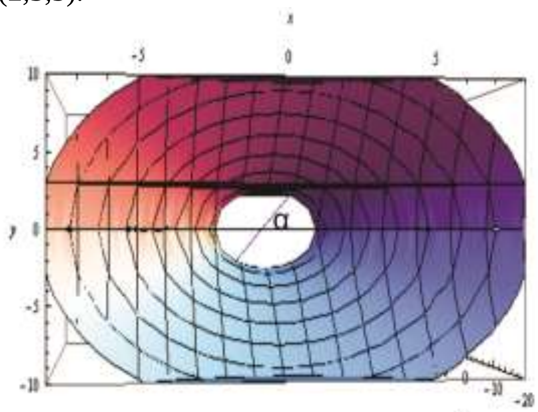


Figure 9. The single hyperboloid

$S:\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{25}=1$, the angle α and the tangent $t: z-5=\frac{5}{2}(x-2), y=3$ at the point M(2,3,5)

III. CONCLUSION

The use of mathematical computer packages, like Mathematica, provides a great opportunity for instructors to develop their own course materials to supplement their traditional teaching methods and tools and, hopefully, to better reach more students. In this paper, it is shown that Mathematica could be used as an additional tool in solving problems in multi variable calculus.

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